Title: Workshop on Kernelization–Schedule

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Area: Algorithms / Complexity

Keywords: Preprocessing, kernelization, fixed-parameter tractability, parameterized algorithmics

Dates: November 8 - November 12 2010.
Monday: Fundamentals

- 9:00-10:00: Registration and coffee
- 10:00-10:15: Introduction by Mieke Schutte, manager of Lorentz Center
- 10:15-10:30: Workshop theme introduction (Fedor V. Fomin)
- 10:30-11:30: **Keynote Lecture 1**: Mike Fellows

*Kernelization is About Efficiently Available Structural Relationships = A Whole New World of Computational Complexity Research*

The equivalence of FPT parameterized complexity and P-time kernelization has for the first time provided a framework for the mathematically-disciplined investigation of pre-processing. This equivalence is expressed: a parameterized problem is FPT if and only if there is a P-time algorithm transforming an instance \((x, k)\) to an equivalent instance \((x', k')\), where \(|x'|\) and \(k'\) are bounded by a function of \(k\). This framework is “intrinsically parameterized” in the sense that the relevant questions are difficult to formulate in the classical setting.

Regarding kernels, the size of \(x'\) is only one possible important measure. Hence, the notion of kernelization generalizes. For example, it can be useful to kernelize to \((x', k')\) where the treewidth of \(x'\) is bounded by a function of \(k\). Better yet if the treewidth bound is constructively delivered in P-time by a bounded width tree decomposition. A well-known example of this possibility is the P-time algorithm that either produces a cycle of length at least \(k\), or produces a tree-decomposition of width at most \(k\), thus kernelizing the parameterized LONG CYCLE problem to bounded treewidth, along with a witnessing tree decomposition.

An even more general perspective is to consider the efficient algorithmic effectivization of existential parameterized structural relationships. This perspective displays elegant connections between kernelization and other themes of algorithms research, such as FPT approximation. It also reveals a host of natural, intrinsically parameterized kinds of complexity questions that have barely begun to be explored.

- 11:30-12:00: Coffee
- 12:00-12:30: Iyad Kanj

*On the independence number of graphs with maximum degree 3*

Let \(G\) be an undirected graph with maximum degree at most 3 such that \(G\) does not contain certain graphs as a subgraph. We prove that the independence number of \(G\) is at least \(n(G)/3 + nt(G)/42\), where \(n(G)\) is the number of vertices in \(G\) and \(nt(G)\) is the number of nontriangle vertices in \(G\). This bound is tight as implied by the well-known tight lower bound of \(5n(G)/14\) on the independence number of triangle-free graphs of maximum degree at most 3.

We show an algorithmic application of the aforementioned combinatorial result to the area of parameterized complexity. We present a linear-time kernelization...
algorithm for the independent set problem on graphs with maximum degree at most 3 that computes a kernel of size at most $420k/141 < 3k$, where $k$ is the given parameter. This improves the known $3k$ upper bound on the kernel size for the problem.

- 12:30-14:00: Lunch
- 14:00-14:30 Hadas Shachnai

**Approximative Kernelization: On the Trade-off between Fidelity and Kernel Size**

Traditionally, kernelization rules have fidelity 1 with respect to lifting an approximate or exact solution for the kernelized instance back to a solution for the original instance. This strong guarantee is at the heart of FPT algorithms that use kernelization. In this work we introduce a definitional framework for *approximative kernelization*, where one can trade off fidelity versus kernel size. Specifically, we define the concept of $\alpha$-fidelity kernelization, which guarantees an $\alpha$-approximation to the optimum when lifting back to the original instance, for any $\alpha \geq 1$. The special case where $\alpha = 1$ is simple kernelization. This enables to improve the best known kernel sizes for FPT algorithms, when we require that the output is an approximate solution for the problem. We demonstrate the tradeoff between kernel size and fidelity on the vertex cover problem, where $\alpha$ can take any value in $[1, 2)$.

