Evolution of Market Heuristics

(An Explanation of an Asset-Pricing Experiment)

Mikhail Anufriev    Cars Hommes

CeNDEF, Faculty of Economics and Business
University of Amsterdam

Workshop on Complexity in Economics and Finance
Lorentz Center, Leiden
24 October 2007
Question

How do people behave (form expectations and learn) in the expectations feedback system?

- expectations are shaped given the market history
- expectations affect the outcome (e.g. price)
Possible Answers

- Fully rational (rational expectations)

- Belief-based learning (e.g. Bayesian learning)
  - Jordan (GEB, 1991), Bray and Savin (E, 1986), Kalai and Lehrer (E, 1993), Cheung and Friedman (GEB, 1997) ...

- Reinforcement learning and learning from regret
  - Arthur (AER, 1991), Arthur (JEE, 1993), Roth and Erev (GEB, 1995), Erev and Roth (AER, 1998), Camerer and Ho (E, 1999), Hart (E, 2005) ...

- Evolutionary selection
  - Hofbauer and Sigmund (1998)
Possible Answers

- Fully rational (rational expectations)
- Belief-based learning (e.g. Bayesian learning)
  - Jordan (GEB, 1991), Bray and Savin (E, 1986), Kalai and Lehrer (E, 1993), Cheung and Friedman (GEB, 1997) ...
- Reinforcement learning and learning from regret
  - Arthur (AER, 1991), Arthur (JEE, 1993), Roth and Erev (GEB, 1995), Erev and Roth (AER, 1998), Camerer and Ho (E, 1999), Hart (E, 2005) ...
- Evolutionary selection
  - Hofbauer and Sigmund (1998)
Here we present...

- a model of “reinforcement” learning in a non-game-theoretic setting with limited information about environment...

  in forecasting agents rely on simple heuristics
  (Tversky and Kahneman, 1974)

  in learning agents update their “active” heuristics on the basis of heuristics’ performances

- explaining the results of a recent experiment where
  subject predicted future price
  price process depended on the expectations
Here we present...

- a model of “reinforcement” learning in a non-game-theoretic setting with limited information about environment...

  in forecasting agents rely on simple heuristics (Tversky and Kahneman, 1974)

  in learning agents update their “active” heuristics on the basis of heuristics’ performances

- ...explaining the results of a recent experiment where
  - subject predicted future price
  - price process depended on the expectations
Experiment

Hommes, Sonnemans, Tuinstra, van de Velden (2005, RFS) during 51 periods participants forecast next realization for an endogenous price process

- two assets in a market
  - riskless with interest $r$
  - risky with dividend $y_t = \bar{y} + \varepsilon_t$ and price $p_t$

- fundamental price of the risky asset $p_f = \frac{\bar{y}}{r}$

- realized price depends (through the demand) on the next-period price forecasts (e.g. Walrasian clearing with mean-variance optimizers)

$$p_t = \frac{1}{1 + r} \left( p_{t+1}^e + \bar{y} + \varepsilon_t \right)$$
Experiment

Hommes, Sonnemans, Tuinstra, van de Velden (2005, RFS) during 51 periods participants forecast next realization for an endogenous price process

- two assets in a market
  - riskless with interest $r$
  - risky with dividend $y_t = \bar{y} + \varepsilon_t$ and price $p_t$

- fundamental price of the risky asset $p^f = \frac{\bar{y}}{r}$

- realized price depends (through the demand) on the next-period price forecasts
  (e.g. Walrasian clearing with mean-variance optimizers)

$$p_t = \frac{1}{1 + r} \left( p^e_{t+1} + \bar{y} + \varepsilon_t \right)$$
Experiment

Hommes, Sonnemans, Tuinstra, van de Velden (2005, RFS) during 51 periods participants forecast next realization for an endogenous price process

- two assets in a market
  - riskless with interest \( r \)
  - risky with dividend \( y_t = \bar{y} + \epsilon_t \) and price \( p_t \)

- fundamental price of the risky asset \( p^f = \frac{\bar{y}}{r} \)

