Investigating Cosmic Defects with Liquid Crystal Experiments

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Outline:

1) Correspondence of cosmic defects with Condensed matter systems

2) From Cosmology to Laboratory:
   - Testing cosmic defect theories in liquid crystal experiments:
     - Topological defects in liquid crystals
     - Quantitative tests: universal properties
       - Defect density per domain,
       - Defect-antidefect correlations

3) From Laboratory to Cosmology:
   - A new insight for cosmic defects from a liquid crystal experiment: String formation with a heat source

   Implications:
   - Infinite string network formation due to cosmic plasma heated by primordial black holes
Correspondence of cosmic defects with Condensed Matter Systems

Many condensed matter systems provide beautiful analogies for the formation and evolution of defects in the universe. Various aspects of cosmic defects investigated in these systems.

In some cases, they can even provide rigorous, quantitative tests of theories of defect formation in the early universe.

This can be achieved by focusing on Universal Predictions, as the two systems are physically very different.

We checked some of these predictions in liquid crystal systems.

NEXT: We explore this correspondence further: Now from Laboratory to Cosmology

Starting with a novel observation in liquid crystal experiment We obtain a new insight for defects in the universe
Topological defect formation in phase transitions

Apart from thermal production during a phase transition, there is a non-equilibrium process for defect production:

This is usually called as the Kibble Mechanism

Kibble had first proposed it for cosmic defect formation (1976)

Now it is applied to all types of cases – Superconductors, Superfluid helium, liquid crystals etc.

Consider a first order phase transition (this will be the case for liquid crystal defect formation)

If supercooled, bubbles with $|\Psi| = \eta$ will nucleate

U(1) symmetry breaking with a complex scalar order parameter
Occasionally, one gets non-trivial windings of $\theta$, hence topological defects. Its probability can be calculated

For a second order transition, $\theta$ varies randomly beyond the correlation length. Determining appropriate correlation length very non-trivial here due to critical slowing down (Zurek’ 1996).

The Kibble – Zurek Mechanism accounts for these aspects

Note: universal predictions (with scaled lengths) unaffected by these complications.
Topological Defects in Nematic Liquid Crystals (NLCs)

Liquid crystals are orientationally ordered liquids formed by elongated (rod like) or flat (disc like) molecules.

We studied Isotropic – Nematic phase transition

Isotropic phase: High temperature phase, no orientational order.

Nematic phase: Orientationally ordered

Order parameter is called “Director” which describes the local axis of orientational order.

Note: Opposite orientations are identified. Hence order parameter space is $S^2/Z_2 = RP^2$ (the projective plane).
String defects:

Determination of winding number of defects:

NLCs are birefringent. Refractive index is different for $E \parallel$ and $E$ normal to the Director. With crossed-polarizers, when $E$ is $\parallel$ or normal to the Director, polarization is maintained, leading to a dark region. Otherwise, polarization changes, leading to bright region.

If sample is rotated, then for $S = 1$ (defect), dark brushes do not rotate, while for $S = -1$ (antidefect), brushes rotate. Thus one can identify defects and antidefects separately.
String formation and evolution in liquid crystals

Zurek first suggested that ideas of cosmic defect formation could be tested in a condensed matter system (superfluid helium).

First experiment in this direction was done with liquid crystals: Chuang, Durrer, Turok, Yurke - Science, 251, 1336 (1991).

A dense network of string defects was produced by carrying out the I-N phase transition using pressure quench.

Evolution of string network was studied, string intercommutation, scaling properties were investigated.

String evolution was observed to be just as predicted for cosmic strings: Strong support for Cosmic defect theories.

However, string evolution (e.g. Intercommutation) depends on system details: So correspondence with cosmology at the qualitative level only.
Experimental verification of the prediction of defect density

Our first experiment: Work done at Syracuse university

Bowick, Chandar, Schiff, AMS – Science, 263, 943 (1994)

Defect density per bubble in good agreement with the prediction.

Note: Defect density per domain is universal:
choice of length scale important.
Another universal prediction: Defect-antidefect correlations

In a given region containing $N$ defects/antidefects on the average, if defects and antidefects were uncorrelated, then expect that,

$$\Delta N = N_d - N_{ad}$$

will be distributed about zero, with width $\sim N^{1/2}$.
(Large number of events, Gaussian distribution).

Kibble mechanism predicts specific correlations so that:
Width of distribution of $\Delta N$ varies as $N^{1/4}$.

![Loop with perimeter L. It goes through L/R domains (R is domain size). q varies randomly from one domain to next.]

Thus, we have a random walk problem with $L/R$ steps. This implies:

Net winding (no. of defects – no. of antidefects) = $\Delta N \sim (L/R)^{1/2} \sim (\text{Area})^{1/4} \sim N^{1/4}$  
(N = total no. of defects+antidefects)

Note: for uncorrelated defects/antidefects, $\Delta N \sim N^{1/2}$
Experimental verification of defect-antidefect correlations


We observe defect-antidefect distributions using crossed polarizer setup. We plot distributions of $\Delta N$ for different values of $N$ and determine the widths $\sigma$ of these distributions.

We check the following relation:

$$\sigma = CN^\nu$$

Theoretical prediction from the Kibble mechanism:

$\nu = \frac{1}{4}$, $C = 0.57$, or $0.71$ (for triangular or square domain)

We find:

$\nu = 0.26 \pm 0.11$

$C = 0.76 \pm 0.21$

Value of $C$, surprisingly Close to prediction by
Rivers and Swarup. Though
Correspondence not clear

Recall: for uncorrelated defects/antidefects, we expect: $\nu = 1/2$
We carry out isotropic-nematic transition very carefully, via Spinodal decomposition, and avoiding nucleation of bubbles. Transition happens roughly uniformly, over the entire region (in a thin top layer).
During transition, sample is rotated. By comparing pictures Before and after Rotation, defects and antidefects are identified.
Identify defects (d) and antidefects (ad) in a region. Find out $N = N_d + N_{ad}$ and $\Delta N = N_d + N_{ad}$.

Plot frequency of $\Delta N$ vs. $\Delta N$ for given $N$.

Determine width $\sigma$ for each $N$.

Crossing of brushes $\rightarrow$ Defects

No rotation of brushes, defect (d)

Brushes rotate, antidefect (ad)

Sample rotated clockwise compared to (a)
Distributions of $\Delta N$ for different $N$ are plotted. Total number of samples analyzed was 179.

We select small portions of defect distributions. Calculate $N$ and $\Delta N$.

By choosing different samples, we generate statistics for $f(\Delta N)$. $\sigma$ is the width of the distribution $f(\Delta N)$. 
\[ \sigma = C N^\nu \]

Theory:
- \( \nu = \frac{1}{2} \) uncorrelated case
- \( \nu = \frac{1}{4} \), \( C = 0.57, 0.71 \) Kibble mechanism

Experiment: \( \nu = 0.26 \pm 0.11 \); \( C = 0.76 \pm 0.21 \)
Density correlation functions for defects and antidefects


Total number of defects and antidefects analyzed = 833

Number of pictures analyzed = 17

We take a defect as origin.

Calculate density of defects and of antidefects as a function of radial distance.

Repeat for other defects at origin, combine statistics.

Defect-antidefect correlation suggests:

Enhancement in antidefect density at small r

Suppression in defect density at small r.
Theoretical prediction:

Peak in antidefect density at 1 domain separation $\xi$

Suppression in defect density at 1 domain separation

Again: These are universal predictions

Power of this analysis best illustrated when domains are not identified (e.g. for second order transition, or for spinodal decomposition)

$\xi$ is proportional to $R_{av}$ (average inter-defect separation, directly measured)

Plot density profiles for $R/R_{av}$, still universal behavior.

