

Qualitative and quantitative aspects of expectation-based emotions: a logical approach

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Abstract. I will first propose a logical representation language which allows to represent the two primitive components of an expectation: the value of the goal and the strength of the belief. Then, I will provide a formal analysis of the intensity of expectation-based emotions such as hope, fear, disappointment and relief.

1 Introduction

The aim of this paper is to propose a logical formalization of expectation-based emotions such as hope, fear, disappointment and relief. Following [7], an expectation-based emotion is an emotion that an agent experiences when having either a positive or a negative expectation about a certain fact φ , that is when: *believing that φ is true with a certain strength, but envisaging the possibility that φ could be false, and either,*

- *having the goal that φ is true (positive expectation) or,*
- *having the goal that φ is false (negative expectation).*

Section 2 is devoted to present a logical representation language for the formalization of expectation-based emotions. I will explain the syntax and the semantics of this logic which allows to represent different kinds of an agent's mental states such as knowledge, graded belief, graded goal, and the notion of epistemic action (as the action of learning that a certain fact φ is true.) In Section 3, the logical framework of Section 2 will be applied to the formalization of expectation-based emotions and of their intensity. I will first consider the concepts of hope and fear. Then, I will consider emotions based on the invalidation of a (positive or negative) expectation: disappointment and relief. In Section 4 I will discuss some related works in the area of computational modeling of emotions emphasizing that current logical models of emotions mostly consider qualitative aspects of emotions, not giving much attention to quantitative aspects related to emotion intensity.

2 A dynamic logic of knowledge, graded beliefs and graded goals

This section presents the syntax and the semantics of the logic DL-KGBG. A complete axiomatization as well as a decidability result for this logic are given in [18].

2.1 Syntax

Assume a countable set of atomic propositions $Prop = \{p, q, \dots\}$ and a finite set of natural numbers $Num = \{x \in \mathbb{N} : 0 \leq x \leq \max\}$ with $\max \in \mathbb{N}$. The language \mathcal{L} of DL-KGBG is defined by the following grammar in Backus-Naur Form (BNF):

$$\begin{aligned} Act : \alpha &::= * \varphi \\ Atm : \chi &::= p \mid \text{after}_{\epsilon|\alpha} \mid \text{exc}_h \mid \text{undes}_h \\ Fml : \varphi &::= \chi \mid \neg \varphi \mid \varphi \wedge \varphi \mid [K]\varphi \mid [\alpha]\varphi \end{aligned}$$

where p ranges over $Prop$, h ranges over Num , α ranges over Act , and ϵ ranges over the set of epistemic action sequences Act^* . The other Boolean constructions \top , \perp , \vee , \rightarrow and \leftrightarrow are defined from p , \neg and \wedge in the standard way. I define the set of objective facts Obj as the set of all Boolean combinations of atomic propositions.

An epistemic action $*\varphi$ in Act is the mental action (or process) of learning that φ is true. As I will show in Section 2.2, technically this amounts to an operation of beliefs' conditionalization in Spohn's sense [26]. Act^* is the set of epistemic action sequences. An epistemic action sequence is of the form $\alpha_1; \dots; \alpha_n$ where “;” stands for concatenation. I note nil the empty sequence of epistemic actions.

Atm contains special constructions of different types which are used to represent both the objective world as well as the mental state of an agent. The formula $\text{after}_{\epsilon|\alpha}$ is a special atom that is read “after the sequence of epistemic actions ϵ , the epistemic action α will occur”. The atomic formula $\text{after}_{nil|\alpha}$ specifies the *actual* epistemic action that is going to occur next. Therefore, I define:

$$\text{occ}_\alpha \stackrel{\text{def}}{=} \text{after}_{nil|\alpha}$$

where occ_α has to be read “the epistemic action α is going to occur”.

The special atoms exc_h are used to identify the degree of *plausibility* of a given world for the agent. Starting from [14], ranking among possible worlds have been extensively used in belief revision theory. I here use the notion of plausibility first introduced by Spohn [26]. Following Spohn's theory, the worlds that are assigned the smallest numbers are the most plausible, according to the beliefs of the individual. That is, the number h assigned to a given world rather captures the degree of *exceptionality* of this world, where the exceptionality degree of a world is nothing but the opposite of its plausibility degree (*i.e.*, the exceptionality degree of a world decreases when its plausibility degree increases.) Therefore, formula exc_h can be read alternatively as “the current world has a degree of exceptionality h ” or “the current world has a degree of plausibility $\max - h$ ”. The special atoms undes_h are used to identify the degree of *desirability* (or the degree of *goodness*) of a given world for the agent. Contrary to plausibility, the worlds that are assigned the biggest numbers are the most desirable for the agent. The degree of undesirability (or degree of a badness) of a given world is the opposite of its desirability degree. Therefore, formula undes_h can be read alternatively as “the current world has a degree of desirability h ” or “the current world has a degree of undesirability $\max - h$ ”.