- realized price depends (through the demand) on the next-period price forecasts (e.g. Walrasian clearing with mean-variance optimizers)

\[
p_t = \frac{1}{1 + r} \left( \overline{p^e_{t+1}} + \bar{y} + \epsilon_t \right)
\]
Experiment

Hommes, Sonnemans, Tuinstra, van de Velden (2005, RFS) during 51 periods participants forecast next realization for an endogenous price process

- two assets in a market
  - riskless with interest $r$
  - risky with dividend $y_t = \bar{y} + \varepsilon_t$ and price $p_t$

- fundamental price of the risky asset $p^f = \frac{\bar{y}}{r}$

- realized price depends (through the demand) on the next-period price forecasts (e.g. Walrasian clearing with mean-variance optimizers)

$$p_t = \frac{1}{1 + r} \left( p^e_{t+1} + \bar{y} + \varepsilon_t \right)$$
Timing in the Experiment

- fix $r = 0.05$ and $\bar{y} = 3$, so that $p^f = 60$

- generate noise $\varepsilon_t \sim N(0, 0.25)$ for periods $t = 0, \ldots, 50$

- 6 human participants in the beginning of period $t$ submit forecasts $p^e_{t+1,h}$

- “robot” traders always predict $p^f$

- price in period $t$ is computed and is announced to subjects

- subjects are paid according to the precision of their forecast
Timing in the Experiment

- fix $r = 0.05$ and $\bar{y} = 3$, so that $p^f = 60$
- generate noise $\varepsilon_t \sim N(0, 0.25)$ for periods $t = 0, \ldots, 50$
- 6 human participants in the beginning of period $t$ submit forecasts $p^{e}_{t+1,h}$
- “robot” traders always predict $p^f$
- price in period $t$ is computed and is announced to subjects
- subjects are paid according to the precision of their forecast
Timing in the Experiment

- fix $r = 0.05$ and $\bar{y} = 3$, so that $p^f = 60$
- generate noise $\varepsilon_t \sim N(0, 0.25)$ for periods $t = 0, \ldots, 50$
- 6 human participants in the beginning of period $t$ submit forecasts $p^e_{t+1,h}$
- “robot” traders always predict $p^f$
- price in period $t$ is computed and is announced to subjects
- subjects are paid according to the precision of their forecast
Formal Presentation of the Experiment

- **price** in period $t = 0, \ldots, 50$ is determined as

  $$p_t = \frac{1}{1 + r} \left( (1 - n_t) \frac{p_{t+1,1}^e + \cdots + p_{t+1,6}^e}{6} + n_t p^f + \bar{y} + \epsilon_t \right)$$

- share of robots $n_0 = 0$ and $n_t$ for $t = 1, \ldots, 50$ is defined as

  $$n_t = 1 - \exp \left( - \frac{1}{200} |p_{t-1} - p^f| \right)$$

- subjects are paid in every period $t = 0, \ldots, 50$ according to

  $$e_{t,h} = \max \left( 1300 - \frac{1300}{49} (p_t - p_{t,h}^e)^2, 0 \right)$$
Formal Presentation of the Experiment

- **price** in period $t = 0, \ldots, 50$ is determined as

$$p_t = \frac{1}{1 + r} \left( (1 - n_t) \frac{p^e_{t+1,1} + \cdots + p^e_{t+1,6}}{6} + n_t p^f + \bar{y} + \varepsilon_t \right)$$

- share of robots $n_0 = 0$ and $n_t$ for $t = 1, \ldots, 50$ is defined as

$$n_t = 1 - \exp \left( - \frac{1}{200} |p_{t-1} - p^f| \right)$$

- subjects are paid in every period $t = 0, \ldots, 50$ according to

$$e_{t,h} = \max \left( 1300 - \frac{1300}{49} (p_t - p^e_{t,h})^2, 0 \right)$$
Formal Presentation of the Experiment