- 15:30 -16:00: Coffee
- 16:00 -17:00: **Keynote Lecture 2**: Gregory Gutin

**Constraint Satisfaction Problems Parameterized Above or Below Tight Bounds.**

In the well-know problem Max-Sat, where given a CNF formula $F$ (with $n$ variables and $m$ clauses) and asked to determine the maximum number of clauses of $F$ that can be satisfied by a truth assignment. This maximum number will be denoted by $\text{sat}(F)$.

In the standard parametrization of Max-Sat, the parameter $k$ is the number of satisfied clauses, and it’s very easy to obtain a linear kernel for the standard parameterization of Max-Sat. Mahajan and Raman (1999) introduced a more useful parameterization: decide whether $\text{sat}(F) \geq m/2 + k$. Here $m/2$ is a tight lower bound on $\text{sat}(F)$: always $\text{sat}(F) \geq m/2$ and for an infinite number of CNF formulas $F$, $\text{sat}(F) = m/2$. Mahajan and Raman obtained a linear kernel for this parameterization of Max-Sat and this result appears to be the first on kernelization of problems parameterized above or below tight bounds.

The aims of the talk are to overview kernelization results obtained for CSPs parameterized above or below tight bounds and to give some ideas of the proof.
techniques. We consider the following problems:

- **Max-Sat above** \(m/2\). We’ll consider the original result by Mahajan and Ramanand and its recent improvement by Crowston, Gutin, Jones and Yeo.

- **Max-UCF-Sat above** \(pm\), where \(p = (\sqrt{5} - 1)/2\) and a CNF formula \(F\) is unit-conflict free (UCF) if it has no pairs of unit clauses, one of which constains a variable and the other its negation. Solving a conjecture of Mahajan and Raman (1999) in affirmative, recently Crowston, Gutin, Jones and Yeo obtained a linear-order kernel (i.e., linear number of variables) for this problem.

- **Max-2-Sat below** \(m\). This problem was proved to be fpt by Razgon and O’Sullivan (2009); we do not know whether the problem admits a polynomial kernel.

- **Max-r-Sat above** \(m(2^r - 1)/2^r\). The parameterized complexity of this problem was stated as an open problem by Mahajan, Raman and Sikdar (2006, 2009) and a quadratic kernel was obtained by Alon, Gutin, Kim, Szeider and Yeo (2010). This was improved to an \(O(k \log k)\)-kernel by Crowston, Gutin, Jones, Kim and Ruzsa (2010) and to an \(O(k)\)-order kernel by Kim and Williams (2010).

We’ll also consider Max-Lin-2 above average (AA), Max-r-CSP AA, and binary and ternary permutation CSPs AA and possibly other problems.

- **17:00-17:30** : Henning Fernau
  
  *Can memoization be used for parameterized problems without small induced kernels?*

  Besides certain mathematical curiosity, there are several algorithmic reasons why people like (induced) kernels of linear size, amongst those the fact that this enables the use of memoization to speed up search tree algorithms, or an easier use of Measure & Conquer techniques for the analysis of such kind of (parameterized) algorithms. We will discuss with various examples (mainly of vertex cover variants) the question whether small induced kernels are really necessary for using memoization. It will become clear that this is not always the case, contrary to earlier stated beliefs. We can interpret our observations also in a different manner, leading to the question if we should always insist on polynomial time for computing ‘kernels’.  

- **17:30-19:30** : Wine and Cheese welcome programme
Tuesday: Applications
This day we focus on kernelization algorithms for concrete problems from various applications.

- 09:00-10:00 : Keynote Lecture 3: Jiong Guo
  *Kernelization for graph-modeled data clustering*
  The graph-modeled data clustering problems ask to modify a given graph into a union of dense subgraphs and find applications in many application fields such as computational biology and data mining. In this talk, I will survey several techniques for designing kernelization algorithms for such clustering problems and provide concrete applications of these techniques.

- 10:00-10:45 : Keynote Lecture 4: Dániel Marx
  *What’s next? Reductions other than kernelization.*
  Kernelization is perhaps the most basic technique for proving fixed-parameter tractability. Research on kernelization discovered beautiful methods and provided information on the limits of this technique. The talk is centered around the following question: what other techniques would benefit from such a systematic investigation of upper and lower bounds?

