Two-dimensional models of early-type fast rotating stars: new challenges in stellar physics

Michel Rieutord and F. Espinosa Lara

Institut de Recherche en Astrophysique et Planétologie, France

2 juillet 2013
Outline

1. Introduction
2. Two-dimensional stellar models
3. The ESTER project and some results
1. Introduction
2. Two-dimensional stellar models
3. The ESTER project and some results
Rotation is important

All the stars rotate and the youngest are the fastest.

For fast rotating stars, we need to understand:

1. The structure, the flows, the atmosphere
2. The oscillation spectrum
3. The mass loss, angular momentum loss and their impact,
4. The rotation as a function of time,
5. The effects of rotation on stellar abundances,
6. The relation rotation-magnetic activity
Indeed they are costly!

- 1D rotating models are valid when \( \Omega \rightarrow 0 \)
- A lot of physics is condensed inside adjustable (transport) coefficients
- 1D models are not usable in asteroseismology of rapid rotators
- New data from optical/IR interferometry require a 2D view...
Achernar

**Figure:** Achernar seen by VLTI (Domiciano de Souza et al. AA, 2003)
**Figure:** Vega seen by NPOI (Peterson et al. ApJ 2006), $\Omega \sim 0.93\Omega_B$
**Altair**

**Figure:** Altair seen by CHARA (Monnier et al. 2007).

- Model of a fast-spinning star
- Actual image of Altair from the CHARA Interferometer

Equator bulges and darkens as star spins faster

2.8 revolutions/day
Oscillations of Altair

**Figure:** Part of the oscillation spectrum of Altair from WIRE (Buzasi et al. 2005).
Outline

1. Introduction

2. Two-dimensional stellar models

3. The ESTER project and some results
An idealization

- We consider a lonely rotating star, of which we would like to know the evolution as a function of the initial conditions.
- We are interested in long time-scales namely those of the order of the life time of the star.
- We discard all magnetic fields.
The equations of the structure

\[
\begin{align*}
\Delta \phi &= 4\pi G \rho \\
\rho T \vec{v} \cdot \vec{\nabla} S &= -\text{Div} \vec{F} + \varepsilon_* \\
\rho (2\vec{\Omega}_* \wedge \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v}) &= -\vec{\nabla} P - \rho \vec{\nabla} (\phi - \frac{1}{2} \Omega_*^2 s^2) + \vec{F}_v \\
\text{Div}(\rho \vec{v}) &= 0.
\end{align*}
\]
The equations of the structure

Microphysics

\[
\begin{align*}
P &\equiv P(\rho, T) \quad \text{OPAL} \\
\kappa &\equiv \kappa(\rho, T) \quad \text{OPAL} \\
\varepsilon_* &\equiv \varepsilon_*(\rho, T) \quad \text{NACRE}
\end{align*}
\]
The energy flux

\[ \vec{F} = -\chi_r \vec{\nabla} T - \chi_{\text{turb}} \frac{T}{R_M} \vec{\nabla} S \]

The transport of momentum

\[ \vec{F}_v = \mu \vec{F}_\mu(\vec{v}) = \mu \left[ \Delta \vec{v} + \frac{1}{3} \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) + 2 \left( \vec{\nabla} \ln \mu \cdot \vec{v} \right) \right. \]
\[ \left. + \vec{\nabla} \ln \mu \times (\vec{\nabla} \times \vec{v}) - \frac{2}{3} \left( \vec{\nabla} \cdot \vec{v} \right) \vec{\nabla} \ln \mu \right] . \]

or any mean-field expression of the Reynolds stress.
Boundary conditions

- On pressure
  \[ P_s = \frac{2}{3} \frac{g}{\kappa} \]

- On the velocity field
  \[ \vec{v} \cdot \vec{n} = 0 \quad \text{and} \quad ([\sigma] \vec{n}) \wedge \vec{n} = \vec{0} \]

- On temperature (black body radiation)
  \[ \vec{n} \cdot \vec{\nabla} T + T/L_T = 0 \]
The last touch

\[ \int r \sin \theta \rho u_\varphi \, dV = L \]

or

\[ \nu_\varphi (r = R, \theta = \pi/2) = V_{\text{Eq}} \]
What should we expect from 2D models?