- **price** in period $t = 0, \ldots, 50$ is determined as

$$p_t = \frac{1}{1 + r} \left( (1 - n_t) \frac{p_{t+1,1}^e + \cdots + p_{t+1,6}^e}{6} + n_t p^f + \bar{y} + \varepsilon_t \right)$$

- **share of robots** $n_0 = 0$ and $n_t$ for $t = 1, \ldots, 50$ is defined as

$$n_t = 1 - \exp \left( - \frac{1}{200} |p_{t-1} - p^f| \right)$$

- **subjects are paid in every period** $t = 0, \ldots, 50$ according to

$$e_{t,h} = \max \left( 1300 - \frac{1300}{49} (p_t - p_{t,h}^e)^2, 0 \right)$$
Information

Subjects know

- the environment (interest rate $r$ and mean dividend $\bar{y}$)
- price is coming from equilibrium between demand and supply
- the positive relation between their forecasts and demand
- on the screen in the beginning of time $t$ every participant $h$ can see the past prices (up to $p_{t-1}$), own past forecasts (up to $p_{t,h}$) and own earnings (up to $e_{t-1,h}$)

Subjects do not know

- exact equilibrium equation
- exact demand schedule of themselves and others
- number and identity of other participants
Information

Subjects **know**

- the environment (interest rate $r$ and mean dividend $\bar{y}$)
- price is coming from equilibrium between demand and supply
- the positive relation between their forecasts and demand
- on the screen in the beginning of time $t$ every participant $h$ can see the past prices (up to $p_{t-1}$), own past forecasts (up to $p_{t,h}$) and own earnings (up to $e_{t-1,h}$)

Subjects **do not know**

- exact equilibrium equation
- exact demand schedule of themselves and others
- number and identity of other participants
Information

Subjects **know**
- the environment (interest rate $r$ and mean dividend $\bar{y}$)
- price is coming from equilibrium between demand and supply
- the positive relation between their forecasts and demand
- on the screen in the beginning of time $t$ every participant $h$ can see the past prices (up to $p_{t-1}$), own past forecasts (up to $p_{t,h}$) and own earnings (up to $e_{t-1,h}$)

Subjects **do not know**
- exact equilibrium equation
- exact demand schedule of themselves and others
- number and identity of other participants
Rational Benchmark

If everybody predicts fundamental price, then

\[ p_t = p^f + \frac{\varepsilon_t}{1 + r} \]
Price in the Experiment

![Graphs showing price movements for different groups.](image)
2 Groups with (Almost) Monotonic Convergence
2 Groups with Constant Oscillations

Mikhail Anufriev, Cars Hommes
CeNDEF, University of Amsterdam
Evolution of Market Heuristics
2 Groups with Damping Oscillations

Mikhail Anufriev, Cars Hommes
CeNDEF, University of Amsterdam
Evolution of Market Heuristics
Summary of the Results of the Experiment

Results are inconsistent with Fundamental Forecasting.

One would like to explain:

- three qualitatively different patterns
- (almost) monotonic convergence
- constant oscillations
- damping oscillations
- coordination of agents in their predictions
Summary of the Results of the Experiment

Results are inconsistent with Fundamental Forecasting.

One would like to explain:

- three qualitatively different patterns
  - (almost) monotonic convergence
  - constant oscillations
  - damping oscillations

- coordination of agents in their predictions
Summary of the Results of the Experiment

Results are inconsistent with Fundamental Forecasting.

One would like to explain:

- three qualitatively different patterns
  - (almost) monotonic convergence
  - constant oscillations
  - damping oscillations

- coordination of agents in their predictions
Estimation of Individual Predictions

...for the past 40 periods

- in converging groups agents use adaptive expectations
  \[ p_{t+1}^e = w p_{t-1} + (1 - w) p_t^e \]

- often agents used simple linear rules
  \[ p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \]

  in particular trend-extrapolating rules
  \[ p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \]
Estimation of Individual Predictions

...for the past 40 periods

▶ in converging groups agents use adaptive expectations

\[ p_{t+1}^e = w p_{t-1} + (1 - w) p_t^e \]

