Vortices and the Berezinskii-Kosterlitz-Thouless Transition in 2D Systems with Competing Order

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Motivation
- Competition between different types of order is an important feature of many strongly correlated systems
- In such systems there may exist a fine-tuned point in parameter space where the symmetry of the order parameter is enhanced
- This symmetry enhancement is interesting in 2D because spontaneous symmetry breaking is forbidden due to Hohenberg-Mermin-Wagner theorem
- \( O(2) \) has a topological phase transition driven by vortices – the BKT transition
- Consider the effect of symmetry enhancement on vortices and the transition they mediate

Competing Orders
- Some examples
  - \( O(2) \times Z_2 \)
  - \( O(2) \times O(2) \)
  - \( O(2) \times O(3) \)

BKT Transition
- Vortex unbinding transition
- Action of a single vortex \( S_{\text{vtx}} = S_{\text{core}} + (\pi T) \ln R/a \)
- Core action \( S_{\text{core}} \sim \pi/2 \)
- Entropy of a single vortex \( s = 2 \ln R/a \)
- Transition temperature \( T_{\text{BKT}} \sim \pi/2 \)
- Fugacity density \( z = y/a^2 = \exp(-S_{\text{core}})/a^2 \)
- RG flow equations:
  \[
  \frac{dT(\ell)}{d\ell} = 4\pi y(\ell)^2, \quad \frac{dy(\ell)}{d\ell} = (2 - \pi T(\ell)^2) y(\ell)
  \]

BKT near Symmetry Enhancement
- We consider the \( O(2) \times O(M) \) EP-NLSM
- For \( \Delta \gg 0 \) this system is basically the \( O(2) \) model so we expect to see a BKT transition
- At \( \Delta = 0 \) vortices are no longer possible so the transition temperature must vanish
- We investigate how the system passes between the two known limits as \( \Delta \) is decreased
  - We take two approaches, starting from the two limits we understand well
  1. Starting from large \( \Delta \), we make a variational ansatz of a modified vortex and minimize the action to find the associated transition temperature
  2. Starting from small \( \Delta \), we treat the anisotropy as a perturbation upon the \( O(2 + M) \) nonlinear sigma model and investigate the RG flow

Symmetry Enhancement
- Not generic but common to the listed examples
- Particular point in parameter space where we can freely rotate between the two sectors of the order parameter
- Enhanced symmetry point has consequences for the rest of the phase diagram

Easy Plane Nonlinear Sigma Model
- Based on general principles of symmetry and universality (or in some cases directly from microscopics) we construct model action for competing phases
- \[
  S = \frac{1}{2\pi} \int dR \left\{ \langle \nabla \vec{\sigma} \rangle^2 + \frac{\lambda}{4!} \vec{\sigma} : D \vec{R} \right\}
  \]
- Lattice length scale = System size \( R \)
- Anisotropy between phases \( \Delta \)
- For \( \Delta = 0 \) symmetry is enhanced \( (O(M) \times O(N)) \rightarrow (O(M + N)) \)

Modified Vortices
- There are two things that make the action of a single vortex big
  - System size \( R \) – disappears when we study a vortex plasma
  - Lattice spacing \( a \) – the size of the vortex core
- There is a formally divergent energy at the centre of the vortex due to the singularity
- For finite \( \Delta \) a vortex can lower its energy by having the field point out of the \( O(2) \) plane into the suppressed phase
- We make the simple ansatz of a modified vortex excitation that looks like a normal vortex beyond some core radius \( \xi \) but points straight into the suppressed phase below \( \xi \)
  - The action contribution of a single such modified vortex is
    \[
    S_{\text{vtx}}(\xi, \ell) = \pi \frac{\Delta}{2a^2} + \pi \frac{R}{\xi} \]
  - Minimizing the action w.r.t. \( \xi \) we find an optimal core size \( \xi_{\text{opt}} = a/\sqrt{\Delta} \)
- The action of the core itself is unaffected by the increased core size \( S_{\text{core}} = \pi \frac{\Delta}{2a^2} = \pi \frac{R}{\xi} \)

Change in Transition Temperature
- The RG flow equations for the standard corrections to the BKT transition temperature
- The factor of \( a \) here comes from starting the RG at the vortex core size so this \( a \rightarrow \xi \)
- \( \Delta \) is the definition of \( a \) as unchanged as it comes from enumerating possible positions of the vortex
- The BKT transition temperature for modified vortices is therefore
  \[
  T_{\text{BKT}} = \frac{\pi}{2 + \pi a^2 z(1/T_{\text{BKT}})} \Rightarrow T_{\text{BKT}} \sim \frac{1}{\ln(1/\Delta)}
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RG Flow of the EP-NLSM
- Treating the anisotropy as a perturbation on the \( O(2 + M) \) NLSM we find RG equations up to one loop to be
  \[
  \frac{dT(\ell)}{d\ell} = T(\ell)^2 M \frac{\Delta(\ell)}{2 + \pi \Delta(\ell)} \Rightarrow T(\ell) = \frac{T(0)}{2 + \pi \Delta(0)} \]
- We flow the RG until \( \Delta \) is of order 1 and then apply regular BKT arguments
- We stop the flow at the scale \( \ell^* \) at which \( \Delta(\ell^*) = 1 \)
- If \( T(\ell^*) < \pi/2 \) then the system flows into the \( O(2) \) ordered regime
- We trace the flow back to find the bare value of temperature \( T_{\text{BKT}} \) corresponding to \( T(\ell^*) \) as a function of \( \Delta \)
- For small \( \Delta \) we find that \( T_{\text{BKT}} \sim 1/\ln(1/\Delta) \)
- Confirms prediction of our variational analysis
- We can use the RG trajectories to sketch a phase diagram

The vortex expands as \( \xi \sim 1/\sqrt{\Delta} \) and is ultimately destroyed from within
- The transition temperature vanishes as \( T \sim 1/\ln(1/\Delta) \)

References

If you have any questions or comments and I am not next to my poster please come and find me!