Gauge Theories in Finite Volume and the Conformal Window

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Alex Bazavov HISQ action (stout smearing is actually used)
Talk is based on new paper posted recently:

I will help to explain things to Referee B!
1. Overview of three coordinated projects of our LHC program
   - SU(3) color, fundamental rep, staggered Nf=4–20
   - 2-index symmetric representation with SU(3) color
   - Running coupling within and without the conformal window
     (talk by Kieran Holland)

2. Chiral symmetry breaking
   - Epsilon-regime, delta-regime, p-regime
   - Taste breaking at Nf=4,8,9,12 and staggered CHPT
   - New results at Nf=12 will be previewed but not released

3. Inside and above the conformal window
   - The importance of zero momentum dynamics at Nf=16
   - An alternative to Schrodinger functional method

4. Conclusions and Outlook
   - Prospects for model building ?
   - Can lattice studies be transformational ?
   - Peta-scale to exa scale ?
Phenomenology goal: nearly conformal gauge theory with minimal realization of the composite Higgs mechanism consistent with ElectroWeak Precision Data?

And they are fun field theories anyway!
Project 3: Important to complement the search for chirally broken phases with running coupling and beta function

Talk by Kieran Holland

Fundamental rep with N_f=4,8,9 should be similar
N_f=10,11,12 under study
N_f=12 border disputes, controversial

How to reach walking scale which is required by ElectroWeak Precision Tests?

Is 2-index symmetric rep nearly conformal?

DeGrand et al. (conformal?)
critically important in model building

would be Banks-Zaks FP
Probing technicolor theories with staggered fermions

Kieran Holland

Figure 1: The conformal window for $SU(N)$ gauge theories with $N_f$ techniquarks in various representations, from [3]. The shaded regions are the windows, for fundamental (gray), 2-index antisymmetric (blue), 2-index symmetric (red) and adjoint (green) representations.

1. Introduction

The LHC will probe the mechanism of electroweak symmetry breaking. A very attractive alternative to the standard Higgs mechanism, with fundamental scalars, involves new strongly-interacting gauge theories, known as technicolor [1, 2]. Such models avoid difficulties of theories with scalars, such as triviality and fine-tuning. Chiral symmetry must be spontaneously broken in a technicolor theory, to provide the technipions which generate the $W^\pm$ and $Z$ masses and break electroweak symmetry. Although this duplication of QCD is appealing, precise electroweak measurements have made it difficult to find a viable candidate theory. It is also necessary to enlarge the theory (extended technicolor) to generate quark masses, without generating large flavor-changing neutral currents, which is challenging.

Technicolor theories have lately enjoyed a resurgence, due to the exploration of various techniquark representations [3]. Feasible candidates have fewer new flavors, reducing tension with electroweak constraints. If a theory is almost conformal, it is possible this generates additional energy scales, which could help in building the extended technicolor sector. There are estimates of which theories are conformal for various representations, shown in Fig. 1. For $SU(N)$ gauge theory, if the number of techniquark flavors is less than some critical number, conformal and chiral symmetries are broken and the theory is QCD-like. For future model-building, it is crucial to go beyond these estimates and determine precisely where the conformal windows are. There have been a number of recent lattice simulations of technicolor theories, attempting to locate the conformal windows for various representations [4, 5, 6, 7, 8].

2. Dirac eigenvalues and chiral symmetry

The connection between the eigenvalues $\lambda$ of the Dirac operator and chiral symmetry breaking is crucial for understanding the conformal window. The eigenvalues are related to the quark masses, and deviations from the chiral symmetry can be detected through the spectrum of the Dirac operator.

Theory space with conformal windows

- **Project 1**: in fundamental rep with $N=3$ colors with $N_f=4, 8, 9, 10, 11, 12, 14, 16, 20$ flavors dynamical staggered
- **Project 2**: 2-index symmetric rep (sextet) $N=3$ colors and $N_f=2$ flavors dynamical overlap

We only run with $N=3$ colors

Running on CPU clusters and GPU clusters Very demanding
**GPU HARDWARE**

**GTX 280**
- Flops: single 1 Tflop, double 80 Gflops
- Memory 1GB, Bandwidth 141 GBs⁻¹
- 230 Watts, $350

**Tesla 1060**
- Flops: single 1 Tflop, double 80 Gflops
- Memory 4GB, Bandwidth 102 GBs⁻¹
- 230 Watts, $1200

**Tesla 1070**
- Flops: single 4 Tflops, double 320 Gflops
- Memory 16GB, Bandwidth 408 GBs⁻¹
- 900 Watts, $8000

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Zoltan Fodor
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Sandor Katz

