Quanto Implied Correlation in a Multi-Lévy Framework

Griselda Deelstra

Université Libre de Bruxelles

Joint work with

Laura Ballotta (Cass Business School, City University) & Grégory Rayée (ULB)

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In this talk we will concentrate upon a flexible multivariate FX model based on Lévy processes capturing the fluctuations in both the FX rates and equity log-returns.

FX risk and its joint evolution with the many other risk factors present in portfolios of financial institutions and insurance companies can adversely affect these portfolios.

Propose/discuss a “market consistent” method of capturing the dependence between FX rate and equities.

A “more realistic” multivariate framework than the standard framework based on the Brownian motion.

The ease of implementation of any given model is a desirable feature to have.

A “realistic” dependence structure in place between the relevant risk factors, based on a systematic risk factor and idiosyncratic risk factors.
Introduction

Objective:

- Fast and accurate calibration procedure: (Semi) Analytical pricing formula for Vanilla Options (and Quanto Products).
- We will use quanto derivatives to extract an implied correlation between FX rates and asset prices.
- We will discuss historical correlation versus implied correlation.
- Implications on tail dependence, risk measures and quanto option pricing.
- A multivariate FX pricing model requires additional properties (compared to equity modelling):
  - If $X=$USDJPY FX rate follows a process $\Rightarrow 1/X=$JPYUSD should follow the same type of process (Symmetry with respect to inversion).
  - If $X_1=$USDJPY and $X_2=$EURUSD follow the same process $\Rightarrow$ the inferred cross rate $X_3 = X_1 X_2 =$EURJPY should follow the same type of process (Symmetry with respect to triangulation).
Pricing Options written on more than one underlying asset:

- **Example: Exchange Option**

\[
E(S_1, S_2, T) = e^{-rT} E^Q[(S_2(T) - S_1(T))^+].
\]

- **Exponential multi-Lévy**

\[
S_1(t) = S_1(0) e^{Y_1(t)}
\]

\[
S_2(t) = S_2(0) e^{Y_2(t)}
\]

where \( Y_1(t), Y_2(t) \) are dependent Lévy processes (BM, VG, NIG,...).

- Restrictions on the range of possible dependencies and the set of attainable values for the correlation coefficient.

Luciano and Schoutens (2006), Baxter (2007), Brigo et al. (2007), Semeraro (2008), Luciano and Semeraro (2010), Guillaume (2013), Wallmeier and Diethelm (2012), Luciano et al. (2014), ...
Multivariate Lévy processes via Linear Transformation:

\[ S_1(t) = S_1(0)e^{Y_1(t)+a_1Z(t)}, \]
\[ S_2(t) = S_2(0)e^{Y_2(t)+a_2Z(t)}. \]

where \( Y_1(t) \), \( Y_2(t) \) and \( Z(t) \) are independent Lévy processes.

- The common process \( Z(t) \) can be considered as the systematic part of the risk
- The processes \( Y_1(t) \) and \( Y_2(t) \) can be seen as capturing the idiosyncratic shock


The literature about univariate FX rate modelling is quite extensive, but only a limited number of papers study multivariate FX models:

- See about flipping and triangular relations: e.g. Flesaker and Hughston (2000), Wystup (2006), Heath and Platen (2006), De Col et al (2013), ...

- Wishart processes: e.g. Branger and Muck (2012), Gnoatto and Grasselli (2014)

- Linear transformations:
  - Kawai (2009) with Lévy processes, but factor processes are assumed to have all the same variance.
  - Kaishev (2013) with linear combinations of independent gamma processes.
Our Multivariate FX model based on Lévy processes

- The Equity, Index,... quoted in the $l$-th currency and under the risk neutral measure $\mathbb{P}^l$:
  \[ S(t) = S(0)e^{\mu_S t + L_S(t)}, \quad S(0) > 0 \]
  with
  \[ L_S(t) = Y_S(t) + a_S Z(t), \quad \text{with triplets } (0, \sigma_S^2, \nu_S) \text{ for } Y_S, \text{ and } (0, \sigma_Z^2, \nu_Z) \text{ for } Z, \]
  \[ \mu_S = r_l - \varphi_L^l(-i) = r_l - \varphi_{Y_S}^l(-i) - \varphi_Z^l(-a_si) \]