▶ often agents used simple linear rules

\[ p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \]

in particular trend-extrapolating rules

\[ p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \]
Estimation of Individual Predictions

...for the past 40 periods

- in converging groups agents use adaptive expectations

\[ p_{t+1}^e = wp_{t-1} + (1-w)p_t^e \]

- often agents used simple linear rules

\[ p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \]

in particular trend-extrapolating rules

\[ p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \]
Dynamics under Homogeneous Expectations

- all the agents use the same rule (adaptive or linear)

\[
\left\{ \begin{array}{l}
p_{t+1}^e = f(p_{t-1}, p_{t-2}, p_t^e) \\
n_t = 1 - \exp\left(-\frac{1}{200}|p_{t-1} - p^f|\right) \\
p_t = \frac{1}{1+r}\left((1 - n_t)p_{t+1}^e + n_t p^f + \bar{y} + \varepsilon_t\right)
\end{array} \right.
\]

- \(\varepsilon_t = 0\): deterministic skeleton

- simulations with \(\varepsilon_t\) from the experiment
Dynamics under Homogeneous Expectations

- all the agents use the same rule (adaptive or linear)

\[
\begin{align*}
    p_{t+1}^e &= f(p_{t-1}, p_{t-2}, p_t^e) \\
    n_t &= 1 - \exp \left( -\frac{1}{200} |p_{t-1} - p^f| \right) \\
    p_t &= \frac{1}{1+r} \left( (1-n_t)p_{t+1}^e + n_t p^f + \bar{y} + \varepsilon_t \right)
\end{align*}
\]

- \( \varepsilon_t = 0 \): deterministic skeleton

- simulations with \( \varepsilon_t \) from the experiment
Adaptive Expectations: \( p_{t+1}^e = w p_{t-1} + (1 - w) p_t^e \)

Dynamics **globally converge** to fundamental price.
Extrapolative Expectations: \( p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \)

Special cases:

- **trend-following heuristic**
  \[ p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \]

- **anchoring and adjustment heuristic**
  \[ p_{t+1}^e = 0.5 (p_{t-1} + p_{t-2}) + (p_{t-1} - p_{t-2}) \]

**Definition**

The extrapolative rule is called **consistent** in the steady-state with price \( p^* \), if it predicts \( p^* \) in this steady-state.
Extrapolative Expectations: \( p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \)

Special cases:

- trend-following heuristic

\[
p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2})
\]

- anchoring and adjustment heuristic

\[
p_{t+1}^e = 0.5 (p^f + p_{t-1}) + (p_{t-1} - p_{t-2})
\]

**Definition**

The extrapolative rule is called **consistent** in the steady-state with price \( p^* \), if it predicts \( p^* \) in this steady-state.
Extrapolative Expectations: \( p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \)

- The unique steady-state where the rule is consistent has \( p^* = p^f \)
- For the trend-following heuristic this is the only steady-state
- Stability conditions
Weak-Trend Extrapolation: $p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2})$

Dynamics converge to fundamental price.

![Price under weak trend following heuristic](image)
Strong-Trend Extrapolation: \( p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \)

Dynamics **dive**rge from fundamental price...

Price under strong trend following heuristic

\[ \gamma = 1.1 \quad \text{red} \quad \gamma = 1.3 \quad \text{blue} \]

Mikhail Anufriev, Cars Hommes
CeNDEF, University of Amsterdam

Evolution of Market Heuristics
Strong-Trend Extrapolation: \( p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \)

...and settles on the quasi-periodic attractor.
Anchoring and Adjustment: $p_{t+1}^e = \frac{p_f + p_{t-1}}{2} + (p_{t-1} - p_{t-2})$

Price under anchoring and adjustment heuristic

with learning of anchor: $p_{t+1}^e = \frac{p_{t-1}^{av} + p_{t-1}}{2} + (p_{t-1} - p_{t-2})$
Model with Homogeneous Expectations

- pattern of **monotonic convergence** can be easily reproduced
  adaptive rule, weak trend extrapolation

