Lattice QCD with 8 and 12 Degenerate Quark Flavors

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Universe in a Box: LHC, Cosmology and Lattice Field Theory
Leiden, 2009
This is interesting!

We will compare extensive simulations with 8 and 12 flavors.
Our approach

- Measure observables from zero temperature lattice simulations.
  - Meson propagators and masses.
  - $\pi$ decay constant, $f_\pi$.
  - Quark potential and string tension.
- DBW2 gauge with naïve staggered fermion action.
- RHMC algorithm — exact.
- Each measurements are separated by 10 trajectories.
- Each trajectory has a length of 0.5 MD unit.
- Simulation parameters of 8 flavors.
  - See our proceedings last year.
  - An additional $32^3 \times 32$ ensemble generated at $m_q = 0.015$ and $\beta = 0.58$.
- Simulation parameters of 12 flavors.
  - $\beta$ ranges from 0.45 to 0.50.
  - $m_q$ ranges from 0.01 to 0.03.
  - Lattice size of $16^3 \times 32$, $24^3 \times 32$ and $32^3 \times 32$.
- Measure observables from finite temperature simulations at $N_\tau = 8$. 
### List of major simulations with 12 flavors at $N_{\tau} = 32$

<table>
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<tr>
<th>$\beta$</th>
<th>$m_q = 0.01$</th>
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<td>$16^3 \times 32$</td>
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Extrapolation of $\langle \bar{\psi} \psi \rangle$

8 flavors

$\langle \bar{\psi} \psi \rangle (\beta = 0.54, m_q = 0) = 0.02555(23)$.  
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$\langle \bar{\psi} \psi \rangle (\beta = 0.58, m_q = 0) = 0.01519(16)$.  

12 flavors

$\langle \bar{\psi} \psi \rangle (\beta = 0.45, m_q = 0) = -0.00382(25)$.  
$\langle \bar{\psi} \psi \rangle (\beta = 0.46, m_q = 0) = 0.00220(27)$.  

Weaker Coupling Side

Stronger Coupling Side

Weaker Coupling Side

Stronger Coupling Side

Rapid change in system. What is it?

Yes, it is consistent!
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Stronger Coupling Side

Transition Region

Weaker Coupling Side

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**12 flavors**

- $\langle \bar{\psi} \psi \rangle (\beta = 0.48, m_q = 0) = -0.00382(25)$.
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Rapid change in system. What is it?

Xiao-Yong Jin (Columbia University)  
LQCD with 8 and 12 Flavors  
Leiden, 2009  
5 / 23
π propagator with 12 flavors

π WALL2Z propagator at $m_q = 0.02$

- $\beta = 0.45$
- $\beta = 0.47$
- $\beta = 0.48$

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LQCD with 8 and 12 Flavors  
Leiden, 2009  
6 / 23
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Exponential Falloff

- $\beta = 0.45$
- $\beta = 0.47$
- $\beta = 0.48$

Xiao-Yong Jin (Columbia University)

LQCD with 8 and 12 Flavors

Leiden, 2009
Extrapolation of $m^2_\pi$

### 8 flavors

- $m^2_\pi(\beta = 0.54, m_q = 0) = 0.01707(20)$
- $m^2_\pi(\beta = 0.56, m_q = 0) = 0.01648(51)$
- $m^2_\pi(\beta = 0.58, m_q = 0) = 0.0106(16)$
- Clear $m^2_\pi \sim m_q$.
- Slope changes little.

### 12 flavors

- $m^2_\pi(\beta = 0.45, m_q = 0) = 0.00482(39)$
- $m^2_\pi(\beta = 0.46, m_q = 0) = 0.00912(55)$
- Strong coupling — Goldstone.
- Weak coupling?
Extrapolation of $m^2_\pi$ at weakest coupling $\beta = 0.49$.

- $m^2_\pi(\beta = 0.49, m_q = 0, \text{all } 16^3\times32) = 0.0457(22)$.
- $m^2_\pi(\beta = 0.49, m_q = 0, \text{w/ } 32^3\times32 @ m_q = 0.01) = 0.01926(66)$.
Extrapolation of $m^2_\pi$ at weakest coupling $\beta = 0.49$

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Finite volume effect with 4 flavors

Goldstone Pion

\[ N_f = 4 \]

\[ \pi \text{ on } 16^3 \times 32 \]

\[ \pi \text{ on } 24^3 \times 32 \]
Extrapolation of $f_\pi$

8 flavors

$2 \times \text{change across the region of rapid evolution of system.}$

12 flavors

$10 \times \text{change!}$
Extrapolation of $f_\pi$

8 flavors

2× change across the region of rapid evolution of system.

12 flavors

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Is this consistent?
Extrapolation of $\langle \bar{\psi} \psi \rangle$

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**4× change**

**100× change**
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$4 \times$ change

$100 \times$ change

$\langle \bar{\psi} \psi \rangle \sim \frac{m^2 \pi f^2}{m_q}$
Extrapolation of $\langle \bar{\psi} \psi \rangle$

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100× change

$\langle \bar{\psi} \psi \rangle \sim \frac{m^2_\pi f^2_\pi}{m_q}$
Extrapolation of $m_\rho$

**8 flavors**

- $\beta = 0.58$
- $\beta = 0.56$
- $\beta = 0.54$

Near 2× change.