- The FX spot rate $X_{m|l}(t)$ (the amount of currency $l$ per unit of currency $m$) under the risk neutral measure $\mathbb{P}^l$ are assumed to be of the form for $m, l = 1, \cdots, N, (m \neq l)$
  \[ X_{m|l}(t) = X_{m|l}(0)e^{\mu_{X_m} t + L_{X_m}(t)}, \quad X_{m|l}(0) > 0 \]
  with
  \[ L_{X_m}(t) = Y_{X_m}(t) + a_{X_m} Z(t), \quad \text{with triplets } (0, \sigma_{X_m}^2, \nu_{X_m}), \]
  \[ \mu_{X_m} = r_l - r_{m} - \varphi_{L_{X_m}}^l(-i) = r_l - r_{m} - \varphi_{Y_{X_m}}^l(-i) - \varphi_Z^l(-a_{X_m}i). \]

- with $Y_S, Y_{X_1}(t), \cdots, Y_{X_N}(t), Z(t)$ independent Lévy processes.
The dependence between components of the multivariate Lévy process 
\( L(t) = (L_S(t), L_{X_i}(t), i = 1, \cdots, N) \) is correctly described by the pairwise linear correlation coefficient (with \( j, k = S, X_i, i = 1, \cdots, N \))

\[
\rho_{jk}^L = \text{Corr} \left( L_j(t), L_k(t) \right) = \frac{a_j a_k \text{Var}(Z(1))}{\sqrt{\text{Var}(L_j(1))} \sqrt{\text{Var}(L_k(1))}},
\]

\( \rho_{jk}^L = 0 \) if and only if either \( a_j a_k = 0 \) or \( \text{Var}(Z(1)) = 0 \), i.e. \( Z \) is degenerate and the margins are independent.

Moreover, \( |\rho_{jk}^L| = 1 \) if and only if \( Y(t) \) is degenerate and there is no idiosyncratic factor in the margins.

Further, \( \text{sign} \left( \rho_{jk}^L \right) = \text{sign} \left( a_j a_k \right) \) and therefore both positive and negative correlation can be accommodated.

Finally, for fixed \( a_j = \bar{a} > 0 \) (resp. \( a_j = \bar{a} < 0 \)), \( \rho_{jk}^L \) is a monotone increasing (resp. decreasing) function of \( a_k \), which can take any value from \(-1\) to \(1\) (resp. from \(1\) to \(-1\)). In particular, \( \rho_{jk}^L = 0 \) if either \( \bar{a} = 0 \), or \( a_k = 0 \) or both, whilst \( |\rho_{jk}^L| = 1 \) as a limit case for \( \bar{a} \to \infty \) and \( a_k \to \infty \).
Tail Dependence

Tail dependence

a) For \( l_j, l_k \downarrow -\infty \ j \neq k, j = 1, \ldots, n \), \( \mathbb{P} ( L_j(t) < l_j, L_k(t) < l_k ) > 0 \) if and only if \( \rho_{L_jL_k} > 0 \) for all \( t > 0 \), and

\[
\mathbb{P} ( L_j(t) < l_j, L_k(t) < l_k ) \simeq \left\{ \begin{array}{ll}
\mathbb{P} \left( Z(t) < \min \left\{ \frac{l_j}{a_j}, \frac{l_k}{a_k} \right\} \right) & \text{if } a_j, a_k > 0 \\
\mathbb{P} \left( Z(t) > \max \left\{ \left| \frac{l_j}{a_j} \right|, \left| \frac{l_k}{a_k} \right| \right\} \right) & \text{if } a_j, a_k < 0.
\end{array} \right. \tag{1}
\]

b) For \( l_j, l_k \uparrow \infty \ j \neq k, j = 1, \ldots, n \), \( \mathbb{P} ( L_j(t) > l_j, L_k(t) > l_k ) > 0 \) if and only if \( \rho_{L_jL_k} > 0 \) for all \( t > 0 \), and

\[
\mathbb{P} ( L_j(t) > l_j, L_k(t) > l_k ) \simeq \left\{ \begin{array}{ll}
\mathbb{P} \left( Z(t) > \max \left\{ \frac{l_j}{a_j}, \frac{l_k}{a_k} \right\} \right) & \text{if } a_j, a_k > 0 \\
\mathbb{P} \left( Z(t) < \min \left\{ -\frac{l_j}{|a_j|}, -\frac{l_k}{|a_k|} \right\} \right) & \text{if } a_j, a_k < 0.
\end{array} \right. \tag{2}
\]