- pattern of **constant oscillations** can be reproduced
  anchoring and adjustment rule without learning

- pattern of **damping oscillations** is reproduced (**very imperfectly**)
  strong-trend extrapolations
Model with Homogeneous Expectations

- pattern of **monotonic convergence** can be easily reproduced
  adaptive rule, weak trend extrapolation

- pattern of **constant oscillations** can be reproduced
  anchoring and adjustment rule **without learning**

- pattern of **damping oscillations** is reproduced **(very imperfectly)**
  strong-trend extrapolations
Model with Homogeneous Expectations

- pattern of **monotonic convergence** can be easily reproduced
  adaptive rule, weak trend extrapolation

- pattern of **constant oscillations** can be reproduced
  anchoring and adjustment rule **without learning**

- pattern of **damping oscillations** is reproduced (**very imperfectly**)
  strong-trend extrapolations
Dynamics for Individual Rules: Converging Groups

Stability region and group 2

Stability region and group 5
Dynamics for Individual Rules: Oscillating Groups

Stability region and group 1

Stability region and group 6
Dynamics for Individual Rules: Damping Groups
Evolution of Individual Predictions

Naive Rule: \( p_{t+1}^e = p_{t-1} \)

Adaptive Rule: \( p_{t+1}^e = 0.25 p_{t-1} + 0.75 p_t^e \)
Evolution of Individual Predictions

Weak trend extrapolation: \( p^e_{t+1} = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \), \( \gamma \approx 0.4 \)
Evolution of Individual Predictions

Anchoring adjustment rule: $p_{t+1}^e = 0.5(p_f^t + p_{t-1}^t) + (p_{t-1}^t - p_{t-2}^t)$
Evolution of Individual Predictions

Strong trend extrapolation: \( p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \), \( \gamma \approx 1.3 \)
Summary

- participants tend to base their predictions on past observations, following **simple routines**

- learning of people has a form of **switching** from one routine to another

- in every group some **heterogeneity** among participants remains despite relatively close predictions

- some **combination** of rules which lead to different outcomes
Informal Description of the Model

- there exist a number of simple **heuristics** (rules mapping the information to the price prediction)

- heuristics are used by agents **unevenly**, so that every heuristic has own **impact** on price determination

- agents evaluate the performances of all heuristics, so that impacts are evolving

- agents tend to **switch** on the more successful heuristics

- **initialization:**
  - prices in the first periods
  - initial impacts
Informal Description of the Model

- there exist a number of simple **heuristics** (rules mapping the information to the price prediction)

- heuristics are used by agents **unevenly**, so that every heuristic has own **impact** on price determination

- agents evaluate the performances of all heuristics, so that **impacts are evolving**

- agents tend to **switch** on the more successful heuristics

**initialization:**
- prices in the first periods
- initial impacts
Informal Description of the Model

- there exist a number of simple **heuristics** (rules mapping the information to the price prediction)

- heuristics are used by agents **unevenly**, so that every heuristic has own **impact** on price determination

- agents evaluate the performances of all heuristics, so that **impacts are evolving**

- agents tend to **switch** on the more successful heuristics

- **initialization:**
  - prices in the first periods
  - initial impacts
Four forecasting heuristics

▶ adaptive rule

**ADA** \( p_{1,t+1}^e = 0.65 p_{t-1} + 0.35 p_{1,t}^e \)

▶ weak trend-following rule

**WTR** \( p_{2,t+1}^e = p_{t-1} + 0.4 (p_{t-1} - p_{t-2}) \)

▶ strong trend-following rule

**STR** \( p_{3,t+1}^e = p_{t-1} + 1.3 (p_{t-1} - p_{t-2}) \)

▶ anchoring and adjustment heuristics with learnable anchor

**LAA** \( p_{4,t+1}^e = 0.5 p_{t-1}^{av} + 0.5 p_{t-1} + (p_{t-1} - p_{t-2}) \)
Stability of four heuristics

\begin{align*}
\beta_2 & = 1.5 \\
\beta_2 & = 1 \\
\beta_2 & = 0.5 \\
\beta_2 & = 0 \\
\beta_2 & = -0.5 \\
\beta_2 & = -1 \\
\beta_2 & = -1.5
\end{align*}