- 10:45-11:15: Coffee

- 11:15-12: 15 : Keynote Lecture 5: Daniel Lokshtanov
  *Protrusions and Finite Integer Index in Kernelization.*
  A $r$-protrusion in a graph $G$ is a vertex set $S$ such that $\text{tw}(G[S]) \leq r$ and there are at most $r$ vertices in $S$ with neighbours outside of $S$. For many optimization problems on graphs one can employ the following reduction rule: when a sufficiently large $r$-protrusion is found, replace it with an equivalent $r$-protrusion of smaller size. In this talk we will cover in detail how such a reduction is performed, and consider concrete problems where protrusion reduction is infeasible.

- 12:15-14:00 Lunch

- 14:00-15:30 : Research Time. Participants will work on open problems.

- 15:30-16:00 : Coffee

- 16:00 : Open problems session by Saket Saurabh. Participants will present open problems to work on during the meeting and later.
Wednesday: Lower bounds

We discuss and review lower bound results.

- 09:00-10:00: **Keynote Lecture 6**: Bart Jansen
  
  **Kernelization Lower Bounds: Review of existing techniques and the introduction of cross-composition**
  
  This presentation focuses on super-polynomial lower bounds on kernel sizes. The first half of the talk treats the existing techniques for proving such lower bounds. We start by considering OR-compositions, which allow us to prove that \( k \)-Path does not admit a polynomial kernel unless the polynomial hierarchy collapses. We then show that the framework can be extended by using polynomial-parameter transformations, which leads to a kernel lower bound for Disjoint Cycles by using the intermediate problem Disjoint Factors. Without going into details we discuss several other applications of this framework to obtain lower bounds for Connected Vertex Cover, Small Universe Set Cover and Small Universe Hitting Set. In the second half of the talk we introduce the new technique of cross-composition. We discuss how cross-composition generalizes and extends the two existing techniques, and give an application of cross-composition by proving that the Clique problem parameterized by size of a vertex cover of the graph does not admit a polynomial kernel (unless ..).

- 10:00-10:30: Michał Pilipczuk
  
  **Kernelization hardness of connectivity problems**
  
  OR-distillation technique, introduced by Fortnow and Santhanam (STOC’08) and Bodlaender et al. (ICALP’08), led to a number of negative results on the existence of polynomial kernels. One of the first problems shown to be hard to kernelize was the \( k \)-path problem. In the proof the composition algorithm was simply taking the disjoint sum of instances, perfect for the connectivity condition involved. Therefore, questions about kernelization of problems that involve connectivity conditions are likely to be negative. I would like to present an elegant technique of encapsulating such conditions in a destillable arbitral problem, Colourful Graph Motif, and thus showing hardness of kernelization. As examples, I will prove that Connected Dominating Set and Steiner Tree do not admit a polynomial kernel even in 2-degenerate graphs. These results come from the paper ”Kernelization hardness of connectivity problems in d-degenerate graphs”, presented during WG 2010.

- 10:30-11:00: Coffee

- 11:00-12:00: **Keynote Lecture 7**: Holger Dell
  
  **Satisfiability Allows No Nontrivial Sparsification Unless The Polynomial-Time Hierarchy Collapses**
  
  In this talk, we discuss tight lower bounds on the absolute size of kernels for certain generic FPT problems under complexity theoretic assumptions. In particular, for any \( d \geq 2 \) and \( \epsilon > 0 \), the vertex cover problem in \( d \)-uniform hypergraphs does not have kernels consisting of \( O(k^{d-\epsilon}) \) edges unless coNP is in NP/poly. For \( d \geq 3 \),
this transfers to a bound of $O(n^{d-\epsilon})$ in the satisfiability problem for $n$-variable $d$-CNF formulas. We discuss this result, the key components of its proof, and a connection between probabilistically checkable proofs and kernelization.