- The tail dependence behaviour is governed by the tail probabilities of the systematic risk process.
- The indices of upper/lower tail dependence are different from zero only when the margin processes are positively correlated, which is consistent with the fact that these coefficients provide a measure of concordance of jumps.
For practical purposes it is at times convenient to change the measure to any other one based on a numeraire denominated in any other of the $N$ currencies included in the FX market.

\[
\eta(t) = \frac{d\mathbb{P}^m}{d\mathbb{P}^l} \bigg|_{\mathcal{F}_t} = \frac{e^{r^t X^m|l}(t)}{e^{r^t X^m|l}(0)} = e^{-\varphi_L X_m (-i)t + LX_m(t)}
\]

This is in fact the density process of an Esscher change of measure with $h = 1$. See e.g. Gerber and Shiu (1994), Hubalek and Sgarra (2006) and Eberlein, Papapantoleon and Shiryaev (2009).

From the invariance of Lévy processes under linear transformation and Esscher change of measure, we can show that the symmetry with respect to inversion and the symmetry with respect to triangulation hold.
Change of measure

The triplets of the idiosyncratic and systematic processes are respectively under the measure $\mathbb{P}^l$ and $\mathbb{P}^m$

\[ Y_{X_m}(t) : \ (0, \sigma_{X_m}^2, \nu_{X_m}) \rightarrow \left( \sigma_{X_m}^2 + \int_{\mathbb{R}} y(e^y - 1)\nu_{X_m}(dy), \sigma_{X_m}^2, e^y\nu_{X_m} \right) \]

\[ Y_j(t) : \ (0, \sigma_j^2, \nu_j) \rightarrow \left( 0, \sigma_j^2, \nu_j \right), \quad j = S, X_k, k \neq m \]

\[ Z(t) : \ (0, \sigma_Z^2, \nu_Z) \rightarrow \left( a_{X_m}\sigma_Z^2 + \int_{\mathbb{R}} z(e^{ax_mz^2} - 1)\nu_Z(dz), \sigma_Z^2, e^{ax_mz^2}\nu_Z \right). \]
The USDJPY FX rate:

- Price today $X(0) \approx 121.52$ JPY/USD
The Nikkei 225:

- Price today $S(0) \approx 20419.77$ JAPANESE YEN (JPY)
- It is structured to reflect the Japanese stock market using the 225 top-rated Japanese companies, including such well-known issues as Honda, Canon and Sony.
A common market practice is to calibrate the model with respect to the Vanilla Options market.

\[
\min_{a_S, a_X, \mathcal{Y}_S, \mathcal{Y}_X, \mathcal{Z}} \sum_{i,j} \left( C^{mod}(S, K^i, T^j) - C^{mkt}(S, K^i, T^j) \right)^2
\]

\[
\min_{a_S, a_X, \mathcal{Y}_S, \mathcal{Y}_X, \mathcal{Z}} \sum_{i,j} \left( C^{mod}(X, K^i, T^j) - C^{mkt}(X, K^i, T^j) \right)^2
\]

And satisfy a correlation condition:

\[
\min_{a_S, a_X, \mathcal{Y}_S, \mathcal{Y}_X, \mathcal{Z}} \left( \rho_{SX}^{mod} - \rho_{SX}^h \right)^2
\]

where the pairwise linear correlation coefficient is given by

\[
\rho_{SX}^{mod} = \text{Corr}(\ln(S(t)), \ln(X(t)))
\]

\[
= \frac{a_S a_X \text{Var}(Z(1))}{\sqrt{\text{Var}(Y_S(1)) + a_S^2 \text{Var}(Z(1))} \sqrt{\text{Var}(Y_X(1)) + a_X^2 \text{Var}(Z(1))}}
\]
## Historical correlation

### Time Window?

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<tr>
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<tr>
<td>$\rho_h^{1m}$</td>
<td>43.15%</td>
<td>49.78%</td>
<td>50.93%</td>
<td>48.29%</td>
<td>43.67%</td>
<td>43.68%</td>
</tr>
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</table>
Dynamic Conditional Correlation (DCC)

Dynamic Conditional Correlation between USDJPY and Nikkei 225 index log-returns, on the basis of multivariate generalized autoregressive conditional heteroskedasticity (Garch). (Using daily data from January 2000-February 2015).
Historical Correlation:

- The market correlation between asset log-returns is usually estimated on a time window of 128 days up to (and including) the valuation date.
  - No rigorous justification for that choice,
  - Sensitive to the length of the time window,
  - Especially if it includes crisis period or not.

