Group Announcement Logic

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joint work with:

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Elevator pitch

Group Announcement Logic extends public announcement logic with:

\[ \langle G \rangle \phi : \ \text{"Group } G \text{ can make an announcement after which } \phi \text{ is true"} \]
Public Announcement Logic (Plaza, 1989)

\[ \varphi ::= p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \langle \varphi_1 \rangle \varphi_2 \]

\( \phi_1 \) is true, and \( \phi_2 \) is true after \( \phi_1 \) is announced

Formally:

\[ M = (S, \sim_1, \ldots, \sim_n, V) \quad \sim_i \text{ equivalence rel. over } S \]

\[ M, s \models K_i \phi \iff \forall t \sim_i s \ M, t \models \phi \]

\[ M, s \models \langle \phi_1 \rangle \phi_2 \iff M, s \models \phi_1 \text{ and } M|\phi_1, s \models \phi_2 \]

The model resulting from removing states where \( \phi_1 \) is false
Adding quantification: APAL

\[ M, s \models \langle \phi_1 \rangle \phi_2 \iff M, s \models \phi_1 \text{ and } M|\phi_1, s \models \phi_2 \]

Idea (van Benthem, Analysis 64, 2004): interpret the modal diamond as “there is an announcement such that..”

Arbitrary announcement logic (APAL) (Balbiani et al., TARK 2007):

\[ \varphi ::= p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \langle \varphi_1 \rangle \varphi_2 \mid \lozenge \varphi \]

\[ M, s \models \lozenge \varphi \iff \exists \psi M, s \models \langle \psi \rangle \varphi \]
Quantification in APAL

\[ M, s \models \lozenge \phi \iff \exists \psi \ M, s \models \langle \psi \rangle \phi \]

Note: the quantification includes announcements that cannot be truthfully made in the system
Quantification: announcements by an agent

$K_i \psi$
Quantification: announcements by an agent

\[ M, s \models \langle i \rangle \phi \iff \exists \psi \ M, s \models \langle K_i \psi \rangle \phi \]
Quantification: announcements by a group

\[ M, s \models \langle G \rangle \phi \iff \exists \{ \psi_i : i \in G \} \ M, s \models \langle \bigwedge_{i \in G} K_i \psi \rangle \phi \]

Group Announcement Logic (GAL):

\[ \phi ::= p \mid K_i \phi \mid \neg \phi \mid \varphi_1 \land \varphi_2 \mid \langle \varphi_1 \rangle \varphi_2 \mid \langle G \rangle \phi \]

Similar to *coalitional ability* operators of Coalition Logic (Pauly, 2002) and ATL (Alur, Henzinger, Kupferman, 1997), with *actions* = public announcements

But GAL is not a Coalition Logic
Example: The Russian Cards Problem

From a pack of seven known cards 0,1,2,3,4,5,6 Anne and Bill each draw three cards and Cath gets the remaining card. How can Anne and Bill openly inform each other about their cards, without Cath learning who holds any of their cards?
Example: **The Russian Cards Problem**

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**Formalisation:** 012\textsubscript{a} : "Ann has cards 0,1 and 2"

\[(\text{one}) \land_{ijk} (ijk_b \rightarrow K_{a}ijk_b) \quad (\text{two}) \land_{ijk} (ijk_a \rightarrow K_{b}ijk_a) \]

\[(\text{three}) \land_{q=0}^6 ((q_a \rightarrow \neg K_c q_a) \land (q_b \rightarrow \neg K_c q_b))\]
Example: The Russian Cards Problem

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Formalisation:

\[
\begin{align*}
(\text{one}) & \quad \wedge_{ijk}(ijk_b \to K_{a}ijk_b) & (\text{two}) & \quad \wedge_{ijk}(ijk_a \to K_{b}ijk_a) \\
(\text{three}) & \quad \wedge_{q=0}^6((qa \to \neg K_{c}qa) \land (qb \to \neg K_{c}qb))
\end{align*}
\]

Known solution: \(\text{anne} \equiv 012_{a} \lor 034_{a} \lor 056_{a} \lor 135_{a} \lor 246_{a}\)

solution: \(\text{bill} \equiv 345_{b} \lor 125_{b} \lor 024_{b}\)
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Formalisation: 012_a : ”Ann has cards 0,1 and 2”

\[ \text{(one)} \land_{ijk} (ijk_b \rightarrow K_a ijk_b) \quad \text{(two)} \land_{ijk} (ijk_a \rightarrow K_b ijk_a) \]
\[ \text{(three)} \land_{q=0} ((q_a \rightarrow \neg K_c q_a) \land (q_b \rightarrow \neg K_c q_b)) \]

Known solution: anne \equiv 012_a \lor 034_a \lor 056_a \lor 135_a \lor 246_a

known solution: bill \equiv 345_b \lor 125_b \lor 024_b

PAL: \langle K_a \text{anne} \rangle \langle K_b \text{bill} \rangle (\text{one} \land \text{two} \land \text{three})
Example: The Russian Cards Problem

From a pack of seven known cards 0,1,2,3,4,5,6 Anne and Bill each draw three cards and Cath gets the remaining card. How can Anne and Bill openly inform each other about their cards, without Cath learning who holds any of their cards?