We determine density profiles by numerical simulation of the Kibble mechanism

Compare with density profiles obtained from experiment.
So far we discussed:

Observation/Investigation of the phenomena predicted for Cosmology in liquid crystals.

We now discuss the reverse:

Start with an observation in a liquid crystal experiment With heated wire tip (motivation for this experiment was very different – to simulate situation of quark-gluon plasma production in Relativistic heavy-ion collisions: string production in a compact region.)

Unexpected observation: expanding string loops

Important implications for cosmic defects produced around evaporating primordial black holes
String formation in liquid crystals with a heat source

Work done at IOP, Bhubaneswar, (AMS)

Experimental setup:

[Diagram showing experimental setup with labels: Lamp for heating, CCD Camera, Microscope, VCR, Drop of liquid crystal (K15) 1–2 mm in size, Crossed Polarizers, Glass slide, 100 micron wire for heating, NLC drop]
Observations:

1. With low current in the resistor, small string loops form and shrink near the wire tip.

   With isotropic region forming near the heated wire tip, string loops can form near the I-N boundary.

   String loops can also form due to turbulence as the Director undergoes non-trivial rotations.

2. As current is increased, heating the wire tip further, string formation increases.

   Remarkably, we see that string loops, instead of shrinking, move away from the wire tip and stretch to large sizes.

Explanation: Non uniform heating leads to convection currents. String loops are carried by convection currents away from the tip leading to stretching of loops.

Confirmed by observation of dust particles in the sample which follow string loops as they repeat cycles of motion away and towards the wire tip.
Implications for the universe

Primordial black holes can heat up the background plasma in the early universe. The temperature of the plasma can be high enough for symmetry restoration. (Nagatani; Rangarajan, Sengupta, AMS; Kapusta)

The heated plasma expands out in convective shells.

This suggests that cosmic string loops may be produced near an evaporating black hole (embedded in background plasma in the early universe with ambient temperature much below the cosmic string scale).

Convective expansion will stretch these loops (as long as they are friction dominated).

Such large string loops from different black holes may percolate, leading to formation of infinite string network.

This is remarkable as infinite string network is thought to arise only when the entire universe undergoes phase transition.
Heating of cosmic plasma by evaporating black holes

(Earlier work on electroweak baryogenesis via black holes: R. Rangarajan, S. Sengupta, AMS, Astropart. Phys. 17, 167 (2002), Also, Nagatani, PRD 59, 041301 (1999), Kapusta, ... )

Black holes evaporate by emitting Hawking radiation:

Temperature of black hole: \( T_{bh} = \frac{M_{PL}^2}{8\pi M_{bh}} \)

Rate of loss of mass: \( \frac{dM_{bh}}{dt} = -\frac{\alpha M_{PL}^4}{M_{bh}^2} \)

\( \alpha \) accounts for the scattering of emitted particles
By the curvature, for our case, \( \alpha \sim 3 \times 10^{-3} \)

Note: In the early universe, most of the energy is emitted when the age of the universe is of same order as black hole life time
Consider a black hole evaporating (mostly) when universe Temperature is $T_U$

Particles emitted by the black hole go through the ambient plasma, and loose energy via interactions.

Using energy loss calculations for, say, quarks, through QGP of a given temperature, (for large black hole temperatures) One can estimate thermalization and resultant heating of Background plasma.

Result: For $T_U$ larger than QCD scale (for our interest), Hawking radiation is absorbed very efficiently by the plasma Resulting in localized heating.

Moreover, resulting temperature (pressure) gradient is Very large: Plasma develops radial flow.

Note: this is just like the liquid crystal experiment we discussed
With expanding plasma shells, temperature profile is:

\[
T(r) = \left[ \frac{L}{(8\pi^3 g^*_*/45)r^2V_p} \right]^{1/4}
\]

Here, \( L = -dM_{bh}/dt \), and \( V_p \) is the plasma flow velocity, which is close to the speed of sound, neglecting shock Formation.

\( g^*_* \sim 100 \) is the number of degrees of freedom.