- Historical correlation is a “real world” measure.

- Vanilla option calibration is done in a risk neutral world.

- In the Gaussian models the correlation is not affected by changes of measures but this is not the case once the assumption of Brownian motion as driving process is abandoned.

Implied Correlation:

- Fitting the implied correlation coming from the market of derivatives written on these two assets.
  1. We need market data of Quanto type products. We use Quanto Futures.
  2. We need (Semi)-analytic pricing formula for these Quanto type products.
Nikkei 225 Index Futures

- CME customers have a choice of currencies for this product:
  1. Yen-denominated CME Nikkei 225 futures (symbol: NIY)
     - Contract Multiplier: 500 JPY
     - Contract Months: Quarterlies and Serials
     - Minimum Price Change (Tick): 5 Index points
  2. Dollar-denominated CME Nikkei 225 futures (symbol: NKD)
     - Contract Multiplier: 5 USD
     - Contract Months: Quarterlies
     - Minimum Price Change (Tick): 5 Index points

- Chicago Mercantile Exchange is the largest and most diverse financial futures and options exchange in the world.
Black Scholes framework

Japan World (foreign)
- Nikkei 225 Index FRN dynamics (JPY) under $\mathbb{P}^f$: $dS(t) = r_f S(t)dt + \sigma_S S(t)dB^\text{FRN}_S(t)$
- Yen-denominated CME Nikkei 225 futures:
  \[ F^f(0, T) = E^f[S(T)] = e^{r_f T} S(0) \]

US World (domestic)
- Nikkei 225 Index dynamics DRN (USD) under $\mathbb{P}^d$: $dS(t) = (r_f + \rho_{SX} \sigma_S \sigma_X) S(t)dt + \sigma_S S(t)dB^\text{DRN}_S(t)$
- USD-denominated CME Nikkei 225 Quanto futures:
  \[ F^d(0, T) = E^d[S(T)] = e^{(r_f + \rho_{SX} \sigma_S \sigma_X) T} S(0) \]
- Moving from the FRN to the DRN world: $\eta_t = \left. \frac{d\mathbb{P}^d}{d\mathbb{P}^f} \right|_{\mathcal{F}_t} = \frac{e^{r_d t} X(t)}{e^{r_f t} X(0)}$
  - where JPY/USD FX rate FRN dynamics (in JPY) under $\mathbb{P}^f$: 
    \[ dX(t) = (r_f - r_d) X(t) dt + \sigma_X X(t) dB^\text{FRN}_X(t) \]
  - $r_f$ is the Japan risk free interest rate
  - $r_d$ is the US risk free interest rate
Correlation Spread:

\[ F^d(0, T) - F^f(0, T) = e^{rfT}S(0)\left(e^{\rho SX\sigma S\sigma X T} - 1\right) \]

- \( \rho SX > 0 \Rightarrow F^d(USD) > F^f(JPY) \),
- \( \rho SX < 0 \Rightarrow F^d(USD) < F^f(JPY) \),
- \( \rho SX = 0 \Rightarrow F^d = F^f \)

BS Quanto adjustment \( q \):

\[ q = \rho SX\sigma S\sigma X \]

See e.g. Jaeckel (2009)
Multi Lévy framework

Japan World (foreign)

- Nikkei 225 index FRN dynamics (i.e. under $\mathbb{P}^f$):

$$S(t) = S(0)e^{(r_f + \omega_S)t + Y_S(t) + a_SZ(t)}$$

- USD-JPY FX spot rate (amount of JPY per unit of USD $X(0) = 121.52\text{ JPY/USD}$) FRN dynamics:

$$X(t) = X(0)e^{(r_f - r_d + \omega_X)t + Y_X(t) + a_XZ(t)}$$

where $Y_X(t)$, $Y_S(t)$ and $Z(t)$ are independent Lévy processes

- The common process $Z(t)$ can be considered as the systematic part of the risk
- The processes $Y_X(t)$ and $Y_S(t)$ can be seen as capturing the idiosyncratic shock

Risk Neutral adjustments:

- $\omega_S = -\varphi Y_S(-i) - \varphi Z(-a_S i)$.
- $\omega_X = -\varphi Y_X(-i) - \varphi Z(-a_X i)$. 