\begin{align*}
\beta_2 & = 2 \\
\beta_2 & = 1 \\
\beta_2 & = 0.5 \\
\beta_2 & = 0 \\
\beta_2 & = -0.5 \\
\beta_2 & = -1 \\
\beta_2 & = -1.5
\end{align*}

\begin{align*}
\gamma & = 0.4 \\
\gamma & = 1.3
\end{align*}

A&A

Mikhail Anufriev, Cars Hommes
CeNDEF, University of Amsterdam
Evolution of Market Heuristics
Price dynamics

- price dynamics

\[ p_t = \frac{1}{1 + r \left( (n_{1,t}p_{1,t+1}^e + n_{2,t}p_{2,t+1}^e + n_{3,t}p_{3,t+1}^e + n_{4,t}p_{4,t+1}^e) \times \right. } \]
\[ \left. \times (1 - n_t) + p^f n_t + \bar{y} + \varepsilon_t \right) \]

- fraction \( n_t \) of robot traders is evolving as

\[ n_t = 1 - \exp \left( - \frac{1}{200} |p_{t-1} - p^f| \right) \]

- impacts of heuristics \( n_{i,t} \) are evolving as in discrete choice model with asynchronous updating
Evolutionary switching

- **performance measure** of heuristic $i$ is

$$U_{i,t-1} = -(p_{t-1} - p_{i,t-1}^e)^2 + \eta U_{i,t-2}$$

parameter $\eta \in [0, 1]$ – the **strength** of the agents’ memory

- **discrete choice** model with **asynchronous updating**

$$n_{i,t} = \delta n_{i,t-1} + (1 - \delta) \frac{\exp(\beta U_{i,t-1})}{\sum_{i=1}^{4} \exp(\beta U_{i,t-1})}$$

parameter $\delta \in [0, 1]$ – the **inertia** of the traders
parameter $\beta \geq 0$ – the **intensity of choice**
Initialization

Model is **initialized** with

- two initial price $p_0$ and $p_1$
- initial impacts of different heuristics $n_{1,t}, n_{2,t}, n_{3,t}, n_{4,t}$

<table>
<thead>
<tr>
<th>Group</th>
<th>$p_0$</th>
<th>$p_1$</th>
<th>$p_0$</th>
<th>$p_1$</th>
<th>ADA</th>
<th>WTR</th>
<th>STR</th>
<th>LAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 2</td>
<td>48.94</td>
<td>51.21</td>
<td>49</td>
<td>50.5</td>
<td>0.25</td>
<td>0.35</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>Group 5</td>
<td>53.78</td>
<td>53.61</td>
<td>54</td>
<td>53.5</td>
<td>0.25</td>
<td>0.35</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>Group 1</td>
<td>53.05</td>
<td>56.45</td>
<td>51</td>
<td>54</td>
<td>0.15</td>
<td>0.35</td>
<td>0.35</td>
<td>0.15</td>
</tr>
<tr>
<td>Group 6</td>
<td>56.54</td>
<td>58.38</td>
<td>56</td>
<td>58</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Group 4</td>
<td>43.72</td>
<td>47.33</td>
<td>42</td>
<td>47</td>
<td>0</td>
<td>0.1</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>Group 7</td>
<td>44.81</td>
<td>49.71</td>
<td>44</td>
<td>48</td>
<td>0</td>
<td>0.17</td>
<td>0.66</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Group 5 (Convergence)

Parameters: $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$
Group 1 (Constant Oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Group 7 (Damping Oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Group 5 (Convergence)

Parameters: $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$
Group 1 (Constant Oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Group 7 (Damping Oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Comparison with Homogeneous Expectations: MSD

Fit of the experiment with parameters $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$