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\begin{align*}
\text{(one)} & \quad \bigwedge_{ijk}(ijk_b \rightarrow K_{a}ijk_b) \\
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PAL: \(\langle K_{a}\text{anne}\rangle \langle K_{b}\text{bill}\rangle (\text{one} \land \text{two} \land \text{three})\)

GAL: \(\langle a\rangle \langle b\rangle (\text{one} \land \text{two} \land \text{three})\)
Quantification: sequences of announcements

APAL: $\Box\Box\phi \rightarrow \Box\phi$

announcing in sequence $\psi, \chi \iff \psi \land [\psi]\chi$

GAL: $\langle G'\rangle\langle G'\rangle\phi \rightarrow \langle G'\rangle\phi$?
Quantification: sequences of announcements

APAL: $\Diamond \Diamond \phi \rightarrow \Diamond \phi$ 

announcing in sequence $\psi, \chi \iff \psi \land [\psi] \chi$

GAL: $\langle G \rangle \langle G \rangle \phi \rightarrow \langle G \rangle \phi$?

$\bigwedge K_i \psi, \bigwedge K_i \chi \iff \bigwedge K_i \psi \land [\bigwedge K_i \psi] \bigwedge K_i \chi$
Quantification: sequences of announcements

APAL: $\Diamond\Diamond \phi \rightarrow \Diamond \phi$

announcing in sequence $\psi, \chi \iff \psi \land [\psi] \chi$

GAL: $\langle G \rangle \langle G \rangle \phi \rightarrow \langle G \rangle \phi$?

$\bigwedge K_i \psi, \bigwedge K_i \chi \iff \bigwedge K_i \psi \land [\bigwedge K_i \psi] \bigwedge K_i \chi$

not a group announcement
Quantification: sequences of announcements

APAL: \[ \Diamond \Diamond \phi \rightarrow \Diamond \phi \] announcing in sequence \[ \psi, \chi \] \iff announcing \[ \psi \land [\psi] \chi \]

GAL: \[ \langle G \rangle \langle G \rangle \phi \rightarrow \langle G \rangle \phi \]\[ \wedge K_i \psi, \wedge K_i \chi \] \iff \[ \wedge K_i \psi \land [\wedge K_i \psi] \wedge K_i \chi \]

not a group announcement

Theorem: Yes.
Quantification: sequences of announcements

\[ \langle G \rangle \langle G \rangle \phi \rightarrow \langle G \rangle \phi \]

\[ M, s \models \langle G \rangle \phi \iff \text{there is an announcement by } G, \text{ after which } \phi \]
Quantification: sequences of announcements

\[ \langle G \rangle \langle G \rangle \phi \rightarrow \langle G \rangle \phi \]

\[ M, s \models \langle G \rangle \phi \iff \text{there is an announcement by } G, \text{ after which } \phi \]
\[ \iff \text{there is a sequence of announcements by } G, \text{ after which } \phi \]
Example: **Russian Cards** (ctnd.)

\[ \langle K_{a}anne \rangle \langle K_{b}bill \rangle (one \land two \land three) \]

\[ \langle a \rangle \langle b \rangle (one \land two \land three) \]

\[ \langle ab \rangle (one \land two \land three) \]
Being able to without knowing it

\[ s \models \langle a \rangle p \land \neg K_a \langle a \rangle p \]
Being able to, knowing that, but not knowing how

\[ \phi = K_b q \land (\neg K_b p \lor \hat{K}_a (K_b p \land \neg K_b q)) \]

\[ s \models \langle K_a q \rangle \phi \rightarrow s \models \langle a \rangle \phi \]

\[ t \models \langle K_a p \rangle \phi \rightarrow t \models \langle a \rangle \phi \]

\[ s \models K_a \langle a \rangle \phi \]
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\[ s \models K_a \langle a \rangle \phi \]

\[ s \models \phi \]
Being able to, knowing that, but not knowing how

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\[t \models \langle K_a p \rangle \phi \rightarrow t \models \langle a \rangle \phi\]

\[s \models K_a \langle a \rangle \phi\]
Being able to, knowing that, but not knowing how

\[
\phi = \Box_b q \land (\neg \Box_b p \lor \Diamond_a (\Box_b p \land \neg \Box_b q))
\]

\[
\begin{align*}
 s \models \langle a \rangle \phi & \quad \text{and} \quad s \models \langle a \rangle \\
 t \models \langle a \rangle \phi & \quad \text{and} \quad s \models \Box_a \langle a \rangle \phi
\end{align*}
\]
Being able to, knowing that, but not knowing how