The formula $[\alpha]\varphi$ has to be read “after the occurrence of epistemic action α , φ will be true”, while $[K]\varphi$ has to be read “the agent knows that φ is true”. This concept

of knowledge is the standard S5-notion, partition-based and fully introspective, that is commonly used both in computer science [11] and economics [4]. As I will show in the next Section 2.2, the operator [K] captures a form of ‘absolutely unrevisable belief’, that is, a form of belief which is stable under belief revision with any new *evidence*. A similar property for the notion of knowledge has been advanced by the so-called *defeasibility* (or *stability*) *theory of knowledge* [15, 27, 25]. According to this theory, a given piece of information φ is part of the agent’s knowledge only if the agent’s justification to believe that φ is true is sufficiently strong that it is not capable of being defeated by evidence that the agent does not possess. As pointed out by [5], two different interpretations of the term ‘evidence’ have been given in the context of this theory, each giving a different interpretation of what *knowledge* is. The first one defines knowledge as a form of belief which is stable under belief revision with ‘any piece of *true* information’, while the second one gives a stronger definition of knowledge as a form of belief which is stable under belief revision with ‘any piece of information’. The concept formalized by the operator [K] captures this latter form of knowledge in a stronger sense.

2.2 Semantics

Definition 1 (Model). *DL-KGBG-models are tuples $M = \langle W, \sim, \kappa_{\text{exc}}, \kappa_{\text{undes}}, \mathcal{H}, \mathcal{V} \rangle$ where:*

- W is a nonempty set of possible worlds or states;
- \sim is an equivalence relation between worlds in W ;
- $\kappa_{\text{exc}} : W \rightarrow \text{Num}$ and $\kappa_{\text{undes}} : W \rightarrow \text{Num}$ are functions from the set of possible worlds into the finite set of natural numbers Num ;
- $\mathcal{H} : W \times \text{Act}^* \rightarrow \text{Act}$ is a partial function called protocol, mapping worlds and action sequences to actions;
- $\mathcal{V} : W \rightarrow 2^{\text{Prop}}$ is a valuation function.

As usual, $p \in \mathcal{V}(w)$ means that proposition p is true at world w . The equivalence relation \sim , which is used to interpret the epistemic operator [K], can be viewed as a function from W to 2^W . Therefore, we can write $\sim(w) = \{v \in W : w \sim v\}$. The set $\sim(w)$ is the agent’s *information state* at world w : the set of worlds that the agent considers possible at world w or, the set of worlds that the agent cannot distinguish from world w . As \sim is an equivalence relation, if $w \sim v$ then the agent has the same information state at w and v (*i.e.*, the agent has the same knowledge at w and v .)

The function κ_{exc} represents a plausibility grading of the possible worlds and is used to interpret the atomic formulas exc_h . $\kappa_{\text{exc}}(w) = h$ means that, according to the agent the world w has a degree of exceptionality h or, alternatively, according to the agent the world w has a degree of plausibility $\text{max} - h$. (Remember that the degree of plausibility of a world is the opposite of its exceptionality degree.) The function κ_{exc} allows to model the notion of belief: among the worlds the agent cannot distinguish from a given world w (*i.e.*, the agent’s information state at w), there are worlds that the agent considers more plausible than others. For example, suppose that $\sim(w) = \{w, v, u\}$, $\kappa_{\text{exc}}(w) = 2$, $\kappa_{\text{exc}}(u) = 1$ and $\kappa_{\text{exc}}(v) = 0$. This means that at world w the agent cannot distinguish the three worlds w , v and u , that is, $\{w, v, u\}$ is the set of worlds that the agent

considers possible at world w . Moreover, according to the agent, the world v is strictly more plausible than the world u and the world u is strictly more plausible than the world w (as $\max - 0 > \max - 1 > \max - 2$.)

The function κ_{undes} is used to interpret the atomic formulas undes_h . $\kappa_{\text{undes}}(w) = h$ means that, according to the agent the world w has a degree of goodness (or desirability) h or, alternatively, according to the agent the world w has a degree of badness (or undesirability) $\max - h$. (Remember that the degree of undesirability of a world is the opposite of its desirability degree.)

Finally, the protocol function \mathcal{H} is used to interpret the atomic formulas $\text{after}_{\epsilon|\alpha}$. A similar notion of protocol has been studied by [30]. For every world $w \in W$, $\mathcal{H}(w, \epsilon) = \alpha$ means that, at world w , the epistemic action α will occur after the sequence of epistemic actions ϵ . For example, imagine that the agent wakes up in the morning. Then $\mathcal{H}(w, *8pm; *rain) = *earthquake$ means that, at world w , after learning in sequence that ‘it is 8 p.m.’ (by looking at the alarm) and that ‘it is raining outside’ (by opening the window), the agent (will switch on the radio) and will learn that ‘during the night an earthquake of magnitude 5 has occurred’. $\mathcal{H}(w, nil) = \alpha$ means that, at world w , the epistemic action α will occur next. \mathcal{H} is supposed to be a partial function as I want to allow states in which no action occurs.

DL-KGBG-models are supposed to satisfy the following additional *normality* constraint for the plausibility grading.

($NORM_{\kappa_{\text{exc}}}$) for every $w \in W$, there is v such that $w \sim v$ and $\kappa_{\text{exc}}(v) = 0$.

