BEC to BCS crossover: Excitons, cold atoms, polaritons

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Vanilla theory of a condensate

BEC Superconductor Density wave Exciton

\[ \langle \Psi \rangle = \langle b_0 \rangle = \sum_k \langle c_k c_{-k} \rangle = \sum_k \langle c_k^\dagger + Q c_k \rangle = \sum_k \langle c_{1,k}^\dagger c_{2,k} \rangle = \Delta e^{i\phi} \]

Energy

Amplitude “Higgs” mode

Phase “Bogoliubov” mode

Superfluid stiffness determines phase mode velocity

Momentum

BEC-BCS
In reality, there are more scales

- Bandwidth $E_F$
- Pairing energy "Gap" $2\Delta$
- Scattering length $\ell^{-1}$
- Interaction range $r_0^{-1}$
- Fermi momentum $k_F$
- Lattice constant $a^{-1}$
- Energy
- Coherence length $1/\xi$
- Momentum

BEC-BCS
Intuitively: two limits

- **BEC** – thermal physics dominated by phase fluctuations
  - soft phase mode, low density, weak interaction between pairs
- **BCS** – thermal physics dominated by pair-breaking
  - stiff phase mode, high density, weak binding of pairs
Outline

- **Excitons** – a toy problem of density-driven crossover
  - following Keldysh, Comte and Nozieres
- **Cold atoms** – interaction-driven crossover
  - following Leggett, Nozieres and Schmitt-Rink
  - generalisation to non-zero density
- **Polaritons**
  - two kinds of crossover
  - dynamics
Hydrogenic Excitons

Conduction Band Electron

Exciton wave-packet

Valence Band Hole

Energy

Momentum

Bohr radius $a_B \sim \text{few nm}$

BEC-BCS
Mean field theory of excitonic insulator

\[ \Phi^\dagger |0\rangle = \sum_k \phi(k) a_{ck}^\dagger a_{vk} |0\rangle \]

\[ e^{\lambda \Phi^\dagger} |0\rangle = \prod_k \left[ u(k) + v(k) a_{ck}^\dagger a_{vk} \right] |0\rangle \]

A coherent state – like a laser
Bose condensation of excitons

\[ \nu(k) = \lambda \phi(k) \]

BCS-like instability of Fermi surfaces

Special features: order parameter; gap

\[ \langle a_{ck}^\dagger a_{vk} \rangle = u_k \nu_k = (\Delta_k / 2E_k); \]
\[ E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2} \]

Wavepacket of bound e-h pair
Composite boson
Excitation spectra

$+(-)E_k$ is energy to add (remove) particle-hole pair from condensate (total momentum zero)

\[
E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}
\]

- **Band energy**
- **Chemical potential** ($<0$ for bound exciton)
- **Correlation energy**

Low density $\mu < 0$
- Chemical potential below band edge

High density $\mu > 0$
- No bound exciton below band edge

Absorption

Emission

BEC-BCS
2D exciton condensate: Mean field solution

Model: 2D quantum wells separated by distance = 1 Bohr radius Zhu et al PRL 74, 1633 (1995)
Crossover from BCS to BEC

Smooth crossover between BCS-like fermi surface instability and exciton BEC

Model: 2D quantum wells separated by distance = 1 Bohr radius Zhu et al PRL 74, 1633 (1995)
2D BEC - no confining potential

Interpolation (by hand) between two limits

$$k_B T_0 = Ry^* \exp(-1/2r_s)$$

$$Ry^* = \frac{m^*/m}{\epsilon^2} \times \text{Rydberg}$$

$$a_0^* = \frac{\epsilon}{m^*/m} \times a_{Bohr}$$

$$r_s^2 = \frac{1}{\pi n a_0^*2}$$

GaAs CQW

T = 4 K

n = 3 x 10^{11} \text{ cm}^{-2}

Plasma

"Preformed pair"

Mean field - should be K-T transition, but OK to estimate energy scales

BEC - no confining potential
BCS-BEC crossover via Feshbach resonance

- Natural parameter in cold atom problem
  \[ \eta = (k_F a_0)^{-1} \]
  - \( a_0 \) is scattering length
- Compare to excitons
  \[ r_s = \left( \frac{9\pi}{4} \right)^{1/3} (k_F a_B)^{-1} \]
- Choose model potential of a short-range gaussian with depth \( V_0 \), and range \( r_0 \)

Well-known physics – Leggett; Nozieres & Schmitt-Rink; Randeria
Occupancy

\[ \eta = (a_0 k_F)^{-1} \]

\[ \eta = \{2.16, 1.63, 1.29, 1.07, 0.80, 0.59, 0.42, 0.15, -0.50, -9.70\} \]
Condensate wavefunction

\[ \eta = (a_0 k_F)^{-1} \]

Values:
- 2.16
- 1.07
- 0.59
- 0.15
- -0.50
- -9.70
Excitation spectrum
Comparison to low density limit

- "Universal" result in terms of single parameter $\eta$ in the low density limit (Leggett)

\[ \eta = (k_F a_0)^{-1} \]

Fix density, vary scattering length

Fix scattering length, vary density
Polaritons: Matter-Light Composite Bosons

\[ |\text{pol}\rangle = c_1 |\text{exc}\rangle + c_2 |\text{ph}\rangle \]

\[ \text{Effective Mass } m^* \sim 10^{-4} m_e \]
\[ T_{\text{BEC}} \sim 1/m^* \]

[C. Weisbuch et al., PRL 69 3314 (1992)]
Polaritons and the Dicke Model - a.k.a. Jaynes-Tavis-Cummings model

Excitons are spins (generalization to e-h liquid by Ogawa et al.)