\( \phi = K_b q \land (\neg K_b p \lor \hat{K}_a (K_b p \land \neg K_b q)) \)

\[ s \models \langle a \rangle \phi \rightarrow s \models \langle a \rangle \phi \]

\[ t \models \langle a \rangle \phi \rightarrow t \models \langle a \rangle \phi \]
Being able to, knowing that, but not knowing how

\[ \phi = K_b q \land (\neg K_b p \lor \hat{K}_a (K_b p \land \neg K_b q)) \]

\[ s \models \langle K_a q \rangle \phi \rightarrow s \models \langle a \rangle \phi \]

\[ t \models \langle K_a p \rangle \phi \rightarrow t \models \langle a \rangle \phi \]

\[ s \not\models \langle K_a p \rangle \phi \]

\[ t \not\models \langle K_a q \rangle \phi \]
Being able to, knowing that, but not knowing how

\[ \phi = \mathcal{K}_b q \land (\neg \mathcal{K}_b p \lor \mathcal{K}_a (\mathcal{K}_b p \land \neg \mathcal{K}_b q)) \]

\[
\begin{align*}
s \models & \langle \mathcal{K}_a q \rangle \phi & \rightarrow & s \models \langle a \rangle \phi \\
t \models & \langle \mathcal{K}_a p \rangle \phi & \rightarrow & t \models \langle a \rangle \phi
\end{align*}
\]

The same announcement will not achieve the goal in both s and t - a does not know how to achieve it
Expressing knowledge *de dicto/de re*

<table>
<thead>
<tr>
<th>Ability</th>
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Expressing knowledge *de dicto/de re*

**Ability**

\[ \exists \psi \ s \models \langle K_a \psi \rangle \phi \]

\[ s \models \langle a \rangle \phi \]

\[ s \models K_a \langle a \rangle \phi \]

\[ \exists \psi \ s \models \langle K_a \psi \rangle K_a \phi \]

**Knowledge of ability, *de dicto***

\[ \forall s \sim_a t \ \exists \psi \ t \models \langle K_a \psi \rangle \phi \]

\[ s \models \langle a \rangle \phi \]

**Knowledge of ability, *de re***

\[ \exists \psi \ \forall s \sim_a t \ t \models \langle K_a \psi \rangle \phi \]

\[ ?? \]

\[ \exists \psi \ s \models \langle K_a \psi \rangle K_a \phi \]
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Depends on
(1) the fact that actions are \textit{announcements}
(2) the S5 properties

\exists \psi \ s \models \langle K_a \psi \rangle K_a \phi
Example: **Russian Cards** (ctnd.)

Ann and Bill *knows how* to execute a successful protocol:

$$\langle a \rangle K_a(two \land three \land \langle b \rangle K_b(one \land two \land three))$$
Some logical properties

\[[G \cup H] \phi \rightarrow [G][H] \phi\]

\[\langle G \rangle[G] \phi \rightarrow [G]\langle G \rangle \phi \quad \text{(Church-Rosser)}\]

\[\langle G \rangle[H] \phi \rightarrow [H]\langle G \rangle \phi \quad \text{(.generalised)}\]
Axiomatisation

$S5_n$ axioms and rules

$PAL$ axioms and rules

$[G]\phi \rightarrow \bigwedge_{i \in G} K_i \psi_i \phi$ where $\psi_i \in \mathcal{L}_{el}$

From $\phi$, infer $[G]\phi$

From $\phi \rightarrow [\theta] \bigwedge_{i \in G} K_i p_i \psi$, infer $\phi \rightarrow [\theta][G] \psi$

where $p_i \notin \Theta_\phi \cup \Theta_\theta \cup \Theta_\psi$

Theorem:
Sound & complete.
Model Checking

The model checking problem:

Given $M, s$ and $\phi$, does $M, s \models \phi$ hold?

**Theorem:**
The model checking problem is PSPACE-complete

(also extends to APAL)
Directions

• More general actions

• Coalition Announcement Logic
  
  • a coalition logic
  
  • there are announcements by G such that for all announcements by the other agents, ...

• Public Announcement Games -> Hans’ talk
For more details:
