Information and Computation of Complex Dynamical Systems

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1. Classical Computational Modeling
G. W. Leibniz: Knowledge Representation and Universal Computation

In his “mathesis universalis” G. W. Leibniz (1646-1716) designed a universal formal language (lingua universalis) to represent human thinking by calculation procedures (“algorithms”) and to implement them on mechanical calculating machines.

An “ars iudicandi” should allow every problem to be decided by an algorithm after representation in numeric symbols. An “ars inveniendi” should allow users to seek and enumerate desired data and solutions of problems. Struggle on preferences of values should be decided “by machines” (“ad abacos”).
Turing’s Theory of Computability

A Turing machine is a formal procedure, consisting of

a) a control box in which a finite program is placed,

b) a potential infinite tape, divided lengthwise into squares,

c) a device for scanning, or printing on one square of the tape at a time, and for moving along the tape or stopping, all under the command of the control box.

Every computable procedure (algorithm) can be realized by a Turing machine (Church’s thesis). Every Turing program can be simulated by a universal Turing machine (general purpose computer).
Computational processes operate on symbolic representations. They refer to a situation in the outside world and obey the correspondence theory of truth (A. Tarski):

If $R_1$ is an internal representation of real-world situation $X_1$, real-world operation $T$ (e.g., by hand) should produce the same real-world result $X_2$, whether performed in the real world or on the internal representation: $\text{decode (encode (T) (encode (X_1)))} = T(X_1) = X_2$. 

Computation and Representation
Representation and Situatedness

A classical robot needs a complete symbol representation of situation which must be updated if the robot’s position is changed. How can the robot handle incomplete knowledge of changing complex situations? How can it distinguish between reality and its relative perspective?

Situated agents (e.g., people) need no representations and updating. They look, talk and interact bodily (e.g., by pointing to things).
2. Natural Evolution of the Embodied Mind
Situation, Adaptation, and Behavior

Adaptive behavior in sudden situations (a) does not depend on formal representations and logical inferences, but on bodily interactions with a situation (e.g., looking, feeling, reacting).

Rational thoughts with logical representations (b) do not guarantee adaptive behavior.
Formal and Embodied Learning

Chess is a *formal game* with complete representations, precisely defined states, board positions, and formal operations.

Soccer is a *nonformal game* with skills depending on bodily interactions, without complete representations of situations and operations which are *never* exactly *identical* – as our life, only much more simple …
Human Cognition and Embodied Mind

Not only “low level” motor intelligence, but also “high level” cognition (e.g., categorization) emerge from complex bodily interaction with an environment by sensory-motor coordination.

An infant learns to categorize objects and to build up concepts by touching, grasping, manipulating, feeling, tasting, hearing, and looking at things, and not by symbolic representations. The categories are fuzzy and may be improved and changed during life.
Dynamics of the Human Brain

Behavior of human persons depends on brain dynamics. Perceptions, emotions, thoughts, and consciousness correspond to neural patterns, self-organizing by neurochemical interactions. Neuropsychology analyzes intentions and preferences as neural states of interacting neural areas.
Nonlinear Dynamics of Neural Clusters

Brain dynamics is not only determined by firing and non-firing of single neurons, but by synchronously firing neural clusters (cell assemblies), clusters of clusters etc. Neural clusters are also characterized by threshold values (control parameters) and order parameters (supervisors) for corresponding brain functions. Interactions of \( n \) clusters with a firing rate \( F_j \) \((1 \leq j \leq n)\) can be represented by a simplified nonlinear model

\[
\frac{dF_1}{dt} = + F_1 (1 - F_1) - \alpha F_2 - \alpha F_3 - \ldots - \alpha F_n
\]

\[
\frac{dF_2}{dt} = -\alpha F_1 + F_2 (1 - F_2) - \alpha F_3 - \ldots - \alpha F_n
\]

\[
\vdots
\]

\[
\frac{dF_n}{dt} = -\alpha F_1 - \alpha F_2 - \alpha F_3 - \ldots + F_n (1 - F_n)
\]

with parameter \( \alpha \) positive for inhibitory interactions of clusters.
The Human Self as Self-Referential Agent

The brain does not only map, interpret and evaluate impressions of the external world, but also of internal states of the whole organism. Many self-monitoring processes of the brain are unaware and unconscious. Self-consciousness is the result of conscious self-awareness, depending on feeling oneself.