Definition 2 (Truth conditions). Given a DL-KGBG-model M , a world w and a formula φ , $M, w \models \varphi$ means that φ is true at world w in M . The rules defining the truth conditions of formulas are:

- $M, w \models p$ iff $p \in \mathcal{V}(w)$
- $M, w \models \text{after}_{\epsilon|\alpha}$ iff $\mathcal{H}(w, \epsilon)$ is defined and $\mathcal{H}(w, \epsilon) = \alpha$
- $M, w \models \text{exc}_h$ iff $\kappa_{\text{exc}}(w) = h$
- $M, w \models \text{undes}_h$ iff $\kappa_{\text{undes}}(w) = h$
- $M, w \models \neg\varphi$ iff not $M, w \models \varphi$
- $M, w \models \varphi \wedge \psi$ iff $M, w \models \varphi$ and $M, w \models \psi$
- $M, w \models [K]\varphi$ iff $M, v \models \varphi$ for all v with $w \sim v$
- $M, w \models [* \varphi]\psi$ iff $M^{*\varphi}, w \models \psi$

where the updated model $M^{*\varphi}$ is defined according to the Definition 9 below.

Following [26], I extend the exceptionality degree of a possible world to a plausibility/exceptionality degree of a formula viewed as a set of worlds.

Definition 3 (Exceptionality degree of a formula). The exceptionality degree of a formula φ at world w , noted $\kappa_{\text{exc}}^w(\varphi)$, is defined as follows:

$$\kappa_{\text{exc}}^w(\varphi) \stackrel{\text{def}}{=} \min\{\kappa_{\text{exc}}(v) : M, v \models \varphi \text{ and } w \sim v\}.$$

As expected, the *plausibility* degree of a formula φ , noted $\kappa_{\text{plaus}}^w(\varphi)$, is defined as $\max - \kappa_{\text{exc}}^w(\varphi)$. I do a similar manipulation for the desirability degree of a formula.

Definition 4 (Desirability degree of a formula). *The desirability degree of a formula φ at world w , noted $\kappa_{\text{undes}}^w(\varphi)$, is defined as follows:*

$$\kappa_{\text{undes}}^w(\varphi) \stackrel{\text{def}}{=} \min\{\kappa_{\text{undes}}(v) : M, v \models \varphi \text{ and } w \sim v\}.$$

As expected, the *undesirability* degree of a formula φ , noted $\kappa_{\text{undes}}^w(\varphi)$, is defined as $\max - \kappa_{\text{undes}}^w(\varphi)$. Again following [26], I define the concept of belief as a formula which is true in all worlds that are maximally plausible (or minimally exceptional).

Definition 5 (Belief). *At world w the agent believes that φ is true if and only if, for every v such that $w \sim v$, if $\kappa_{\text{exc}}(v) = 0$ then $M, v \models \varphi$.*

The following concept of graded belief is taken from [16]. I say that at world w the agent believes that φ with strength at least h if and only if, all possible worlds in which φ is false are exceptional at least degree h (or all possible worlds in which φ is false are plausible at most degree $\max - h$.)

Definition 6 (Graded belief). *At world w the agent believes that φ with strength at least h if and only if, $\kappa_{\text{exc}}^w(\neg\varphi) \geq h$.*

I define the following concept of strong (or certain) belief: an agent has the strong belief that φ if and only if either he knows that φ is true (*i.e.*, he has an unrevisable belief that φ is true) or he believes that φ is true with maximal strength \max .

Definition 7 (Strong belief). *At world w the agent strongly believes that φ (or at w the agent is certain that φ is true) if and only if, $M, v \models \varphi$ for all v with $w \sim v$ or $\kappa_{\text{exc}}^w(\neg\varphi) = \max$.*

The following concept of graded goal is the motivational counterpart of the notion of graded belief. This concept has not been studied before in the logical literature. I say that at world w the agent wants (or wishes) φ to be true with strength at least h if and only if, all possible worlds in which φ is true are desirable at least degree h (or all possible worlds in which φ is true are undesirable at most degree $\max - h$.) This implies that, all possible worlds which are desirable at most degree $h - 1$ satisfy $\neg\varphi$ (or all possible worlds which are undesirable at least degree $\max - (h - 1)$ satisfy $\neg\varphi$.)

Definition 8 (Graded goal). *At world w the agent wants/wishes φ to be true with strength at least h (or the agent has the goal that φ with strength at least h) if and only if, $\kappa_{\text{undes}}^w(\varphi) \geq h$.*

The reason why the definition of graded goal is not symmetric to the definition of graded belief is that these two concepts satisfy different logical properties. As I will show below in Section 2.3, Definition 6 and Definition 8 allow to capture interesting differences between graded belief and graded goal, especially on the way they distribute over conjunction and over disjunction. As the following proposition highlights, the concepts of belief, graded belief, strong belief and graded goal semantically defined in Definitions 5-8 are all syntactically expressible in the logic DL-KGBG.

Proposition 1. *For every DL-KGBG-model M and world w :*

1. *at w the agent believes that φ is true if and only if*
 $M, w \models [K](\text{exc}_0 \rightarrow \varphi)$

2. at w the agent believes that φ is true with strength at least h if and only if
 $M, w \models \bigvee_{l \in \text{Num}: l \geq h} ((K)(\text{exc}_l \wedge \neg\varphi) \wedge \bigwedge_{k \in \text{Num}: k < l} [K](\text{exc}_k \rightarrow \varphi))$
3. at w the agent strongly believes that φ is true (or the agent is certain that φ is true) if and only if
 $M, w \models \bigwedge_{k \in \text{Num}: k < \max} [K](\text{exc}_k \rightarrow \varphi)$
4. at w the agent wants φ to be true with strength at least h if and only if
 $M, w \models \bigvee_{l \in \text{Num}: l \geq h} ((K)(\text{undes}_l \wedge \varphi) \wedge \bigwedge_{k \in \text{Num}: k < l} [K](\text{undes}_k \rightarrow \neg\varphi))$

In the light of Proposition 1, I provide the following four syntactic abbreviations:

$$\begin{aligned}
\text{Bel}\varphi &\stackrel{\text{def}}{=} [K](\text{exc}_0 \rightarrow \varphi) \\
\text{Bel}^{\geq h}\varphi &\stackrel{\text{def}}{=} \bigvee_{l \in \text{Num}: l \geq h} ((K)(\text{exc}_l \wedge \neg\varphi) \wedge \bigwedge_{k \in \text{Num}: k < l} [K](\text{exc}_k \rightarrow \varphi)) \\
\text{SBel}\varphi &\stackrel{\text{def}}{=} \bigwedge_{k \in \text{Num}: k < \max} [K](\text{exc}_k \rightarrow \varphi) \\
\text{Goal}^{\geq h}\varphi &\stackrel{\text{def}}{=} \bigvee_{l \in \text{Num}: l \geq h} ((K)(\text{undes}_l \wedge \varphi) \wedge \bigwedge_{k \in \text{Num}: k < l} [K](\text{undes}_k \rightarrow \neg\varphi))
\end{aligned}$$

where $\text{Bel}\varphi$, $\text{Bel}^{\geq h}\varphi$, $\text{SBel}\varphi$ and $\text{Goal}^{\geq h}\varphi$ respectively mean: “the agent believes that φ ”, “the agent believes that φ with strength at least h ”,¹ “the agent strongly believes that φ ” and “the agent wants/wishes φ to be true with strength at least h ”.

Definition 9 (Model update). Given a DL-KGBG-model $M = \langle W, \sim, \kappa_{\text{exc}}, \kappa_{\text{undes}}, \mathcal{H}, \mathcal{V} \rangle$, $M^{*\varphi}$ is the model such that:

$$\begin{aligned}
W^{*\varphi} &= W \\
\sim^{*\varphi} &= \sim \\
\text{for all } w, \kappa_{\text{exc}}^{*\varphi}(w) &= \begin{cases} \kappa_{\text{exc}}(w) - \kappa_{\text{exc}}^w(\varphi) & \text{if } M, w \models \varphi \\ \text{Cut}_{\max}(\kappa_{\text{exc}}(w) + \Delta) & \text{if } M, w \models \neg\varphi \end{cases} \\
\text{for all } w, \kappa_{\text{undes}}^{*\varphi}(w) &= \kappa_{\text{undes}}(w) \\
\text{for all } w, \mathcal{H}^{*\varphi}(w, \epsilon) &= \begin{cases} \alpha & \text{if } \mathcal{H}(w, *\varphi; \epsilon) = \alpha \\ \text{undefined} & \text{if } \mathcal{H}(w, *\varphi; \epsilon) \text{ is undefined} \end{cases} \\
\mathcal{V}^{*\varphi} &= \mathcal{V}
\end{aligned}$$

where $\Delta \in \text{Num} \setminus \{0\}$ and

$$\text{Cut}_{\max}(x) = \begin{cases} x & \text{if } 0 \leq x \leq \max \\ \max & \text{if } x > \max \end{cases}$$

An epistemic action $*\varphi$ makes the protocol \mathcal{H} to advance one step forward (see the definition of $\mathcal{H}^{*\varphi}$.) That is, if α will occur after the sequence of events $*\varphi; \epsilon$ then, after the occurrence of the epistemic action $*\varphi$, it is the case that α will occur after the sequence of events ϵ . As epistemic actions only affect the agent’s beliefs and do not affect the objective world, the valuation function \mathcal{V} is not altered by them (see the definition

¹ Similar operators for graded belief are studied in [16, 3, 31].

of $\mathcal{V}^{*\varphi}$.) Moreover, the epistemic action $*\varphi$ modifies the plausibility ordering (see the definition of $\kappa_{\text{exc}}^{*\varphi}$) but does not modify the agent's information state (see the definition of $\sim^{*\varphi}$) nor the desirability ordering (see the definition of $\kappa_{\text{undes}}^{*\varphi}$.) In particular, the epistemic action of learning that φ is true induces a kind of belief conditionalization in Spohn's sense [26]. The plausibility ranking over possible worlds is updated as follows.

- For every world w in which φ is true, the degree of exceptionality of w decreases from $\kappa_{\text{exc}}(w)$ to $\kappa_{\text{exc}}(w) - \kappa_{\text{exc}}^w(\varphi)$, which is the same thing as saying that, degree of plausibility of w increases from $\max - \kappa_{\text{exc}}(w)$ to $\max - (\kappa_{\text{exc}}(w) - \kappa_{\text{exc}}^w(\varphi))$. (Note that, by Definition 3, we have $\kappa_{\text{exc}}(w) - \kappa_{\text{exc}}^w(\varphi) \leq \kappa_{\text{exc}}(w)$.)
- For every world w in which φ is false, the degree of exceptionality of w increases from $\kappa_{\text{exc}}(w)$ to $\text{Cut}_{\max}(\kappa_{\text{exc}}(w) + \Delta)$, which is the same thing as saying that, degree of plausibility of w decreases from $\max - \kappa_{\text{exc}}(w)$ to $\max - \text{Cut}_{\max}(\kappa_{\text{exc}}(w) + \Delta)$.

Cut_{\max} is a minor technical device, taken from [3], which ensures that the new plausibility assignment fits into the finite set of natural numbers Num . The parameter Δ is a *conservativeness* index which captures the agent's disposition to radically change his beliefs in the light of a new evidence. More precisely, the higher is the index Δ , and the higher is the agent's disposition to decrease the plausibility degree of those worlds in which the learnt fact φ is false. (When $\Delta = \max$, the agent is minimally conservative.) I assume that Δ is different from 0 in order to ensure that, after learning that p is true, the agent will believe p . That is, from $\Delta \in Num \setminus \{0\}$ it follows that $[*p]\text{Bel}p$ is valid for every proposition $p \in Prop$. Note that the operation $*\varphi$ preserves the constraints on DL-KGBG-models: if M is a DL-KGBG-model then $M^{*\varphi}$ is a DL-KGBG-model too.

In the sequel I write $\models_{\text{DL-KGBG}} \varphi$ to mean that φ is *valid* in DL-KGBG (φ is true in all DL-KGBG-models.)

2.3 Some properties of mental attitudes

The following are some interesting examples of validity. For every $h, k \in Num$ we have:

- (1) $\models_{\text{DL-KGBG}} \text{Bel}^{\geq h} \varphi \rightarrow \text{Bel}^{\geq k} \varphi$ if $h \geq k$
- (2) $\models_{\text{DL-KGBG}} \text{Goal}^{\geq h} \varphi \rightarrow \text{Goal}^{\geq k} \varphi$ if $h \geq k$
- (3) $\models_{\text{DL-KGBG}} [K]\varphi \rightarrow \neg \text{Bel}^{\geq h} \varphi$
- (4) $\models_{\text{DL-KGBG}} \text{SBel}\varphi \leftrightarrow ([K]\varphi \vee \text{Bel}^{\geq \max} \varphi)$
- (5) $\models_{\text{DL-KGBG}} \neg(\text{Bel}\varphi \wedge \text{Bel}\neg\varphi)$
- (6) $\models_{\text{DL-KGBG}} [* \varphi]\text{Bel}\varphi$ if $\varphi \in Obj$
- (7) $\models_{\text{DL-KGBG}} [K]\varphi \rightarrow [* \psi][K]\varphi$ if $\varphi \in Obj$
- (8) $\models_{\text{DL-KGBG}} (\text{Bel}^{\geq h} \varphi \wedge \text{Bel}^{\geq k} \psi) \rightarrow \text{Bel}^{\geq \min\{h,k\}}(\varphi \wedge \psi)$
- (9) $\models_{\text{DL-KGBG}} (\text{Goal}^{\geq h} \varphi \wedge \text{Goal}^{\geq k} \psi) \rightarrow (\text{Goal}^{\geq \max\{h,k\}}(\varphi \wedge \psi) \vee [K](\neg\varphi \vee \neg\psi))$
- (10) $\models_{\text{DL-KGBG}} (\text{Bel}^{\geq h} \varphi \wedge \text{Bel}^{\geq k} \psi) \rightarrow (\text{Bel}^{\geq \max\{h,k\}}(\varphi \vee \psi) \vee [K](\varphi \vee \psi))$
- (11) $\models_{\text{DL-KGBG}} (\text{Goal}^{\geq h} \varphi \wedge \text{Goal}^{\geq k} \psi) \rightarrow \text{Goal}^{\geq \min\{h,k\}}(\varphi \vee \psi)$

According to the validity (1), if the agent believes that φ with strength at least h and $h \geq k$, then he believes that φ is true with strength at least k . Validity (2) is the corresponding property for graded goals. The validity (3) highlights that knowledge and graded belief are distinct concepts. According to the validity (4), the agent has the strong belief that φ (*i.e.*, the agent is certain that φ) if and only if, either he knows that φ is true (*i.e.*, he has an unrevisable belief that φ is true) or he believes that φ with maximal strength max . According to the validity (5) (which follows from the normality constraint $NORM_{\kappa_{\text{exc}}}$ given in Section 2.2), an agent cannot have inconsistent beliefs.

The validity (6) highlights a basic property of belief revision in the sense of AGM theory [2]: if φ is an objective fact then, after learning that φ is true, the agent believes that φ is true. The validity (7) highlights a basic property of knowledge as *unrevisable belief* (see Section 2.1 for a discussion): if φ is an objective fact and the agent knows that φ is true then, after learning a new fact ψ , he will continue to know that φ is true. In this sense, knowledge is stable under belief revision. According to the validity (8), if the agent believes that φ with strength at least h and believes that ψ with strength at least k , then the strength of the belief that $\varphi \wedge \psi$ is at least $\min\{h, k\}$. According to the validity (10), if the agent believes that φ with strength h and believes that ψ with strength k , then either he believes $\varphi \vee \psi$ with strength at least $\max\{h, k\}$ or he knows that $\varphi \vee \psi$. (Remember that knowledge and graded belief are distinct concepts.)

The validities (9) and (11) are corresponding principles for goals. According to the validity (9), if the agent wishes φ to be true with strength at least h and wishes ψ to be true with strength at least k , then either he wishes $\varphi \wedge \psi$ to be true with strength at least $\max\{h, k\}$ or he knows that φ and ψ are inconsistent facts. According to the validity (11), if the agent wishes φ to be true with strength at least h and wishes ψ to be true with strength at least k , then he wishes $\varphi \vee \psi$ to be true with strength at least $\min\{h, k\}$.

The interesting aspect is that graded goals distribute over conjunction and over disjunction in the opposite way as graded beliefs. Consider for instance the validity (9) and compare it to the validity (8). The joint occurrence of two events φ and ψ is less plausible than the occurrence of a single event. This is the reason why in the right side of the validity (8) we have the *min*. On the contrary, the joint occurrence of two desired events φ and ψ is more desirable than the occurrence of a single event. This is the reason why in the right side of the validity (9) we have the *max*. For example, suppose Peter wishes to go to the cinema in the evening with strength at least h (*i.e.*, $\text{Goal}^{\geq h} \text{goToCinema}$) and, at the same time, he wishes to spend the evening with his girlfriend with strength at least k (*i.e.*, $\text{Goal}^{\geq k} \text{stayWithGirlfriend}$.) Then, according to the validity (9), either Peter wishes to go the cinema with his girlfriend with strength at least $\max\{h, k\}$ (*i.e.*, $\text{Goal}^{\geq \max\{h, k\}}(\text{goToCinema} \wedge \text{stayWithGirlfriend})$) or Peter knows that going to the cinema with his girlfriend is impossible (*i.e.*, $[\text{K}](\neg \text{goToCinema} \vee \neg \text{stayWithGirlfriend})$), perhaps because he knows that his girlfriend has the flu and cannot go out.

2.4 Exact strength of belief and exact strength of goal

The operators of type $\text{Bel}^{\geq h}$ and $\text{Goal}^{\geq h}$ introduced above enable to specify a concept of graded belief of the form “the agent believes that φ with strength h ” in which the *exact* strength of the agent’s belief is specified. I assume that “the agent believes that φ exactly with strength h ”, noted $\text{Bel}^h \varphi$ if and only if, the agent believes that φ with

strength at least h and it is not the case that the agent believes that φ with strength at least $h+1$. Formally, for every $h < \max$, I define:

$$\text{Bel}^h \varphi \stackrel{\text{def}}{=} \text{Bel}^{\geq h} \varphi \wedge \neg \text{Bel}^{\geq h+1} \varphi$$

In order to have a uniform notation, I will use $\text{Bel}^{\geq \max} \varphi$ and $\text{Bel}^{\max} \varphi$ as interchangeable expressions. I do a similar manipulation for goals. For every $h < \max$, I define:

$$\text{Goal}^h \varphi \stackrel{\text{def}}{=} \text{Goal}^{\geq h} \varphi \wedge \neg \text{Goal}^{\geq h+1} \varphi$$

where $\text{Goal}^h \varphi$ has to be read “the agent wants/wishes φ to be true exactly with strength h ”. As for beliefs, I will use $\text{Goal}^{\geq \max} \varphi$ and $\text{Goal}^{\max} \varphi$ as interchangeable expressions.

3 Formalization of expectation-based emotions and their intensity

The modal operators of graded belief and graded goal defined in Section 2.2 are used here to provide a logical analysis of expectation-based emotions such as hope, fear, disappointment and relief and their intensities.

3.1 Hope and fear

According to some psychological models [24, 17, 23] and computational models [13, 6] of emotions, the intensity of hope with respect to a given event is a monotonically increasing function of: the degree to which the event is desirable and the likelihood of the event. That is, the higher is the desirability of the event φ , and the higher is the intensity of the agent’s hope that this event will occur; the higher is the likelihood of the event φ , and the higher is the intensity of the agent’s hope that this event will occur.² Analogously, the intensity of fear with respect to a given event is a monotonically increasing function of: the degree to which the event is undesirable and the likelihood of the event. There are several possible merging functions which satisfy these properties. For example, I could define the merging function *merge* as an average function, according to which the intensity of hope about a certain event φ is the average of the strength of the belief that φ will occur and the strength of the goal that φ will occur. That is, for every $h, k \in \text{Num}$ representing respectively the strength of the belief and the strength of the goal, I could define *merge*(h, k) as $\frac{h+k}{2}$. Another possibility is to define *merge* as a product function $h \times k$ (also used in [13]), according to which the intensity of hope about a certain event φ is the product of the strength of the belief that φ will occur and the strength of the goal that φ will occur. Here I do not choose a specific merging function, as this would require a serious experimental validation and would much depend on the domain of application in which the formal model has to be used.

² According to Ortony et al. [23] the intensity of hope and fear is determined by a third parameter: the (temporal and spatial) *proximity* to the expected event (the higher is the proximity to the expected event, and the higher is the intensity of hope/fear.) This third dimension is not considered in the present analysis.

Let me now define the notions of hope and fear with their corresponding intensities. An agent is experiencing a hope with with intensity i about φ if and only if there are $h, k \in Num$ such that such that $h < \max$, h is the strength to which the agent believes that φ is true, k is the strength to which the agent wishes φ to be true and $merge(h,k) = i$:

$$\text{Hope}^i \varphi \stackrel{\text{def}}{=} \bigvee_{h,k \in Num: h < \max \text{ and } merge(h,k)=i} (\text{Bel}^h \varphi \wedge \text{Goal}^k \varphi)$$

The notion of fear can be defined in a similar way, after assuming that if an agent wishes φ to be true, then the situation in which φ is false is undesirable for him.³ An agent is experiencing a fear with with intensity i about φ if and only if there are $h, k \in Num$ such that $h < \max$, h is the strength to which the agent believes that φ is true, k is the strength to which the agent wishes φ to be false and $merge(h,k) = i$:

$$\text{Fear}^i \varphi \stackrel{\text{def}}{=} \bigvee_{h,k \in Num: h < \max \text{ and } merge(h,k)=i} (\text{Bel}^h \varphi \wedge \text{Goal}^k \neg \varphi)$$

In the preceding definitions of hope and fear, the strength of the belief is supposed to be less than \max in order to distinguish hope and fear, which imply some form of uncertainty, from happiness and sadness (or unhappiness) which are based on certainty. In order to experience hope (resp. fear) about φ , the agent should have a minimal degree of uncertainty that φ might be false. Indeed, we have that:

$$(12) \quad \models_{DL-KGBG} \text{Hope}^i \varphi \rightarrow \neg \text{SBel} \varphi$$

$$(13) \quad \models_{DL-KGBG} \text{Fear}^i \varphi \rightarrow \neg \text{SBel} \varphi$$

This means that if an agent hopes (resp. fears) φ to be true, then he is not certain that φ (*i.e.*, the agent does not have the strong belief that φ .) For example, if I hope that my paper will be accepted at the next SEM workshop, then it means that I am not certain that my paper will be accepted. The preceding two validities are consistent with Spinoza's quote "Fear cannot be without hope nor hope without fear". Indeed, if an agent hopes that φ will be true then, according to the validity (12), he envisages the possibility that φ will be false. Therefore, he experiences some fear that φ will be false. Conversely, if an agent fears that φ will be true then, according to the validity (13), he envisages the possibility that φ will be false. Therefore, he experiences some hope that φ will be false.

Remark 1. It has to be noted that hope and fear, and more generally expectations, are not necessarily about a *future* state of affairs, but they can also be about a *present* state of affairs or a *past* state of affairs. For example, I might say 'I hope that you feel better now!' or 'I fear that you did not enjoy the party yesterday night!'.

On the contrary, to feel happy (resp. sad) about φ , the agent should be *certain* that φ is true. For example, if I am happy that my paper has been accepted at the SEM workshop, then it means that I am certain that my paper has been accepted. More precisely,

³ If an agent wishes φ to be true, then in the situation in which φ is false he will be frustrated. This is the reason why the latter situation is undesirable for the agent.

an agent is experiencing happiness with intensity h about φ if and only if, the agent strongly believes (is certain) that φ is true and h is the strength to which the agent wishes φ to be true:

$$\text{Happiness}^h \varphi \stackrel{\text{def}}{=} \text{SBel} \varphi \wedge \text{Goal}^h \varphi$$

Moreover, an agent is experiencing sadness with intensity h about φ if and only if, the agent strongly believes (is certain) that φ is true and h is the strength to which the agent wishes φ to be false:

$$\text{Sadness}^h \varphi \stackrel{\text{def}}{=} \text{SBel} \varphi \wedge \text{Goal}^h \neg \varphi$$

The following validities highlight some relationships between hope and fear, and happiness and sadness. Suppose $\Delta = \max$ and $\varphi \in \text{Obj}$, then:

$$(14) \quad \models_{\text{DL-KGBG}} \text{Hope}^i \varphi \rightarrow [* \varphi] \bigvee_{k \in \text{Num}} \text{Happiness}^k \varphi$$

$$(15) \quad \models_{\text{DL-KGBG}} \text{Fear}^i \varphi \rightarrow [* \varphi] \bigvee_{k \in \text{Num}} \text{Sadness}^k \varphi$$

This means that, if the agent is minimally conservative and hopes φ to be true then, after learning that φ , his sadness will transform into happiness about φ (with a certain strength.) On the contrary, if he is minimally conservative and fears φ to be true then, after learning that φ , his fear will transform into sadness about φ (with a certain strength.) As the following two validities highlight, if the agent is minimally conservative and learns that φ is false, then his hope about φ will transform into sadness about $\neg \varphi$ and his fear about φ will transform into happiness about $\neg \varphi$. That is, suppose $\Delta = \max$ and $\varphi \in \text{Obj}$, then:

$$(16) \quad \models_{\text{DL-KGBG}} \text{Hope}^i \varphi \rightarrow [* \neg \varphi] \bigvee_{k \in \text{Num}} \text{Sadness}^k \neg \varphi$$

$$(17) \quad \models_{\text{DL-KGBG}} \text{Fear}^i \varphi \rightarrow [* \neg \varphi] \bigvee_{k \in \text{Num}} \text{Happiness}^k \neg \varphi$$

3.2 Disappointment and relief

An interesting class of expectation-based emotions are those based on the invalidation of either a positive or a negative expectation. In agreement with Ortony et al.'s psychological model [23], I distinguish two types of these emotions: disappointment and relief. Disappointment is the emotion due to the invalidation of an agent's hope, while relief is the emotion due to the invalidation of an agent's fear. On the one hand, the higher is the intensity of the invalidated hope that φ is true, and the higher is the intensity of the agent's disappointment. On the other hand, the higher is the intensity of the invalidated fear that φ is true, and the higher is the intensity of the agent's relief.