<table>
<thead>
<tr>
<th>Specification</th>
<th>Group 2</th>
<th>Group 5</th>
<th>Group 1</th>
<th>Group 6</th>
<th>Group 4</th>
<th>Group 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Prediction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADA – exp prices</td>
<td>0.841</td>
<td>0.200</td>
<td>7.676</td>
<td>8.401</td>
<td>330.101</td>
<td>51.526</td>
</tr>
<tr>
<td>WTR – exp prices</td>
<td>4.419</td>
<td>1.983</td>
<td>8.868</td>
<td>6.252</td>
<td>308.549</td>
<td>30.298</td>
</tr>
<tr>
<td>STR – exp prices</td>
<td>585.789</td>
<td>478.525</td>
<td>638.344</td>
<td>509.266</td>
<td>1231.064</td>
<td>698.361</td>
</tr>
<tr>
<td>LAA – exp prices</td>
<td>5.475</td>
<td>3.534</td>
<td>5.405</td>
<td>14.404</td>
<td>307.605</td>
<td>69.749</td>
</tr>
<tr>
<td>ADA – fitted prices</td>
<td>0.514</td>
<td>0.199</td>
<td>6.832</td>
<td>7.431</td>
<td>312.564</td>
<td>36.436</td>
</tr>
<tr>
<td>WTR – fitted prices</td>
<td>4.222</td>
<td>1.844</td>
<td>8.670</td>
<td>6.228</td>
<td>292.150</td>
<td>19.764</td>
</tr>
<tr>
<td>STR – fitted prices</td>
<td>413.435</td>
<td>42.488</td>
<td>182.284</td>
<td>29.200</td>
<td>580.543</td>
<td>579.141</td>
</tr>
<tr>
<td>LAA – fitted prices</td>
<td>2.055</td>
<td>1.859</td>
<td>4.236</td>
<td>13.433</td>
<td>284.880</td>
<td>45.153</td>
</tr>
<tr>
<td>4 heuristics (plots)</td>
<td>0.449</td>
<td>0.302</td>
<td>8.627</td>
<td>14.755</td>
<td>526.417</td>
<td>29.520</td>
</tr>
<tr>
<td>4 heuristics (fitted)</td>
<td>0.313</td>
<td>0.245</td>
<td>7.227</td>
<td>7.679</td>
<td>235.900</td>
<td>18.662</td>
</tr>
</tbody>
</table>
Comparison with Homogeneous Expectations: AR2

Fit of the experiment with parameters $\beta = 0.4, \eta = 0.7, \delta = 0.9$

<table>
<thead>
<tr>
<th>Specification</th>
<th>Group 2</th>
<th>Group 5</th>
<th>Group 1</th>
<th>Group 6</th>
<th>Group 4</th>
<th>Group 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Prediction</td>
<td>0.946</td>
<td>0.671</td>
<td>2.673</td>
<td>3.610</td>
<td>2.311</td>
<td>2.002</td>
</tr>
<tr>
<td>ADA – exp prices</td>
<td>0.239</td>
<td>0.006</td>
<td>2.182</td>
<td>2.898</td>
<td>1.691</td>
<td>1.494</td>
</tr>
<tr>
<td>WTR – exp prices</td>
<td>0.066</td>
<td>0.529</td>
<td>0.383</td>
<td>0.627</td>
<td>0.203</td>
<td>0.165</td>
</tr>
<tr>
<td>STR – exp prices</td>
<td>1.494</td>
<td>2.583</td>
<td>0.112</td>
<td>0.020</td>
<td>0.240</td>
<td>0.342</td>
</tr>
<tr>
<td>A&amp;A – exp prices</td>
<td>1.095</td>
<td>1.848</td>
<td>0.010</td>
<td>0.038</td>
<td>0.045</td>
<td>0.094</td>
</tr>
<tr>
<td>LAA – exp prices</td>
<td>0.747</td>
<td>1.544</td>
<td>0.003</td>
<td>0.050</td>
<td>0.003</td>
<td>0.013</td>
</tr>
<tr>
<td>ADA – fitted prices</td>
<td>0.100</td>
<td>0.000</td>
<td>1.584</td>
<td>2.159</td>
<td>1.385</td>
<td>1.157</td>
</tr>
<tr>
<td>WTR – fitted prices</td>
<td>0.068</td>
<td>0.343</td>
<td>0.262</td>
<td>0.435</td>
<td>0.174</td>
<td>0.139</td>
</tr>
<tr>
<td>STR – fitted prices</td>
<td>1.358</td>
<td>2.192</td>
<td>0.078</td>
<td>0.001</td>
<td>0.147</td>
<td>0.242</td>
</tr>
<tr>
<td>A&amp;A– fitted prices</td>
<td>1.036</td>
<td>1.755</td>
<td>0.005</td>
<td>0.029</td>
<td>0.038</td>
<td>0.083</td>
</tr>
<tr>
<td>LAA – fitted prices</td>
<td>0.640</td>
<td>1.277</td>
<td>0.000</td>
<td>0.033</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>4 heuristics (plots)</td>
<td>0.383</td>
<td>0.744</td>
<td>0.011</td>
<td>0.008</td>
<td>0.157</td>
<td>0.239</td>
</tr>
<tr>
<td>4 heuristics (fitted)</td>
<td>0.144</td>
<td>0.499</td>
<td>0.009</td>
<td>0.003</td>
<td>0.121</td>
<td>0.048</td>
</tr>
</tbody>
</table>
Results and further directions