The “self” is an order parameter, emerging from a recursive process of multiple self-reflections, supervising our conscious actions.
3. Technical Innovation of Cognitive Systems
Evolution as Standard for Robots and Computers

Quantum Physics (atoms, particles) \( \rightarrow \) QUANTUM COMPUTING (quantum gates, quantum logic)

Nanotechnology (nanomolecules) \( \rightarrow \) MOLECULAR COMPUTING (molecular switch)

Genetotechnology (DNA-strands) \( \rightarrow \) DNA-COMPUTING (DNA as information strand)

Evolutionary Biology (cells, organism) \( \rightarrow \) BIOCOMPUTING (Evolutionary algorithms)

Brain Research (neurons, synapses) \( \rightarrow \) NEUROCOMPUTING (neuronal nets)

Cognitive Science (thinking, emotions) \( \rightarrow \) SOFT COMPUTING (fuzzy logic, learning algorithms, affective computing)

Sociobiology (swarm intelligence) \( \rightarrow \) SOCIONICS (distributed AI, Multi-agents-systems)
Cellular organisms and cellular automata

Cellular automata are complex systems of finite automata ('cells'), with states (e.g., numbers) which change in dependence of neighboring cells according to simple local rules. There is no central processor, but self-organization.

Special cellular automata can reproduce themselves in sequential generations. Every computer can be simulated by an appropriate cellular automaton and vice versa (Church’s thesis).
With Simple Rules to Complex Structures

1-dimensional cellular automata with 2 states and 3 preceding cells are determined by $2^3 = 8$ rules.
Cellular automata simulate phase transitions and attractors like in nature: equilibria (1) and periodic oscillations (2), independent of initial conditions, chaos and turbulence (3) and growth of complex structures (4) with sensitive dependence on initial conditions ("butterfly effect").
Random, Noise, and Uncontrollable Growth

Different increasing complex and random patterns can be generated by the same simple rules of cellular automata with different initial conditions. In some cases, there is no finite program, in order to forecast the development of random patterns. The algorithmic information content is incompressible because of the computational irreducibility. Thus, the development is irreversible (Turing’s Halting Problem, Hilbert’s Entscheidungsproblem).
Cellular Growth and Electronic Circuits

The interaction of living cells and nano materials leads to tissue engineering, e.g., cells on nano fibres of synthetic materials (left) or on a nano-structuralized surface as material of implantation (right).

A future trend will be biohybride systems with combined electronic circuits and biological functions, in order to improve the interface of brains and machines (e.g., neurons of a rat’s brain on a silicon chip).
A CNN is a spatial network of locally-coupled cells, where each cell is a dynamical system (analog processor) with an input, an output, and a state evolving according to dynamical laws (analogic neural computer).
CNN Cell with its Associated State and Output Equation

State Equation: \[ \dot{x}_{ij} = f\left( x_{ij}, z_{ij}, u_{ij} \right) \]

Output Equation: \[ y_{ij} = g\left( x_{ij}, z_{ij}, u_{ij} \right) \]
Standard CNN and CNN Genes

A standard CNN with a 3 x 3 sphere of influence is completely specified by one threshold $z_{ij}$, nine control coefficients $b_{kl}$ (‘feedforward template’) and nine coefficients $a_{kl}$ (‘feedback template’):

Thus, the corresponding CNN is completely determined by a CNN gene of 19 real numbers.
CONTOUR EXTRACTION CNN

Task Prescription: Extract contours which resemble edges (resulting from big changes in gray level intensities) from gray-scale images.

Boundary Condition
Fixed: $x_{ij} = 0$, $u_{ij} = 0$  
($i^*j^*$ denotes boundary cells)

Initial State
$x_{ij}(0) = u_{ij} = $ input image

Example 1: Array Size = 30 x 48

Input Image

Initial State

Output Image

Example 2: Array Size = 256 x 256

$a = 0.5 \{ \text{sgn}(u_{ij} - u_{kl} - 0.18) - \text{sgn}(u_{ij} - u_{kl} + 0.18) + 1 \}$
Conway’s ‘game of life’ is a cellular automaton (CA) with simple local rules (e.g., birth, death by overcrowding, death by exposure, survival). The evolution of CA configurations corresponds to the CNN image flow.