• 12:00-13:30 : Lunch
• 13:30-15:00 : Research Time: Participants will work on open problems.
• 15:00-15:30 : Coffee
• 15:30-16:30 : Panel discussion on the future of the Kernelization by Saket Saurabh
• 16:45 : Departure to boat by bus
• 17:00-21:00 : Boat trip
• 21:00 : Departure by bus to Leiden Central Station, Hotel Van der Valk and Lorentz Center
Thursday: Meta-results

- 09:00-10:00: **Keynote Lecture 8**: Stephan Kratsch
  
  *Polynomial kernels for constant-factor approximable problems*
  
  It is known that many constant-factor approximable (APX) problems also admit polynomial kernels (i.e., their std. parameterizations do). However, examples like Bin Packing or Connected Vertex Cover rule out the possibility that all APX problems might have polynomial kernels. We will consider syntactical subclasses of APX, where all problems provably admit polynomial kernels. Furthermore, we will consider some Boolean CSP variants and relate approximability and admittance of polynomial kernels, depending on the constraint language.

- 10:00-10:30: Magnus Wahlström
  
  *Parameterized complexity and kernelizability of Max Ones and Exact Ones problems*
  
  We consider the FPT and kernelization properties of constraint Max Ones problems (i.e., the CSP generalization of problems such as Independent Set), and the kernelization properties of constraint Exact Ones problems (i.e., the CSP generalization of Weighted \(k\)-SAT). For the latter, a characterization of the problems as being either FPT or \(W[1]\)-hard was previously produced by Dániel Marx.

  We present the resulting classification, which divides the Max Ones problems into five levels: Polynomial time, polynomial kernel, FPT but kernelization hard, \(W[1]\)-hard, and finally \(NP\)-complete for constant parameter values. In particular, we observe the existence of a maximization problem which is FPT but has no polynomial kernel, something that is otherwise not so common.

  To the extent of time, we will also say something of the results on the Exact Ones side. (Joint work with Stefan Kratsch and Dániel Marx.)

- 11:00-12:00: **Keynote Lecture 9**: Dimitrios Thilikos
  
  *Graph Minors and Kernelization*
  
  We present a series of recent results that use techniques from the Graph Minors series in order to derive linear or polynomial size kernels for generic families of problems on minor closed graph classes. We reveal a strong connection between bidimensionality theory and the existence of linear kernels. Also, we describe how to use the irrelevant vertex technique for the dominating set problem and its variants on \(H\)-minor free graphs.

- 12:00-12:30: Christophe Paul
  
  *The use of modular decomposition in kernelization*
  
  Modular decomposition is a graph decomposition technique which appears in the early 70’s. Since then it has been used in many different contexts of combinatorics and algorithmic theory. More recently, modular decomposition based reduction rule leads to polynomial kernelizations for a number of parameterized problems. In
this talk we will first briefly introduce the concept of modular decomposition. We’ll then present an overview of its recent use in the context of kernelization. We’ll end with a series of questions around the application of modular decomposition (or its generalizations) in kernelization.

• 12:30-14:00 : Lunch
• 14:00-15:30 : Informal discussions in groups.
• 15:30-16:00 : Coffee
• 16:00 : **Keynote Lecture 10**: Fedor V. Fomin

*More on Protrusions*

Informally, a protrusions in a graph is a subgraph of small treewidth that is separated from the remaining part of the graph by a small set of vertices. Protrusions were used to obtain linear and polynomial kernels on topological graphs, like planar graphs, graphs of bounded genus, or graphs excluding some fixed graph as a minor. In this talk we discuss new applications of protrusions in kernelization of problems on “non-topological” graphs, like graphs of bounded degree, or more generally, graphs containing no claw $K_{1,t}$ as an induced subgraph, and on general graphs.