Griselda Deelstra (ULB) Griselda.Deelstra@ulb.ac.be
Multi Lévy framework

- JPY Nikkei 225 Index Futures (NIY) with maturity $T$:

$$F^f(0, T) = E^f[S(T)] = e^{rf \cdot T} S(0)$$

- USD Nikkei 225 Index Quanto Futures (NKD):

$$F^d(0, T) = E^d[S(T)] = E^f[\eta_T S(T)]$$

$$\eta_t = \frac{d\mathbb{P}^d}{d\mathbb{P}^f} \bigg|_{\mathcal{F}_t} = \frac{e^{rd \cdot t} X(t)}{e^{rf \cdot t} X(0)} = e^{-\varphi YX(-i)t - \varphi Z(-a_X)i t} e^{YX(t) + a_X Z(t)}$$

$$F^d(0, T) = S(0) e^{(r_f + \varphi Z(-i(a_S + a_X)) - \varphi Z(-a_X)i - \varphi Z(-a_Si)) T}$$

- Adjustment only dependent on the $Z(t)$ parameters and $a_S$ and $a_X$. 
Quanto adjustment

\[ F^d(0, T) = S(0)e^{(r_f + q)T} \]

\[ q = \varphi_Z(-i(as + ax)) - \varphi_Z(-axi) - \varphi_Z(-asi) \]

\[ = asax\sigma_Z^2 + \int_{\mathbb{R}} \left( e^{(as+ax)z} - e^{axz} - e^{asz} + 1 \right) \nu_Z(dz) \]

\[ = \text{Cov}^f(L_S, L_X) + \sum_{n=3}^{\infty} \sum_{k=1}^{n-1} \frac{a_s^{n-k} a_x^k}{k!(n-k)!} \int_{\mathbb{R}} z^n \nu_Z(dz) \]

- In the case in which the driving processes for \( S \) and \( X \) are all Brownian motions:

\[ q = asax\sigma_Z^2 = \rho_{XS} \sqrt{\text{Var}(L_X(1))\text{Var}(L_S(1))} \]

- In the more general case of any other Lévy process, higher order cumulants of the jump part of the systematic risk process also have an impact.
Quanto futures - based calibration

Calibration procedure:

- **Step 1**: The $Z$ parameters and the $a_S$ and $a_X$ coefficients can be calibrated with respect to the USD Nikkei 225 Index Quanto Futures market,

\[
\min_{a_S, a_X, Z} \sum_i \left( F_{mod}^d(0, T^i) - F_{mkt}^d(0, T^i) \right)^2
\]

- **Step 2**: Once the $Z$ parameters and the $a_S$ and $a_X$ coefficients are calibrated, we can efficiently obtain the $Y_S$, $Y_X$ parameters by calibration to Vanilla Options on individual stock $S$ (Nikkei 225) and $X$ (USDJPY FX spot rate),

\[
\min_{Y_S} \sum_{i,j} \left( C_{mod}^d(S, K^i, T^j) - C_{mkt}^d(S, K^i, T^j) \right)^2
\]
\[
\min_{Y_X} \sum_{i,j} \left( C_{mod}^d(X, K^i, T^j) - C_{mkt}^d(X, K^i, T^j) \right)^2
\]
Historical correlation - based calibration

Calibration procedure:

\[
\min_{a_S, a_X, Y_S, Y_X, Z} \sum_{i,j} \left( C_{mod}^i (S, K^i, T^j) - C_{mkt}^i (S, K^i, T^j) \right)^2
\]

\[
\min_{a_S, a_X, Y_S, Y_X, Z} \sum_{i,j} \left( C_{mod}^i (X, K^i, T^j) - C_{mkt}^i (X, K^i, T^j) \right)^2
\]

\[
\min_{a_S, a_X, Y_S, Y_X, Z} \left( \rho_{mod}^{SX} - \rho_{h}^{SX} \right)^2
\]

where the pairwise linear correlation coefficient is given by

\[
\rho_{SX}^{mod} = Corr(\ln(S(t)), \ln(X(t)))
\]

\[
= \frac{a_S a_X Var(Z(1))}{\sqrt{Var(Y_S(1)) + a_S^2 Var(Z(1))} \sqrt{Var(Y_X(1)) + a_X^2 Var(Z(1))}}
\]
European vanilla options:
FFT method of Carr-Madan (1999),