For a game of life with binary states and 3 x 3 spheres of influence, there are $2^9 = 512$ input-out rules which must be realized by a CNN.
The CNN Universal Machine

A CNN Universal Machine is a CNN of cells with programming features:

- *local analog memory* for storing continuous external or internal signals
- *local logic memory* for storing *digital* external or internal signals
- *local control and communication circuitry*
- *global unit with communication channel to each cell and to outside communication.*

The prototype CNN universal chip consists of 16 x 16 cells where *any algorithm of logical operations* and CNN genes can be processed as in a *von Neumann general purpose computer.*
The CNN universal machine (implemented in a chip) is a universal Turing machine.

Proof: (1) The CNN GAME-OF-LIFE program can be executed on the CNN universal machine.

(2) The CA GAME-OF-LIFE is a universal Turing machine (Conway/Berlekamp 1982).

(1) and (2) \(\Rightarrow\) Theorem.
Simulation of Nonlinear PDEs via Autonomous CNNs

Nonlinear Partial Differential Equations (PDEs)

\[ \text{discretized system} \]
\[ \text{decomposed systems} \]
\[ \text{with local neighborhoods} \]

Autonomous CNN Equations

**Consequence:** In the majority of cases it is sufficient to consider computer simulations of autonomous CNNs in order to study PDEs, representing complex dynamical systems.
Systems Biology and Bioinformatics

Systems biology integrates the molecular, cellular, organic, human and ecological levels of life with models of complex systems. They are characterized by mathematical (nonlinear differential) equations.

In bioinformatics, mathematics and informatics grow together with biology, in order to explain and forecast the complexity of life.
In systems biology, computational modeling and simulation ("in silico experiment") and technology-driven high-throughput lab ("wet") experiments are combined to generate new knowledge, which is used to fine tune models and design new experiments.

Increasing accumulation of biological data ranging from DNA and protein sequences to metabolic pathways results in the development of computational models of cells, organs, and organisms with complex metabolic and gene regulatory networks.
The goal of systems biology is to develop models to describe and predict cellular behavior at the whole-system level. The genome project was still a reductionist research program with the automatic analysis of DNA-sequences by high speed supercomputers (e.g., 2000 bases per second).

The paradigm shift from molecular reductionism to the whole-system level of cells, organs and organisms needs an immense increase of computational capacity in order to reconstruct integrated metabolic and regulatory networks at different molecular levels and to understand complex functions of regulation, control, adaptation, and evolution (e.g., computational metabolic network of E. Coli with power law connection degree distribution and scale-free property).
Complexity of Genetic Regularity Networks

Machine learning algorithms are powerful tools for identifying causal gene regularity networks from observational gene expression data. Dynamic Bayesian network (DBN) algorithms (C++) infer cyclic feedback loops, strength and direction of regulatory influence:

Nodes represent genes, directed links represent conditional statistical dependence of the child node on the parent node. Parents may be activators (arrow), repressors (flat heads), or neutral. Search heuristics are e.g., genetic algorithms, simulated annealing, or greedy search.

In bioinformatics, true genetic causal systems are simulated by Gene Sim with gene expression data. They are compared with the recovered DBN networks. To evaluate the accuracy of a recovered network, the percentage of links in the causal network that also exist in the recovered network and the percentage of links in the recovered networks that do not exist in the true network are estimated.
Cognition in Technical Systems

In the research project „Cognition in Technical Systems“ (COTESYS), cognitive and life sciences, information processing and mathematical sciences, and engineering sciences work systematically together to explore cognition for technical systems.
Robotic agents cannot be fully programmed for every application. The program learns from causal interaction where to stand when taking a glass out of a cupboard, how to best grab particular kitchen utensils, where to look for particular cutlery, etc. This requires the control system to know the parameters of causal routines and to have models for how the parameters change the behavior.
The sensor data of a robot’s environment (“experience”) are stored in a relational database system (“memory”). The data in the database together with causal structure on domain relations (“cognitive category”) imply a joint probability distribution over relations in the activity domain. This distribution is represented using Markov logic, which allows inferring the conditional probability of logical (first-order) statements (“probabilistic estimation of possible worlds”).
Neural networks are complex systems of firing and non-firing neurons with topologies like brains. There is no central processor (‘mother cell‘), but a self-organizing information flow in cell-assemblies according to rules of synaptic interaction (‘synaptic plasticity‘).

