3-D Lagrangian Voronoï analysis for clustering of particles and bubbles in turbulence

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Preferential concentration of particles

- Inertial particles do not distribute homogeneously in turbulent flows but cluster.
- A better understanding of the clustering behavior is important in many fields.

Research questions:
- How do (light, neutral, heavy) particles distribute in homogeneous isotropic turbulence?
- How do we quantify the clustering?
- How do the clusters change along particle trajectories? (in Lagrangian temporal view)

Approach: **Voronoï** analysis

- DNS simulations + 3D Experiments
Clustering of inertial particles in turbulence

Eulerian analysis (Minkowski functionals)

Light
Filamentary structures

Neutral (Tracer)
No clustering

Heavy
wall-like topology around interconnected tunnels

iCFDdatabase: http://cfd.cineca.it/

E. Calzavarini, M. Kerscher, D. Lohse & F. Toschi, JFM, 607, 2008
D. Lohse, Physics 1, 18, 2008
Voronoï analysis

Monchaux R., Bourgoin M. & Cartellier A., Phys. Fluids (2010), 22, 103304

**Voronoï tessellation**

Spatial tesselation based on particle positions

Each area (or volume) of Voronoï cells is inverse of local concentration

- **Advantages:**
  - Easy
  - Requires no arbitrary length scale
  - Local concentration defined at each particle position at each timestep

→ **Lagrangian temporal analysis of clustering**
Objective of present work

Three-dimensional Voronoi analysis to study particle clustering in turbulence

(Two approaches of Voronoi analysis)

✓ **Eulerian** Voronoi analysis
   ✷ PDFs of Voronoi volumes

✓ **Lagrangian** Voronoi analysis
   ✷ Temporal auto-correlation of Voronoi volumes

- DNS numerical data (iCFD database: http://cfd.cineca.it/)
- Micro-bubble experimental data from Twente Water Tunnel
Numerical scheme for particles in isotropic turbulence

Dilute suspension (neglect particle collisions) of particles in homogeneous and isotropic turbulence:

**Particle dynamics:**

\[
\frac{d}{dt} x = v, \quad \frac{d}{dt} v = \beta D u + \tau_p^{-1} (u - v),
\]

\( u \) : fluid velocity

\( v \) : particle velocity

\( \beta \): density ratio

\( \beta = 3\rho_f / (\rho_f + 2\rho_p) \)

\( \beta < 1 \): heavy particles

\( \beta > 1 \): light particles

\( \beta = 1 \): neutral particles

Stokes number: particle response time

\( St = \tau_p / \tau_\eta = a^2 / (3\beta\eta^2) \)

Special cases

\( \beta = 0 \): Very heavy particles

\( \beta = 1 \): Neutrally buoyant particles

\( \beta = 3 \): Bubbles in water


Mean flow carrying microbubbles

Active grid

Mean flow carrying microbubbles

LASER + Volume optics

3D PTV

Test section

(See afternoon talk by Yoshiyuki Tagawa)
Data sets for 3D Voronoï analysis

Numerical Simulations

\[ \text{Re}_\lambda = 75, 180 \]
\[ \beta = 0, 1, 3 \]

24 cases of St for each \( \beta \)

iCFD database: http://cfd.cineca.it/

3D micro-bubble experiments in Twente Water Tunnel

\[ \text{Re}_\lambda = 162 \]
\[ \beta = 3 \]
\[ \text{St} = 0.04 \pm 0.02 \]
Eulerian Voronoï analysis
Particle density effect

(fixed St=0.6)

- Light and heavy particles show clustering
- Light particles show more intense clustering
- Trends for both $Re_\lambda$ are similar
Stokes number effect

$\text{Re}_\lambda = 180$

Light particles ($\beta=3$)

- Probability of finding clusters and voids increases up to a value of $St = 1.6$
- Experimental result agrees well with the trend of numerical data

Heavy particles ($\beta=0$)

- Clustering of heavy particles is weaker as compared to light particles
- Probability of finding clusters and voids increases with increasing $St$
Clustering indicator

Gamma distribution described by standard deviation $\sigma$

$$f(x) = \frac{1}{\sigma^2 \Gamma(\sigma^{-2})} x^{(\sigma^{-2} - 1)} \exp(-x\sigma^{-2})$$

$\sigma$ provides a proper statistical quantification of Voronoï volumes

Single parameter fit

$\sigma$: Standard deviation of the Voronoï volumes

$\sigma/\sigma_\Gamma = 2.5$

$\sigma/\sigma_\Gamma = 1.5$

$\sigma/\sigma_\Gamma = 1$

Light particles

Heavy particles

Neutrally buoyant particles

$\sigma$ provides a proper statistical quantification of Voronoï volumes
Normalized standard deviation $\sigma/\sigma_\Gamma$

- Stronger clustering for light particles at $St 1-2$
- Stronger clustering for larger $Re_\lambda$

Experimental result agrees within the experimental errorbar
Lagrangian Voronoï analysis
Temporal Lagrangian autocorrelation at \( \text{St}=0.6 \)

Clustering of light particles lasts for longer time

Decorrelation time of the Voronoi volume is much longer than that of enstrophy

Lagrangian time series of the Voronoi volumes

Decorrelation time of the Voronoi volume is much longer than that of enstrophy
Decorrelation time of Voronoï volume $\tau_V$ and enstrophy $\tau_\Omega$

- Light particles cluster for a longer time
  - Morphology of the clustered light particles is different from that of heavy particles

- Life-times of the clustered particles much longer than the life-time of the trapping flow structures
  - Clustered particles are constrained in different regions of the flow
  - Due to their inertia, particles need time to reorganize themselves in the flow
Summary

3D Voronoi analysis was used to study particle clustering in homogenous isotropic turbulence in numerics and experiments.

• **Eulerian Voronoi analysis**
  – Particle density effect ($\beta$ effect): light particles show more clustering
  – Stokes number effect: most clustering for $St = 1-2$
  – Experimental results and numerical results agree
  – Normalized standard deviation is used as an indicator of clustering

• **Lagrangian Voronoi analysis**
  – Clustering of light particles last much longer than that of heavy and neutrally buoyant particles
  – Light and heavy particles remain clustered for a much longer time than the flow structures

Thank you for your attention!