Example: the Carr-Madan type formula:

\[
C(K, T) = e^{-rf^T} E^f [(S(T) - K)^+]
= \frac{e^{-\alpha \ln(K)}}{\pi} \int_0^{+\infty} e^{(-i\nu \ln(K))} \psi(\nu) d\nu
\]

where

\[
\psi(\nu) = \frac{e^{-rf^T}}{\alpha^2 + \alpha - \nu^2 + i(2\alpha + 1) \nu} = \frac{e^{-rT} \phi_{ln S}^f (\nu - (\alpha + 1)i)}{\alpha^2 + \alpha - \nu^2 + i(2\alpha + 1) \nu}
\]

\[
\phi_{ln S}^f (u) = E^f [e^{iu \ln(S(T))}]
= E^f [e^{iu \ln(S(0)) + (rf + \omega S) T + Y_T(S(T)) + a_s Z(T)}]
= e^{i u (\ln(S(0)) + (rf + \omega S) T)} E^f [e^{iu Y_T(S(T))}] E^f [e^{iu a_s Z(T)}]
\]
**Table**: Synopsis of market data. Source: Bloomberg, CME free web platform (see http://www.cmegroup.com/). Observation date: June 13, 2014.
## Quanto futures – based calibration : VG case

### JPY Futures

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### Graphs

- **USDJPY implied volatility**
- **Nikkei 225 implied volatility**
Historical correlation - based calibration : VG case

\[
\begin{align*}
\text{USDJPY} & \\
\theta_{YX} & = -0.1362 & \theta_Z & = 0.5978 & \theta_{YS} & = -0.3825 \\
\sigma_{YX} & = 0.0294 & \sigma_Z & = 0.0956 & \sigma_{YS} & = 0.1661 \\
k_{YX} & = 0.0456 & k_Z & = 0.0307 & k_{YS} & = 0.0750 \\
\end{align*}
\]

\[
\begin{align*}
a_X & = 0.2776 & a_S & = 0.6158 \\
\text{RMSRE} & = 6.33E-03 & \rho_{h, VG} & = 28.00\% \\
\end{align*}
\]
### Numerical results

#### Calibration 13/06/2014

#### Quanto futures - based calibration

<table>
<thead>
<tr>
<th>nikkei 225 Margin process</th>
<th>USDJPY</th>
<th>Systematic process</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbb{E}L_S(1) )</td>
<td>-0.3491</td>
<td>( \mathbb{E}L_X(1) ) 0.0781</td>
</tr>
<tr>
<td>( \sqrt{\text{Var}L_S(1)} )</td>
<td>0.2129</td>
<td>( \sqrt{\text{Var}L_X(1)} ) 0.0573</td>
</tr>
<tr>
<td>( s(L_S(1)) )</td>
<td>-0.2324</td>
<td>( s(L_X(1)) ) -0.0495</td>
</tr>
<tr>
<td>( k(L_S(1)) )</td>
<td>0.1920</td>
<td>( k(L_X(1)) ) 0.1165</td>
</tr>
</tbody>
</table>

#### Historical correlation - based calibration

<table>
<thead>
<tr>
<th>nikkei 225 Margin process</th>
<th>USDJPY</th>
<th>Systematic process</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbb{E}L_S(1) )</td>
<td>-0.0144</td>
<td>( \mathbb{E}L_X(1) ) 0.0297</td>
</tr>
<tr>
<td>( \sqrt{\text{Var}L_S(1)} )</td>
<td>0.2149</td>
<td>( \sqrt{\text{Var}L_X(1)} ) 0.0571</td>
</tr>
<tr>
<td>( s(L_S(1)) )</td>
<td>-0.2814</td>
<td>( s(L_X(1)) ) -0.0393</td>
</tr>
<tr>
<td>( k(L_S(1)) )</td>
<td>0.2379</td>
<td>( k(L_X(1)) ) 0.1030</td>
</tr>
</tbody>
</table>