Learning algorithms:
• supervised
• non-supervised
Complex Neural Networks and Attractors of Self-Organization

The dynamics of neural nets is modeled in the phase space of synaptic weights with trajectories converging to attractors (e.g., prototypes of patterns):

Example: Neural net is learning to distinguish between two types of sonar echoes ('time series') – mine or rock?
Neural Networks Embodied in a Cognitive Robot

A simple robot with diverse sensors (e.g., proximity, light, collision) and motor equipment can generate complex behavior by a self-organizing neural network:

In the case of collision, the connections between the active nodes of proximity and collision layer are reinforced by Hebbian learning: A behavioral pattern emerges!
Nonlinear Dynamics of Agents and Environment

Agents (e.g., robots, humans) and their environment can be modeled by dynamical systems. The behavior of an agent \( A \) and its environment \( E \) is described by time-depending differential equations of their state variables \( x_a \) and \( x_e \) (e.g., state of a body, mental state, state of an ecological niche).

The environment influences the agent through a sensory function \( S \), the agent influences its environment through a motor function \( M \). The interaction is modeled by coupling both differential equations

\[
\frac{dx_a}{dt} = A(x_a, S(x_e), u_a) \quad \frac{dx_e}{dt} = E(x_e, M(x_a), u_e)
\]

with parameters \( u_a \) and \( u_e \) not involved in the coupling (e.g., thresholds of the system).

The agent’s behavior is only simple in the case of linear situations. Complex nonlinear interactions lead to dynamical attractors of periodic, quasi-periodic, and chaotic attractor.
Embodied Robotics

A robot with visual, haptic, and motor systems (e.g., camera, gripper, wheels) has the task to collect certain objects and to bring them to a home base. Therefore, he/she/it must categorize conductive and non-conductive objects with strongly or slightly textured surfaces.

Sensory networks receive inputs from the sensors. These sensory networks are connected to attention and feature maps of corresponding networks which together with the effectors form an attentional sensory-motor loop, modulated by a value map, according to the robot’s task.
Self-Organization of Embodied Robotics

- visual system
  - visual sensors
  - visual attention map
  - visual feature map
  - effectors
  - haptic sensors
  - haptic attention map
  - haptic feature map
  - modulation
  - value map

- attentional sensory-motor loop
Self-Organization of Prerational Motor Intelligence

Walking is a complex bodily self-organization, largely without central control of brain and consciousness: Motor intelligence emerges without internal representations.

A robot walks down a shallow slope very natural and humanlike, only driven by system-environment interaction of gravity, inertia, and collision, rather than an internal central controller. It is a complex dynamical system, driven into the equilibrium of a limit cycle with steady periodic motion.
Cognitive states of persons depend on emotions. Robots (e.g., Mark II) can recognize emotional expressions of a human face (e.g., happiness, anger, aggression, surprise) with pattern recognition of a neural network and react by generating an appropriate facial expression (non-verbal communication) in proper time.
Complex Network of Emotions

Connectionistic models (e.g., Cathexis) connect complex emotions from basic emotion types \( p = 1, \ldots, P \) (e.g., fear, anger). Their intensity \( I_p(t) \) at time \( t \) depends on excitatory and inhibitory influences of other emotion types and elicitors \( i = 1, 2, 3, 4 \) (neural, sensorimotor, motivational, cognitive):

\[
I_p(t) = g(f(I_p(t-1)) + \sum_{i=1}^{4} \varepsilon_{pi} + \sum_{m=1}^{P} (\alpha_{pm} - \beta_{pm})I_m(t))
\]

with \( \varepsilon_{pl} \) (value of prototype \( p \) depending on elicitor \( l \)), \( \alpha_{pm} \) (excitatory influence of prototype \( p \) on prototype \( m \)), \( \beta_{pm} \) (inhibitory influence of prototype \( p \) on prototype \( m \)), \( f \) (control function of decreasing emotion intensity), and \( g \) (control function of emotion intensity between zero and its saturation value).

