Critical and Glassy Spin Dynamics in Non-Fermi-Liquid Heavy-Fermion Metals

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Comparison of muon relaxation with resistivity & specific heat in NFL systems.

Local (single-ion or cluster) vs. cooperative dynamics, critical and (or?) glassy dynamics.

Conclusions and questions.

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- Quantum rather than thermal critical fluctuations,
- New low-lying excitations not Fermi-liquid quasiparticles.
Muon Spin Relaxation in NFL metals

Results: in f-electron NFL metals studied to date, low-frequency spin fluctuations at low temperatures are singular [with a (very) low-frequency cutoff] as $\tau_0$; in disordered NFL materials spin fluctuations are slow (enhanced low-frequency spectral weight) and inhomogeneous (broad spatial distributions of coupling strengths and/or fluctuation amplitudes), but cooperative: Form of correlation function is homogeneous, so not a simple distribution of fluctuation rates. Properties suggest glassy dynamical behavior, but no spin freezing for $T < 20 \text{ mK}$. 
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Longitudinal-field $\mu$SR
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Schematic diagram of longitudinal-field $\mu$SR.
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Obtain time histogram of
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Determined by longitudinal field $H \parallel P_\mu$: $\omega_\mu = \gamma_\mu H$. 

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A local probe—sums over all $q$. 
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(Distributed) muon relaxation rate $W(\mathbf{r}, H)$ related to local spin autocorrelation function $q(t) = \langle \mathbf{S}(\mathbf{r}, t) \cdot \mathbf{S}(\mathbf{r}, 0) \rangle$: 
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W(\mathbf{r}, H) \propto T\chi''(\mathbf{r}, \omega_\mu)/\omega_\mu, \quad \omega_\mu = \gamma_\mu H.
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Time-field scaling

Near a critical point (thermal or quantum) expect
$q(t) = \alpha t^\eta$ (scaling form).

Then $W(r; H) = V(r) = V(r; \beta)$, for $\beta > 1$.

Coefficient $V(r)$ spatially distributed but exponent the same for all spins.

Then $G(t; H) / Z = G(t = H)$ (\beta = H).

This is time-field scaling. First seen in LFSR in spin-glass AgMn, T > T_g (Keren et al. 96).

No need to assume any particular form for $G(t)!$
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Time-field scaling in NFL UCu₄Pd
Time-field scaling in NFL UCu$_4$Pd

LF-$\mu$SR relaxation functions in UCu$_{5-x}$Pd$_x$, $x = 1.0$ and 1.5 (not shown), obey time-field scaling (DEM et al. 01).

LF-$\mu$SR scaling in UCu$_4$Pd (DEM et al. 01, 02).
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LF-$\mu$SR relaxation behavior at low temperatures:
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LF-μSR relaxation behavior at low temperatures:

- CePtSi\textsubscript{1-x}Ge\textsubscript{x},
- UCu\textsubscript{5-x}Pd\textsubscript{x},
- UCu\textsubscript{5-x}Pt\textsubscript{x}*: relaxation *subexponential* (⇒ inhomogeneous) and strong.

*not shown

LF-μSR relaxation functions $G(t)$ at low temperatures in NFL materials (DEM et al. 03).
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LF-μSR relaxation behavior at low temperatures:

- CePtSi$_{1-x}$Ge$_x$,
  UCu$_{5-x}$Pd$_x$,
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- CeNi$_2$Ge$_2$, YbRh$_2$Si$_2$, CeCu$_{5.9}$Au$_{0.1}$,*
  Ce(Ru$_{0.5}$Rh$_{0.5}$)$_2$Si$_2$*:
  relaxation nearly *exponential* (⇒ homogeneous) and *much weaker*.

*not shown
LF-$\mu$SR, resistivity, and specific heat

Effect of disorder? Compare with residual resistivities $(\rho(0))$.

Materials-dependent differences in fluctuation energy scales? Compare with low-temperature specific heat coefficients $(C(T))$ as measures of these scales. (Choose $T = 1 \text{ K}$.)
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(Parameterization only; no *ab initio* justification).

- \(\Lambda^{-1} = \) characteristic \(1/e\) time for relaxation,
- \(K < 1\) measure of spread in rates (broad distribution \(\Rightarrow\) reduced \(K\)).

LF-\(\mu\)SR relaxation functions \(\overline{G}(t)\) at low temperatures in NFL materials (DEM *et al.* 03).
Correlation of normalized $\Lambda$ and $K$ with residual resistivity: with increasing $\rho(0)$
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Correlation is good (smooth).
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Correlation of normalized $\Lambda$ and $K$ with residual resistivity: with increasing $\rho(0)$

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Distributed low-temperature rate, but unique value of time-field scaling exponent for all spins.

Form of spin correlation function $S(r; t)$ is the same for all spins. Strongly suggests homogeneous dynamics (but inhomogeneous fluctuation amplitudes and hyperfine couplings), cooperative rather than single-ion or independent-cluster behavior. Evidence against inhomogeneous local single-ion or cluster dynamics (distributed fluctuation rates) of Kondo disorder/Grifths-phase pictures.
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Conclusions

LF-SR relaxation in ordered and disordered f-electron NFL materials: Field dependence of relaxation rate gives $\gamma(H)$; divergence of $\gamma(H)$ down to very low (but not zero) frequencies for both ordered and disordered systems. Disorder gives rise to much stronger low-frequency fluctuations than homogeneous quantum criticality. Correlation function same form for all spins (cooperative rather than local (single-ion or cluster) dynamics).

Glassy behavior in disordered systems?
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Are these slow "glassy" spin fluctuations ordered-system quantum fluctuations strongly modified by disorder, or new "quantum spin glass" excitations created by disorder?

Do "glassy" NFL spin dynamics depend universally on residual resistivity? What does this mean?

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