**12 flavors**

- $\beta = 0.47$
- $\beta = 0.475$
- $\beta = 0.48$
- $\beta = 0.49$

Close to 10× change. We don’t have measurements at stronger coupling. We’ll see more about it later.
The Coulomb gauge fixing method has larger finite V effects (Min Li, Lattice 2006)
Shape of the quark potential with 12 flavors

Quark potential at $m = 0.02$

The Coulomb gauge fixing method has larger finite $V$ effects (Min Li, Lattice 2006)
String tension vs. $m_q^2$

8 flavors

12 flavors

$\sigma(\beta = 0.48, m_q = 0) = 0.0043(15)$.

$\sigma(\beta = 0.49, m_q = 0) = 0.0014(17)$.

100× change.
Parity doubling

8 flavors

12 flavors

No visible parity doubling.

Parity doubling! Does it indicate finite volume effect or the system is above $T_c$?
Parity doubling is eliminated in a larger volume, which indicates that our system might also suffer from finite volume effects.

$m_\rho$ may drop in our 12 flavors simulations.
What about $T_c$?

- Expect $T_c \sim m_\rho$ with $N_f$ dependent coefficient.
- $N_f = 8$ has $m_\rho \simeq 0.4$ in weak coupling. And it is below $T_c$ since it has nonzero $\sigma$ and $m_\pi^2 \sim m_q$.
- $N_f = 12$ has $m_\rho \simeq 0.16$, which is $2.5 \times$ smaller. It can still be below $T_c$.
- Apparently below $T_c$ as shown by comparing $m_\pi^2 \sim m_q$ with $N_f = 4$ on two volumes.
Evolution of $\langle \bar{\psi}\psi \rangle$ at finite temperature ($N_\tau = 8$)

### 8 flavors

$\beta = 0.56$

- $v \simeq 0.02629(27)$
- $v \simeq 0.02793(52)$

### 12 flavors

$\beta = 0.49$

- $v \ll 0.02008(30)$
- $v < 0.01291(12)$

- $v = 0.02555(23)$ by linear extrapolation.
- $\beta = 0.49$ may be influenced by the transition.
- Distance from transition depends on the quark mass.

- Changed $N_\tau$ from 32 to 8 at fixed $\beta$.
- Ordered and disordered start at both $N_\tau$ values.
$m_\pi^2$ versus $m_q$ at finite temperature ($N_\tau = 8$)

### 8 flavors

- Changed $N_\tau$ from 32 to 8 at fixed $\beta$.
- Screening mass of $\pi$ at $N_\tau = 8$.
- Very different behavior — Not Goldstone Boson.

At $\beta = 0.56$, $m_q = 0.024$ point lies on both lines.

### 12 flavors

- $\beta = 0.54$
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- $\beta = 0.58$
- $24^3 \times 8, \beta = 0.56$

- $\beta = 0.45$
- $\beta = 0.46$
- $\beta = 0.47$
- $\beta = 0.475$
- $\beta = 0.48$
- $\beta = 0.49$
- $32^3 \times 8, \beta = 0.49$
Unrenormalized Polyakov loop

Clearly $\langle P \rangle \neq 0$ at $N_\tau = 8$

8 flavors

$\beta = 0.56, m_q = 0.008$

$24^3 \times 32$ $24^3 \times 8$

$0.00013(19)$ $0.05717(50)$

$\beta = 0.56, m_q = 0.016$

$24^3 \times 32$ $24^3 \times 8$

$0.00013(23)$ $0.04738(42)$

12 flavors

$\beta = 0.49, m_q = 0.01$

$32^3 \times 32$ $32^3 \times 8$

$0.00020(13)$ $0.07642(28)$

$\beta = 0.49, m_q = 0.02$

$16^3 \times 32$ $32^3 \times 8$

$-0.00031(36)$ $0.06799(18)$
Renormalized Polyakov loop should be small in confined phase, and large in deconfined.
Polyakov loop

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- We can renormalize it from the quark potential, since this gives self energy.
- At $N_\tau = 8$, renormalized Polyakov loops are about $1 \sim 2$.
- $N_\tau = 8$ is above $T_c$.
- At $N_\tau = 32$, we cannot prove that it is below $T_c$ from the Polyakov loops.
12 flavors results comparison with Deuzeman et al., 2009
arXiv:0904.4662v1

$m_q = 0.05$, spatial volume $16^3$, circles for $N_\tau = 16$ and crosses for $N_\tau = 8$.

- Bulk transition observed.
Weaker coupling side of the transition.

- Very small $\langle \bar{\psi} \psi \rangle$ at chiral limit.
12 flavors results comparison with Deuzeman et al., 2009
arXiv:0904.4662v1

Weaker coupling side of the transition.

- Very small \( \langle \bar{\psi}\psi \rangle \) at chiral limit.
- Our simulations run in smaller \( m_q \)'s.
- We studied more observables.
Summary

- 8 flavors is clearly in a chiral symmetry breaking phase with rapid crossover, which changes scale by $\sim 2\times$.
- 12 flavors clearly has a marked change in system.
  - Looks like approximately $\sim 10\times$ scale change.
  - Non-zero $f_\pi$, $m_\rho$, $\sigma$, etc. argues against conformal phase.
  - Significant differences between $N_\tau = 32$ and $N_\tau = 8$.
  - No evidence for $m_\pi^2 \sim m_q$ at $N_\tau = 8$, which implies $N_\tau = 8$ is above $T_c$.
  - Plausible arguments that it is chiral symmetry breaking phase with some finite volume effects.
  - Simulations with larger volumes at zero temperature should address remaining finite volume questions.

Acknowledgment

- Simulations are done using NYBlue BG/L and QCDOC at Columbia University and BNL.
- Many thanks to all RBC members and especially to Norman Christ for insightful discussions.