More precisely, an agent *is going to* experience a disappointment with intensity i about φ if and only if the agent hopes φ to be true with intensity i and the agent is going to learn that φ is false. That is:

$$\text{Disappointment}^i \varphi \stackrel{\text{def}}{=} \text{Hope}^i \varphi \wedge \text{occ}_{* \neg \varphi}$$

Moreover, an agent *is going to* experience a relief with intensity i about φ if and only if the agent fears φ to be true with intensity i and the agent is going to learn that φ if false. That is:

$$\text{Relief}^i\varphi \stackrel{\text{def}}{=} \text{Fear}^i\varphi \wedge \text{occ}_{*\neg}\varphi$$

It has to be noted that disappointment and relief always implies some form of surprise, where surprise is defined as the invalidation of an agent's beliefs and its intensity depends on the strength of the invalidated belief, that is:

$$\text{Surprise}^h\varphi \stackrel{\text{def}}{=} \text{Bel}^h\varphi \wedge \text{occ}_{*\neg}\varphi$$

where $\text{Surprise}^h\varphi$ has to be read the agent *is going to* feel surprised about φ with intensity h . This definition of surprise has been elaborated elsewhere (see [19]).

4 Related works

Emotion is a very active field not only in psychology but also in AI. Several computational architectures of affective agents have been proposed in the last few years (see *e.g.* [10, 8, 9]). The cognitive architecture EMA (Emotion and Adaption) proposed by Gratch & Marsella [13] is one of the best example of research in this area. EMA defines a domain independent taxonomy of appraisal variables stressing the many different relations between emotions and cognition, by enabling a wide range of internal appraisal and coping processes used for reinterpretation, shift of motivations, goal reconsideration *etc.* EMA also considers emotion intensity based on appraisal variables like desirability of an event and its likelihood. However, there is an important difference between Gratch & Marsella's work and the work presented here. Gratch & Marsella do not provide a logical model of emotion intensity, while this is one of the main contribution of the present work. The advantage of having a logical model of emotion is that one can study affective phenomena in a rigorous way by deriving interesting properties of different types of emotion such as hope, fear, disappointment and relief; checking whether the assumptions of the theory of emotion are consistent; *etc.* Furthermore, a logical model of emotion can be useful in practice because it can be used as the formal specification of an implemented cognitive architecture in which one can verify whether the implemented model satisfies certain properties or not (*e.g.*, whether an artificial agent can feel fear and sadness at the same time.)

There are other researchers who have developed logical formalisms for modelling emotions. One of the most prominent logical analysis of emotions is the one proposed by Meyer and coll. [21, 29]. In order to formalize emotions, they exploit the logical framework KARO [22]: a framework based on a blend of dynamic logic with epistemic logic, enriched with modal operators for motivational attitudes such as desires and goals. Meyer and coll. formalize generation rules (*i.e.* triggering conditions) for different kinds of emotions such as joy, sadness, anger and fear [21], and also guilt and shame [29]. In a more recent work [28] Meyer et al. have integrated quantitative aspects in their logical model of emotions. However, different from the present approach, they do not study the variables determining intensities but instead focus on the integration of

intensities into a qualitative model of emotions (for example, they propose a function describing how the intensity of an emotion decreases over time.)

Lorini & Schwarzenruber [20] have recently proposed a logical model of counterfactual emotions such regret, *i.e.* those emotions based on counterfactual thinking about agents' choices and actions. Adam et al. [1] have exploited a BDI logic in order to provide a logical formalization of the emotion types defined in Ortony, Clore and Collins's model (OCC model) [23]. Similar to the approach presented in this paper, in Lorini & Schwarzenruber's approach and in Adam et al.'s approach emotion types are defined in terms of some primitive concepts (and corresponding modal operators) such as the concepts of belief, desire, action and responsibility which allow to capture the different appraisal variables of emotions. However, Lorini & Schwarzenruber's approach and Adam et al.'s approach are purely qualitative and do not consider emotion intensity.

5 Conclusion

The logical analysis of expectation-based emotions presented in this paper misses some important psychological aspects. For example, the fact that mental states on which emotions such as hope, fear, disappointment and relief are based are usually joined with bodily activation and components, and these components shape the whole subjective state of the agent and determine his action tendencies [12]. I have only focused on the cognitive structure of emotions, without considering the *felt* aspect of emotions. This is of course a limitation of the model presented in this paper, as the intensity of emotion also depends on the presence of these somatic components (*e.g.* the intensity of fear is amplified by the fact that, when experiencing this emotion I feel my stomach contracted, my throat dry, *etc.*) I postpone to future work a formal analysis of the basic relationships between emotion and action (*i.e.* how an emotion with a given intensity determines the agent's future reactions), and of the relationships between cognitive structure and somatic aspects of emotions (*i.e.* how somatic components affect emotion intensity).

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