Example: \( T_U = 1 \text{ GeV}, \ M_{bh} = 4 \times 10^{11} \text{ M}_{\text{PL}}, \ T_{bh} = 10^6 \text{ GeV} \)

Life time of black hole \( \sim 5 \times 10^{17} \text{ GeV}^{-1} = 3 \times 10^{-7} \text{ sec.} \)
General picture:

- Plasma flow
- String formation
- String loops expand
- Flowing out with plasma

Symmetry restored region

Here $T$ drops to ambient value

String loop stretching with plasma flow possible only when friction dominates over string tension
\[ F_{tension} \approx \frac{\mu}{R} = \frac{\eta^2}{R} \]

\[ F_{friction} \approx \beta T^3 \frac{V}{\sqrt{1-V^2}} \]

(R is the size of string loop
(Assume, circular string loop around black hole)

\[ \eta \quad \text{Cosmic string scale} \]

\[ \beta \quad \text{Parameter of order 1, V, plasma flow velocity relative to string} \]

Using \( T(r) \), and the condition that \( F_{friction} > F_{tension} \) we get
Maximum distance \( R_{max} \) up to which string loops can be stretched

\[ R_{max} = 10^{-8} \times \frac{x^3 M_{PL}^6}{M_{bh}^3 \eta^4} \]

This is for later stages of black hole evaporation when its mass Has reduced to \( M_{bh} /x \)
For a uniform density of black holes, expanding string loops from different black holes will intersect if:

$$R_{\text{max}} > d_{\text{bh}}$$, typical inter-black hole separation

d_{bh} determined by fraction \( f \) of universe energy density in the form of black holes (at the stage of their evaporation)

Intersecting strings always intercommute, (unless they are ultra-relativistic, which is not the case here)

Intercommutation of different string loops will lead to percolation, thereby forming an infinite string network

Note: This is remarkable, as such an infinite string network is thought to arise only when the entire universe undergoes phase transition

Also, note that here this happens when \( T_U \) is much below the cosmic string scale \( \eta \)
String percolation can be achieved with very low black hole density fraction $f$, and for $\eta$ as large as $10^{14}$ GeV. For large $\eta$, $M_{bh}$ not too large compared to $M_{pl}$, we quote results for small $\eta$.

**TABLE I: Sample parameter values for percolation of strings**

<table>
<thead>
<tr>
<th>$f$</th>
<th>$T_U$ (GeV)</th>
<th>$\eta$ (GeV)</th>
<th>$M_0/M_{pl}$</th>
<th>$M_x/M_{pl}$</th>
<th>$r_\eta$ (GeV$^{-1}$)</th>
<th>$R_{stretch}$ (GeV$^{-1}$)</th>
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<tbody>
<tr>
<td>1</td>
<td>$2 \times 10^7$</td>
<td>$10^{10}$</td>
<td>$5 \times 10^6$</td>
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<td>$10^{-7}$</td>
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<td>$10^{10}$</td>
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<td>550</td>
<td>$6 \times 10^{-7}$</td>
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<td>33000</td>
<td>$10^8$</td>
<td>$10^{12}$</td>
</tr>
</tbody>
</table>
This new possibility of formation of infinite string network Without a phase transition may be important for situations Where conventional string formation via Phase transition may not occur. For example:

1. In models of inflation with low reheat temperature
2. Models of gravity with low scale

Eventually the string network should approach scaling regime. For subsequent stages, the network resulting from black holes may not be different from the conventional one.

Question: Is it possible that the initial network may have some important differences?
Conclusions:

1. Correspondence between cosmology and condensed matter systems can provide rigorous, quantitative tests of certain aspects of theories of cosmic defects (No way of directly experimentally investigating cosmic defects).

2. From observations in condensed matter systems, by suitable interpreting analogues in cosmology, new insights can be obtained for the universe.

3. Correspondence with other systems, e.g. quark-gluon plasma production in relativistic heavy ion collisions should also be explored. (Work in progress)