Spins are flipped by absorption/emission of photon

\[ H = \omega \psi^\dagger \psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}} \sum_i \left[ S_i^+ \psi + \psi^\dagger S_i^- \right] \]

\[ N \sim \left( \frac{\text{photon wavelength}}{\text{exciton radius}} \right)^d \gg 1 \]

Mean field theory – i.e. BCS coherent state – expected to be good approximation

\[ |\lambda, w_i\rangle = \exp \left[ \lambda \psi^\dagger + \sum_i w_i S_i^+ \right] |0\rangle \]

\[ T_c \approx g \exp \left( -\frac{1}{gN(0)} \right) \]

Transition temperature depends on coupling constant

BEC-BCS
Condensation in the Dicke model (g/T = 2)

Increasing excitation density

Upper polariton

Lower polariton

Chemical potential (normal state)

Excitation energies (condensed state)

Coherent light

Chemical potential (condensed state)

No inhomogeneous broadening

Δ = ω − ε = 0

(Δ/ω)_{ex} = g/\sqrt{\rho_{ex}}

PR Eastham and PBL, PRB, 64, 235101 (2001)
Compare condensed polaritons to superconductor

\[ \omega \]

Particle-hole continuum

\[ 2\Delta \]

Amplitude mode

Phase mode

\[ \frac{1}{\xi} \]

\[ k \]

NB $2\Delta/E_F \ll 1; \ k_F\xi \ll 1$

Phase mode – LP
Amplitude mode – UP
Continuum – inhom. broadening

Keeling 2006

BEC-BCS
Beyond mean field: Interaction driven or dilute gas?

- Conventional “BEC of polaritons” will give high transition temperature because of light mass $m^*$
- Single mode Dicke model gives transition temperature $\sim g$

Which is correct?

$$k_B T_0 \approx \frac{\hbar^2}{2m} n$$

$$\left( \frac{g}{\hbar^2/2ma_0^2} \right) \times \left( \frac{m^*}{m} \right) \approx 10^{-4}$$

$a_o = \text{characteristic separation of excitons}$

$$a_o > \text{Bohr radius}$$

Dilute gas BEC only for excitation levels $< 10^9 \text{ cm}^{-2}$ or so

A further crossover to the plasma regime when $na_B^2 \sim 1$
Phase diagram:

- $T_c$ suppressed in low density (polariton BEC) regime and high density (renormalised photon BEC) regimes
- For typical experimental polariton mass $\approx 10^{-5}$ deviation from mean field is small

Keeling et al. PRL 93, 226403 (2004)
Dictionary of broken symmetries

- Connection to excitonic insulator generalises the BEC concept – different guises

\[ e^{\lambda \sum_k \phi_k a_{ck}^{\dagger} a_{vk}} = \prod_k \left[ 1 + \lambda \phi_k a_{ck}^{\dagger} a_{vk} \right] \]

- Rewrite as spin model

\[ S_i^+ = a_{ci}^{\dagger} a_{vi} ; \quad S_i^z = a_{ci}^{\dagger} a_{ci} - a_{vi}^{\dagger} a_{vi} \]

- XY Ferromagnet / Quantum Hall bilayer / triplons (BaCuSiO)

\[ |w_i\rangle = e^{\sum_i w_i S_i^+} |0\rangle \]

- Couple to an additional Boson mode:
  photons -> polaritons;
  molecules -> cold fermionic atoms near Feshbach resonance

\[ |\lambda, w_i\rangle = \exp[\lambda \psi^{\dagger} + \sum_i w_i S_i^+] |0\rangle \]
Spontaneous dynamical coherence

Paul Eastham and Richard Phillips PRB 79 165303 (2009)

Pump generates non-eq. distribution of excitons

\[ \langle P_{k=0} \rangle \]
\[ \langle \psi_{k=0} \rangle \]

are macroscopic - scaling with \( N^{1/2} \)

⇒ A condensate of both photons and k=0 excitons
⇒ Ringing produced by dynamical amplitude oscillations
⇒ Mean field assumed: i.e. keep only momenta of pump and k=0
Full nonlinear semiclassical dynamics ....

Quasienergy spectrum of oscillating system

- **Red lines** – derived from phase modes (LP)
- **Black lines** – amplitude modes (UP)
- **Unstable regimes** when $\text{Im} \lambda$ nonzero (Blue crosses)
Unstable regimes

OPO – like instability
amplitude modes pump phase

Attractive interaction
between amplitude fluctuations

Spectrum of dilute Bose gas with
weak attractive interactions
Ginzburg - Landau analysis

\[
\begin{align*}
\frac{i}{\partial t} \psi &= \left( \omega_0 - \frac{\hbar^2}{2m_{ph}} \nabla^2 \right) \psi + \frac{\Omega_R}{2} \left( 1 - \lambda |P|^2 \right) P \\
&\quad - i \gamma \psi + \xi + F, \\
\frac{i}{\partial t} P &= E P + \frac{\Omega_R}{2} \left( 1 - \lambda |P|^2 \right) \psi.
\end{align*}
\]

Lower and upper polariton resonantly pumped

Upper polariton resonantly pumped

Long-wavelength instability appears to develop spatio-temporal chaos
Conclusions

Main distinction between two regimes is in dynamics

- **Cold atoms**
  - no density driven crossover: dilute gas limit except at unitarity

- **Excitons**
  - crossover from BEC to BCS (i.e. electron-hole pair) when $r_s \sim 1$

- **Polaritons**
  - crossover from polariton BEC to polariton coherent state when $r_s = 10^{-4}$
  - crossover to electron-hole polariton when $r_s \sim 1$
  - however, dissipation will likely drive weak coupling laser before this ..... 

- **Electron superconductors**
  - still no confirmed examples

- **Charge and spin density waves**
  - considerable debate over role of pair-breaking vs. decoherence
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