CUDA code:
Kalman Szabo
Sandor Katz

also USQCD CPU cluster support

UCSD Tesla cluster
just funded by DOE
Chiral regimes to identify in theory space:

Goldstone dynamics is different in each regime

We study $\delta / \epsilon$ -regimes (RMT) and $p$-regime (probing chiral loops) complement each other

interpretation of rotator levels in $m_q \to 0$ limit:

$m_q = 0$

$m_q \neq 0$

Not to misidentify rotator gaps as evidence of chirally symmetric phase !!
One-loop expansion in our analysis of p-regime:

\[ M^2 = M^2 \left[ 1 - \frac{M^2}{8\pi^2 N_f F^2} \ln\left( \frac{\Lambda_3}{M} \right) \right], \]

\[ F_\pi = F \left[ 1 + \frac{N_f M^2}{16\pi^2 F^2} \ln\left( \frac{\Lambda_4}{M} \right) \right], \]

\[ \lambda = M L_s \]

\[ M_\pi(L_s, \eta) = M_\pi \left[ 1 + \frac{1}{2N_f} \frac{M^2}{16\pi^2 F^2} \cdot \tilde{g}_1(\lambda, \eta) \right], \]

\[ F_\pi(L_s, \eta) = F_\pi \left[ 1 - \frac{N_f}{2} \frac{M^2}{16\pi^2 F^2} \cdot \tilde{g}_1(\lambda, \eta) \right], \]

We use staggered action with stout smearing

Taste breaking included in staggered perturbation theory!

structure changing as Nf grows
Goldstone spectra

\[
\begin{align*}
\Delta(\xi_5) &= 0, \\
\Delta(\xi_\mu) &= \frac{8}{F^2}(C_1 + C_2 + C_3 + 3C_4 + C_5 + 3C_6), \\
\Delta(\xi_{5\mu}) &= \frac{8}{F^2}(C_1 + C_2 + 3C_3 + C_4 - C_5 + 3C_6), \\
\Delta(\xi_{\mu\nu}) &= \frac{8}{F^2}(2C_3 + 2C_4 + 4C_6).
\end{align*}
\]

\[\begin{align*}
\mathcal{L}^{(4)}_\chi &= \frac{F^2}{4} \text{Tr}(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - \frac{1}{2} B m_q F^2 \text{Tr}(\Sigma + \Sigma^\dagger) + V_\chi^{(6)} \\
M_\pi(T_a)^2 &= 2 B m_q + a^2 \Delta(T_a) + O(a^2 m_q) + O(a^4)
\end{align*}\]

\[
\left(\frac{m_{\pi^+}^{1\text{-loop}}}{2m}\right)^2 = \mu \left\{ 1 + \frac{1}{16\pi^2 f^2} \left( \frac{2}{N_F} \left[ \ell(m_{\eta'_V}^2) - \ell(m_{\pi_V}^2) \right] \\
+ \frac{2}{N_F} \left[ \ell(m_{\eta'_A}^2) - \ell(m_{\pi_A}^2) \right] + \frac{1}{2N_F} \ell(m_{\pi_1}^2) \right) \\
+ \frac{16\mu}{f^2} (2L_8 - L_5) (2m) + \frac{32\mu}{f^2} (2L_6 - L_4) (4N_F m) + a^2 C \right\}
\]
Random Matrix Theory tests in epsilon regime:

- Dirac spectrum
- Integrated eigenvalue distributions of RMT
  --> quartet degeneracy
  --> RMT
Addendum to Beijing preview on Nf=12:

Goldstone Spectrum $V=24^3 \times 32$
taste breaking like Nf=8

L=24  gap almost collapsed
good quartet degeneracies

1-loop $M_{pi}^2$ fit
parameters $B$, $F$
sensitive to $m_q$ range
fit like Nf=8

Nf=12 analysis is not released yet
This preview expands on our Beijing talks
Dirac Spectrum at $L=16$ over large beta range

massive non-pion Goldstones begin to separate from hadron spectrum very sensitive to $L$

$L=16 \rightarrow L=24$

$L=16$ large gap good quartet degeneracies

$L=24$ gap almost collapsed good quartet degeneracies

V=$20^3 \times 32$
Goldstones barely begin to peel off

$L=32$ should be definitive
Inside the conformal window  Nf=16 case study

Nf=16 is most accessible to analysis

What is the finite volume spectrum?

How does the running coupling $g^2(L)$ evolve with L?