Connectionistic models are only behavioral. They must be embodied into, e.g., neurochemical systems to produce feelings.
Thoughts and emotions correspond to complex patterns of neural cell-assemblies generated by simple synaptic rules (e.g., Hebb’s rule). Although the local rules are computable, emerging patterns may be too complex to be forecast. Thus, an artificial computational system with the complexity of the human brain may be not always controllable (like a human brain).
Evolution of a Global Superbrain?

In a technical evolution, a *global communication network* (*World Wide Web*) is emerging with surprising similarity to *self-organizing neural networks* of the *human brain*. Its *increasing complexity* needs *intelligent strategies of information retrieval* and *learning algorithms*, according to the *synaptic plasticity* of a brain.
Complex Networks as Random Graphs

Complex Networks with no apparent design principles have been described as random graphs. They start with $N$ nodes, and connect every pair of nodes with probability $p$, creating a graph with approximately $pN(N-1)/2$ edges distributed randomly.

In most networks there is a relatively short path between any two nodes despite their often large size (“small-world property”). But, in complex systems of, e.g., molecular, cellular, social, or technological networks, we also observe emerging clusters of, e.g., molecular structures, cellular assemblies, social cliques and groups, wiring diagrams, and wireless patterns. What are the underlying organizing principles?
Clustering and Degree Distribution in Complex Networks

A node $i$ of a network has $k_i$ edges connecting it to $k_i$ other nodes. The total number of edges with the nearest neighbors in a cluster is $k_i(k_i-1)/2$. The clustering coefficient of node $i$ is the ratio between the number $E_i$ of actually existing edges and the total number, i.e.

$$C_i = \frac{2E_i}{k_i(k_i - 1)}$$

The clustering coefficient $C$ of the whole network is the average of all individual $C_i$’s. In a random graph, since the edges are distributed randomly, $C = p$. Further on, the majority of nodes has nearly the same degree (number) of edges without (local) structures. Therefore, the degree distribution $P(k)$ (i.e., probability that any node has $k$ edges) is a Poisson distribution. But, most realistic networks has a degree distribution with a power-law tail, i.e., $P(k) \sim k^{-\gamma}$ without a characteristic scale (scale-free networks), indicating highly organized structures developed in evolution, technology or society.
Information Complexity in the World Wide Web (WWW)

The WWW is the largest information network with web pages as nodes and hyperlinks as edges. The directed edges are characterized by two degree distributions of outcoming and incoming distributions with power-law tails:

\[ P_{out}(k) \sim k^{-\gamma_{out}} \quad P_{in}(k) \sim k^{-\gamma_{in}} \]

Example: a sample of 200 million web pages with \( \gamma_{out} = 2.72 \) and \( \gamma_{in} = 2.1 \)

Despite the large number of nodes, WWW displays the small-world property (e.g., as a sample of 800 million nodes with path length of around 19). Clustering coefficients need undirected edges. With modifications for bidirectional edges, one gets ca. \( C = 0.1078 \), orders of magnitude higher than \( C_{rand} = 0.00023 \) corresponding to a random graph of the same size and average degree.
The Internet links computers and other telecommunication devices. At the router level, the nodes are the routers, and the edges are their physical connections. At the interdomain level, each domain of hundreds of routers is represented by a single node with at least one route as connection with other nodes. At both levels, the degree distribution follows a power law (e.g. $\gamma_r^i \sim 2.3, \gamma_d^i \sim 2.2$) of a scale-free network (compare systems biology).

Measurements of the clustering coefficient deliver values between $C = 0.18$ and $C = 0.3$, to be compared with $C_{rand} = 0.0001$ for random networks. The average paths at the domain level ranges between 3.70 and 3.77 and at the router level it is 9 (small-world property).
Increasing Complexity of Information Retrieval

Information Retrieval in the WWW of today must test billions of websites with unstructuralized contents (left). In the future, websites will contain tags with elements of meaning which are automatically achieved, read, and understood by software agents (right).