Basically: the knowledge of anisotropies of rotating stars

- Range of validity of rotating 1D models
- How pole-on stars differ from equator-on stars
- A more thorough view of the core-envelope interface
- An interpretation of oscillation spectra
Outline

1. Introduction
2. Two-dimensional stellar models
3. The ESTER project and some results
The ESTER code: a summary

- A spectral code (Chebyshev polynomials & spherical harmonics) written in C++ & Python
- This is an open source code (http://code.google.com/p/ester-project)
- Solves the structure equations in the asymptotic limit $E \ll 1$
- Tested through comparison with 1D codes at $\Omega = 0$
- Internal tests (virial, energy integral, spectral convergence, roundoff)
A note on critical rotation

At critical angular velocity

\[ \frac{GM}{R_e^2} = \Omega_c^2 R_e \quad \text{so} \quad \Omega_c^2 = \frac{GM}{R_e^3} \]

Roche model says

\[ R_e = \frac{3}{2} R_p \]

So the critical angular velocity is usually taken as:

\[ \Omega_c = \sqrt{\frac{8}{27} \frac{GM}{R_p^3}} \]

with \( R_p = R(\Omega = 0) \).

Interferometrists usually give \( \omega_{\text{interfero}} = \Omega / \Omega_c \).
But another definition is also used (in modeling):

\[ \Omega_k = \sqrt{\frac{GM}{R^3_e(\Omega)}} \]

that is the keplerian angular velocity associated with the equatorial radius at the actual angular velocity. This is closer to the true critical angular velocity (which is unknown).

Example:
if \( \omega_{\text{interfero}} = 0.88 \) then \( \Omega / \Omega_k = 0.63 \).

My recommendation: forget about \( \omega \) and use

\[ \varepsilon = \text{flatness} = \frac{R_e - R_p}{R_e} \]

which directly gives the idea of distortion. Example: \( \varepsilon = 0.17 \).
Mappings

**Figure**: The mapping.
Convergence of iterations
Gravity darkening of Achernar ($\alpha$ Eri)
Gravity darkening exponent

\[ \varepsilon = 1 - \frac{R_p}{R_e} \]

\[ T_{\text{eff}} \propto g^\beta \]

Fitted values of $\beta$ in $T_{\text{eff}} \propto g^\beta$

- : New model
+ , \( \triangle \) : ESTER models

\( (M = 2.5 - 4 M_\odot) \)

Von Zeipel’s value:

\[ \beta = 0.25 \]

(see Espinosa Lara & Rieutord 2011)
Gravity darkening exponent

\[ \epsilon = 1 - \frac{R_p}{R_e} \]

Fitted values of \( \beta \) in
\[ T_{\text{eff}} \propto \epsilon^\beta \]

- : New model
+,: ESTER models
(\( M = 2.5 - 4 M_\odot \))

Von Zeipel's value :
\[ \beta = 0.25 \]

(see Espinosa Lara & Rieutord 2011)
We have modeled 8 stars of intermediate mass:

<table>
<thead>
<tr>
<th>Star</th>
<th>M (M☉)</th>
<th>V&lt;sub&gt;eq&lt;/sub&gt; (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altair α Aql</td>
<td>1.9</td>
<td>286</td>
</tr>
<tr>
<td>Alderamin α Cep</td>
<td>1.9</td>
<td>265</td>
</tr>
<tr>
<td>Ras Alhague α Oph</td>
<td>2.2</td>
<td>242</td>
</tr>
<tr>
<td>δA Vel</td>
<td>2.27 &amp; 2.43</td>
<td>150 &amp; 143</td>
</tr>
<tr>
<td>Vega α Lyr</td>
<td>2.4</td>
<td>205</td>
</tr>
<tr>
<td>Regulus α Leo</td>
<td>4.1</td>
<td>335</td>
</tr>
<tr>
<td>Achernar α Eri</td>
<td>6.5</td>
<td>339</td>
</tr>
</tbody>
</table>
δ Vel seen by Kervella et al. 2013