From 2-loop beta function $g^* \approx 0.5$

$g^2(L) \rightarrow g^* \ , \ as \ L \rightarrow \infty$

Nontrivial small volume dynamics in QCD turns into large volume
dynamics around weak coupling fixed point of conformal window

At small $g^2(L)$ the zero momentum components of the gauge field
dominate the dynamics: Born–Oppenheimer approximation

Originally it was applied to pure-gauge system  Luscher, van Baal
SU(3) pure-gauge model: 27 inequivalent vacua

Low excitations of Hamiltonian (Transfer Matrix) scale with \( \sim g^{2/3}(L)/L \) will evolve into glueball states for large L

Three scales of dynamics on smallest scale WF is localized on one vacuum tunneling across vacua on second scale over the barrier: confinement scale (third)

\[
A_i(x) = T^a C_i^a / L \quad \text{ <-- zero momentum mode of gauge field}
\]

For \( SU(3) \), \( T_1 = \lambda_3/2 \) and \( T_2 = \lambda_8/2 \)

\[
V_{\text{eff}}^k(C^b) = \sum_{i>j} V(C^b [\mu_b^{(i)} - \mu_b^{(j)}]) - N_f \sum_i V(C^b \mu_b^{(i)} + \pi k) \quad \mu^{(1)} = (1,1,-2)/\sqrt{12} \quad \text{and} \quad \mu^{(2)} = \frac{1}{2}(1,-1,0)
\]

Effective potential shows the effects of massless fermions

Fermions develop a gap in the spectrum \( \sim 1/L \) \( k=(0,0,0) \) periodic \( k=(1,1,1) \) antiperiodic

van Baal
Quantum vacuum is at minimum of $V_{\text{eff}}(C)$ when massless fermions are turned on. Early work by van Baal, Kripfganz and Michael.

Fermions develop a gap $\sim \pi /L$ in the spectrum:

$k= (1,1,1)$ antiperiodic  minimal when $l = 0 \mod 2 \pi$  $A = 0$

$k= (0,0,0)$ periodic  minimal when $\vec{l} \neq 0$  nontrivial vacua

Polyakov loop distributions probe the vacua.

$k= (0,0,0)$ antiperiodic  $A = 0$ ($P_j = 1$)

$k= (0,0,0)$ periodic  $P_j = \exp(\pm 2\pi i /3)$

16$^4$ lattice simulation at $\beta = 18$

If there is strong coupling inside the conformal window, transition over the barrier into third regime (confinement in QCD) where this picture qualitatively changes.

\[ \mu_{\phi}^{(n)} C_{b} = 2\pi I /N \mod 2\pi \]

\[ V^k_{\text{eff}} = -N_f N V(2\pi I /N + \pi k) \]
2-stout, $N_f=16$, $12^2 \times 36$, $\beta=30.0$

2-stout, $N_f=16$, $8^3 \times 48$, $\beta=30.0$

Nf=16 inside conformal window: femto volume and tunneling volume

Nf=20 above conformal window: femto/tunnel volume and over barrier strong coupling volume

2-stout, $N_f=20$, $12^4$, $\beta=20.0$

2-stout, $N_f=20$, $12^8$, $\beta=20.0$

2-stout, $N_f=20$, $12^4$, $\beta=3.0$
Project 3: New method for running coupling and beta-function
Pilot study at \( N_f=16 \) talk by Kieran Holland also Bilgici et al

Why not Schrödinger Functional?

1. WL simple to implement with dynamical fermions
2. No staggered fermion boundary condition issues
3. Free of tunneling problems?
4. Good signal to noise ratio from smearing procedures

define renormalized coupling from second derivative of Wilson loops running with \( L \) if \( R/L \) is kept fixed:

\[
k \cdot g^2_R(L_0, \frac{R}{L_0}) = -R^2 \left. \frac{\partial^2}{\partial R \partial T} \ln \langle W(R, T; L_0, T_0) \rangle \right|_{T=R}
\]

\( k \) is geometric factor (cutoff dependent on lattice) defined from tree level relation with the bare coupling

\[
-R^2 \left. \frac{\partial^2}{\partial R \partial T} \ln \langle W(R, T; L_0) \rangle^{\text{tree}} \right|_{T=R} = kg^2_0
\]

Lattice implementation requires the study of the step function together with its cutoff dependence

Wilson loops could be replaced by Polyakov loop correlators in twisted volume (fermions?)
Conclusions and Outlook

- Our focus will shift to $N_f=10-16$ range (and beyond?)
  Is $N_f=12$ chirally broken?

- Zero-mode $\rightarrow$ Low lying glueball spectrum relative to mesons!

- $N_f=12$ might be close enough to realize walking technicolor

- What is the fate of the $N_f=2$ sextet model?

- Reliable lattice studies will be very demanding on computing

- Time to think about EW precision quantities (S/T/U)