- the model with evolutionary switching is built
- three qualitatively different patterns of the experiment have been reproduced
- model has path-dependence feature

- Code Evexex: http://www.cafed.eu/evexex
- Software Package E&F Chaos
Results and further directions

▶ the model with evolutionary switching is built
▶ three qualitatively different patterns of the experiment have been reproduced
▶ model has path-dependence feature

▶ Code Evexex: http://www.cafed.eu/evexex
▶ Software Package E&F Chaos
Evolutionary Model

- deterministic skeleton

\[
\begin{align*}
    p_{i,t+1}^e &= f_i(p_{t-1}, p_{t-2}, p_{i,t}^e) \\
    p_t - p^f &= \frac{1}{1 + r} \exp \left( -\frac{1}{200} |p_{t-1} - p^f| \right) \sum_{i=1}^{4} n_{i,t} (p_{i,t+1}^e - p^f) \\
    n_{i,t} &= \delta n_{i,t-1} + (1 - \delta) \frac{\exp(\beta U_{i,t-1})}{\sum_{i=1}^{4} \exp(\beta U_{i,t-1})} \\
    U_{i,t-1} &= -(p_{t-1} - p_{i,t-1}^e)^2 + \eta U_{i,t-2}
\end{align*}
\]

- parameters

- \( \beta \geq 0 \) – the intensity of choice
- \( \eta \in [0, 1] \) – the strength of the agents’ memory
- \( \delta \in [0, 1] \) – the inertia of the traders
Stability for the Model with **Fixed Impacts**

Stability region for model with fixed fractions

\[ s_3, \text{fraction of STR} \]

Mikhail Anufriev, Cars Hommes
CeNDEF, University of Amsterdam
Coexisting Attractors in Model “STR vs. LAA”

Parameters: $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$
Coexisting Attractors in Model “STR vs. LAA”

Parameters: $\beta = 0.4$, $\eta = 0$, $\delta = 0$
Time Series in Model “STR vs. LAA”

Initial Prices: 58, 59, . . .

Parameters: $\beta = 0.4$, $\eta = 0$, $\delta = 0$ and $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$
Group 5 (Convergence)
Group 1 (Constant Oscillations)
Group 7 (Damping Oscillations)

![Graph showing experiment and simulation results for Group 7. The graph compares AAA+WTR and AAA+WTR+ADA with 4 heuristics. The x-axis represents time, and the y-axis shows the value of the oscillations. The graph illustrates the damping effect over time.]

Mikhail Anufriev, Cars Hommes
CeNDEF, University of Amsterdam
Evolution of Market Heuristics