• 17:00-17:30: René van Bevern

*Kernelization Through Approximation and Tidying*

We introduce the NP-hard graph-based data clustering problem \emph{s-Plex Cluster Vertex Deletion}, where the task is to delete at most $k$ vertices from a graph so that the connected components of the resulting graph are $s$-plexes. In an $s$-plex, every vertex has an edge to all but at most $s - 1$ other vertices; cliques are 1-plexes. We propose a new method for kernelizing vertex deletion problems in undirected graphs. The corresponding kernelization method of “approximation and tidying” exploits polynomial-time approximation results and thus provides a useful link between approximation and kernelization. We illustrate our method by presenting data reduction rules that, in $O(ksn^2)$ time, transform an \emph{s-Plex Cluster Vertex Deletion} instance with $n$ vertices into an equivalent instance comprising $O(k^2s^3)$ vertices.
Friday: Recent developments
A Special Session on Multi-Cuts

- 09:00-10:00: Dániel Marx/Stéphan Thomassé
  **Multi-Cut is FPT - I**
  In the multi-cut problem we are given a graph $G$, $t$ pair of terminals $(s_i, t_i)$ and a positive integer $k$. Objective is to find a set of $k$ edges (vertices) such that after its deletion there is no path between any pair of given terminals. We show that this problem is FPT and thereby resolving one of the most outstanding open problem in the area. This has been shown independently by the team of Dániel Marx and Igor Razgon and by the team of Stéphan Thomassé, Jean Daligault and Nicolas Bousquet.

- 10:00-10:30: Coffee

- 10:30-11:30: Dániel Marx/Stéphan Thomassé
  **Multi-Cut is FPT - II**

- 11:30-12:00: Mathias Weller
  **Two-Layer Planarization Parameterized by Feedback Edge Set**
  Given an undirected graph $G$ and an integer $k$, the NP-hard 2-Layer Planarization problem asks whether $G$ can be transformed into a forest of caterpillar trees by removing at most $k$ edges. Since transforming $G$ into a forest of caterpillar trees requires breaking every cycle, the size $f$ of a minimum feedback edge set is a natural parameter with $f < k$. We improve on previous fixed-parameter tractability results with respect to $k$ by presenting a problem kernel with $O(f)$ vertices and edges and a new search-tree based algorithm, both with about the same worst-case bounds for $f$ as the previous results for $k$, although we expect $f$ to be smaller than $k$ for a wide range of input instances.

- 12:00-12:30: Neeldhara Misra
  **Expansions for Reductions**
  The $c$-expansion lemma is a natural generalization of Hall’s theorem, which provides a necessary and sufficient condition for a bipartite graph to admit a perfect matching. It turns out that there are many problems for which polynomial kernels may be obtained by applications of this lemma. In particular, we discuss how the expansion lemma plays a crucial part in the kernelization procedure for the problem of finding a hitting set for minor models of the $\theta_c$ graph. The lemma is used in obtaining an equivalent instance where maximum degree is bounded by a polynomial function in the parameter.

- 12:30-14:00: Lunch

- 14:00-15:00: **Keynote Lecture 11**: Hans Bodlaender
  **Kernels and nonstandard parameters**
  Many results on kernelization and fixed parameter tractability look at problems
with a 'standard' parameterization: the parameter is the target value of the optimization problem, e.g., the 'k' in: does G have a vertex cover of size at most 'k'? In this talk, a number of results on kernelization are surveyed where problems are considered with a nonstandard parameterization: e.g., find a vertex cover of a graph that has a feedback vertex set of size k. We look at parameters like the size of a minimum vertex cover or feedback vertex set, and problems like vertex cover, clique, treewidth, graph coloring, and weighted variants of these and ask if the problem has a kernel of size, polynomial in the considered parameter.

Several interesting insights on the complexity of these problems will be obtained. An example is a known heuristic for treewidth for which we now can show that it gives a cubic kernel for treewidth parameterized by vertex cover. Another example is that vertex cover (or, equivalently, clique or independent set) has a cubic kernel when parameterized by the size of a feedback vertex set; when we look at the weighted variant of this problem, no polynomial kernel exists unless NP is a subset of coNP/poly. This is joint work with Bart Jansen and Stefan Kratsch.

• 15:00-15:30 : Summary and Closing remarks by Saket Saurabh
• 15:30 : Conclusion with Coffee