Ontologies define the meaning of tags. Software agents create and use the semantics. The WWW becomes more intelligent by agent-based self-organization.
Many energy providers of central generators and decentralized renewable energy resources lead to power delivery networks with increasing complexity.

Smart grids mean the integration of the power delivery infrastructure with a unified communication and control network, in order to provide the right information to the right entity at the right time to take the right action. It is a complex information, supply and delivery system, minimizing losses, self-healing and self-organizing.
Complexity and Self-Organization of Cyber-Physical Systems

Classical computer systems separate physical and virtual worlds. Cyber-physical systems (CPS) observe their physical environment by sensors, process their information and influence their environment with actuators according to communication devices: CPS are embodied networks.

CPS is a complex system of many self-organizing net components, dramatically increasing the adaptability, autonomy, reliability and usability of automotive, aerospace, energy, healthcare, manufacturing, transportation, and consumer appliances.
4. Handling a World with Increasing Complexity
During evolution new forms of information storage have been developed:

- **genetic information**
- **neural information**
- **extrasomatic information**

In human beings $10^{10}$ bit **genetic information** are surpassed by $10^{14}$ bit **neural information**.

Since $10^3$ years mankind develops **extrasomatic information storage** (e.g., libraries, data bases, internet, robots) surpassing the **information capacity of single human brains**.
Information Complexity and 1/f-Noise

Complex patterns of signals and data correspond to a spectrum with frequency $f$, approximately proportional to $1/f^b(b>0)$, called 1/f – noise:
e.g., spectrum with white noise ($b=0$) with statistical independent and uncorrelated data (Gaussian distribution), pink noise ($b=1$), red noise ($b=2$), and black noise ($b=3$).

The degree of irregularity decreases with increasing $b$. Pink and red noise characterize self-organization of complex structures in evolution, technology, and society between complete randomness (white noise) and regularity (black noise):

Non-Gaussian distribution with power laws and scale-free networks
Computational Complexity

*Computability and decidability* can be measured with *computational degrees*:

- **Turing-computability:** computable by a *universal Turing machine* (Church’s thesis)

- **degrees of complexity and computational time:**
  - compute different computational times (i.e. number of elementary operations) depending on length of *n* inputs (e.g., *linear, quadratic, polynomial, exponential* functions of computational time)

- **P-problems:** computable by *deterministic* Turing-machines in *polynomial time*

- **NP-problems:** computable by *non-deterministic* Turing-machines in *polynomial time*

- **P ≠ NP:** \( P \subset NP \), but \( NP \subset P \)?

- **relative computability:** computable by *ψ-oracle machine*, i.e., among the instructions of a Turing-machine, there is an operation *ψ* with unknown computability

- **higher recursion theory:** computational degrees (*hierarchies*) of *arithmetical* and *analytical predicates*
According to Church’s thesis, every effective procedure can be simulated by a (universal) Turing machine. K. Gödel and R. Feynman already discussed the extension of Church’s thesis that every process in nature is equivalent to a kind of computation.

Actually, natural processes are modeled by dynamical systems with different degrees of computational complexity (e.g., deterministic and non-deterministic Turing machines, CA, CNN, quantum computers, relative computability).
Complexity

Computational Complexity
- computational time (e.g., P, NP-time)
- program size

Information Complexity
- white noise
- pink noise
- red noise
- black noise

Dynamical Complexity
- random
- chaos attractor
- quasi-periodic attractor
- periodic attractor
- fixed point attractor
The theory of nonlinear, complex systems has become by now a proven problem-solving approach in the natural sciences. And it is now also recognized that many if not most of our social, ecological, economical and political problems are essentially of a global, complex and nonlinear nature. And it is now further accepted than any holistic perspective of the human mind and brain can hardly be achieved by any other approach. In this wide-ranging, scholarly but very concise treatment, physicist, computer scientist and philosopher Klaus Mainzer discusses, in essentially non-technical language, the common framework behind these ideas and challenges. Emphasis is given to the evolution of new structures in natural and cultural systems and we are lead to see clearly how the new integrative approach can give insights not available from traditional reductionistic methods. The fifth edition enlarges and revises almost all sections and supplements an entirely new chapter on the complexity of economic systems.