Rieutord et al. Two-dimensional models of early-type fast rotating stars
### δ Velorum A and Achernar
An eclipsing binary and a Be star

<table>
<thead>
<tr>
<th>Star</th>
<th>Delta Velorum Aa</th>
<th>Delta Velorum Ab</th>
<th>Achernar (α Eri)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs. Model</td>
<td>Obs. Model</td>
<td>Obs. Model</td>
</tr>
<tr>
<td>Mass (M_☉)</td>
<td>2.43 ± 0.02</td>
<td>2.27 ± 0.02</td>
<td>8.20</td>
</tr>
<tr>
<td>R_{eq} (R_☉)</td>
<td>2.97 ± 0.02</td>
<td>2.52 ± 0.03</td>
<td>11.5</td>
</tr>
<tr>
<td>R_{pol} (R_☉)</td>
<td>2.79 ± 0.04</td>
<td>2.37 ± 0.02</td>
<td>7.9</td>
</tr>
<tr>
<td>T_{eq} (K)</td>
<td>9450</td>
<td>9560</td>
<td>9955±115</td>
</tr>
<tr>
<td>T_{pol} (K)</td>
<td>10100</td>
<td>10120</td>
<td>18013±141</td>
</tr>
<tr>
<td>L (L_☉)</td>
<td>67±3</td>
<td>51±2</td>
<td>4500±300</td>
</tr>
<tr>
<td>V_{eq} (km/s)</td>
<td>143</td>
<td>150</td>
<td>298±9</td>
</tr>
<tr>
<td>P_{eq} (days)</td>
<td>1.045</td>
<td>0.832</td>
<td>1.72</td>
</tr>
<tr>
<td>P_{pol} (days)</td>
<td>1.084</td>
<td>0.924</td>
<td>1.68</td>
</tr>
<tr>
<td>X_{env}</td>
<td>0.70</td>
<td>0.70</td>
<td>0.74</td>
</tr>
<tr>
<td>X_{core}/X_{env}</td>
<td>0.10</td>
<td>0.30</td>
<td>0.05</td>
</tr>
<tr>
<td>Z</td>
<td>0.011</td>
<td>0.011</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Inside the stars: internal differential rotation

$M = 30M_\odot$ at 98% of critical angular velocity

Espinosa Lara & Rieutord (2013)
Inside the stars: meridian circulation

$M = 5M_\odot$ at 70% of critical angular velocity

Espinosa Lara & Rieutord (2013)
Towards evolution
HR diagram track of a $7M_{\odot}$ star of constant angular momentum, $\varepsilon = 0.116 \rightarrow 0.200$
Towards evolution: increasing impact of rotation

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure}
\caption{Graph showing the relationship between $\Omega/\Omega_c$ and $X_c$.}
\end{figure}
\end{center}
Evolution of a $5M_\odot$ star on the main sequence
Outlooks

- Extend to lower masses
- take into account anisotropic mass loss (hence aml)
- take into account gravitational contraction (PMS and beyond MS)
Outlooks

But presently we can

- do asteroseismology of MS stars at any rotation rate
- invert interferometric visibilities
- determine the validity of 1D models
- monitor evolution on the MS at constant angular momentum
- ...

Rieutord et al. Two-dimensional models of early-type fast rotating stars
Introduction
Two-dimensional stellar models
The ESTER project and some results

Rieutord et al.
Two-dimensional models of early-type fast rotating stars