- Very similar volatility (meant as the square root of the process variance).
- The Nikkei 225 index shows a more pronounced left skew with thicker tails under the Historical correlation-based calibration.
- The USDJPY FX rate distribution presents these features under the Quanto futures-based calibration.
- We also note that the skewness of the distribution of systematic risk process \( Z(t) \) changes sign under the two calibration procedures.
Both calibrated VG models produce a non negligible tail dependence effect.
The upper tail dependence index is significantly higher under the Historical correlation-based calibration, indicating a stronger correlated upwards jump effect.
On the other hand, correlated downward jumps seem more likely when the parameters of the systematic risk process are recovered from Quanto futures.
This effect is a consequence of the different sign of the skewness of the distribution of the process Z observed.
95% VaR for a short position in one call option on the Nikkei 225 index over both a 1 day and a 10 days exposure periods (inspired by Eberlein et al. (1998)).

The 95% VaR is higher under the Quanto futures-implied calibration procedure, due to the fact that the resulting distribution of the Nikkei 225 index has heavier right tail, implying more likely upwards movements in the index.

Similar conclusions hold for Quanto call options.
Quanto Options

The arbitrage free price of a Quanto Call Option on the USD denominated Nikkei 225 Index Quanto Futures is given by

\[ QC(F^d(T_1; T_2), K, T_1) = e^{-r_d T_1} E^d[(F^d(T_1; T_2) - K)^+] \]
\[ = e^{-r_d T_1} Q_{adj} E^d[(S(T_1) - K^*)^+] \rightarrow (USD) \]

with

\[ Q_{adj} = e^{(r_f + q)(T_2 - T_1)}, \]
\[ K^* = \frac{K}{Q_{adj}}. \]

The arbitrage free price of a standard Quanto Call Option on the Nikkei 225 Index is given by putting \( Q_{adj} = 1 \)

The arbitrage free price of a standard Call Option on the Nikkei 225 Index is given by

\[ C(K, T) = e^{-r_f T} E^f[(S(T) - K)^+] \rightarrow (JPY) \]
### Quanto Options

<table>
<thead>
<tr>
<th>$K$</th>
<th>$C^{VG}$</th>
<th>$QC^{VG} (\rho_{SX}^{i, VG})$</th>
<th>$QC^{VG} (\rho_{SX}^{h, VG})$</th>
<th>$QC^{BS}<em>{V1} (\rho</em>{SX}^{h, BS})$</th>
<th>$QC^{BS}<em>{V2} (\rho</em>{SX}^{h, BS})$</th>
<th>$QC^{BS}<em>{V1} (\rho</em>{SX}^{i, BS})$</th>
<th>$QC^{BS}<em>{V2} (\rho</em>{SX}^{i, BS})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14625</td>
<td>639.3</td>
<td>647.4</td>
<td>645.3</td>
<td>616.0</td>
<td>643.2</td>
<td>621.4</td>
<td>649.0</td>
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<tr>
<td>14750</td>
<td>547.7</td>
<td>555.3</td>
<td>552.7</td>
<td>530.1</td>
<td>551.3</td>
<td>535.1</td>
<td>556.6</td>
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<tr>
<td>14875</td>
<td>462.1</td>
<td>469.0</td>
<td>466.1</td>
<td>451.3</td>
<td>465.4</td>
<td>455.9</td>
<td>470.1</td>
</tr>
<tr>
<td>15000</td>
<td>383.3</td>
<td>389.5</td>
<td>386.4</td>
<td>380.0</td>
<td>386.2</td>
<td>384.2</td>
<td>390.4</td>
</tr>
<tr>
<td>15125</td>
<td>312.1</td>
<td>317.6</td>
<td>314.8</td>
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<td>320.0</td>
<td>318.4</td>
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<tr>
<td>15250</td>
<td>249.4</td>
<td>254.2</td>
<td>252.0</td>
<td>260.2</td>
<td>251.6</td>
<td>263.5</td>
<td>254.7</td>
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<tr>
<td>15375</td>
<td>195.8</td>
<td>199.9</td>
<td>198.6</td>
<td>211.5</td>
<td>197.6</td>
<td>214.3</td>
<td>200.3</td>
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<tr>
<td>15500</td>
<td>151.7</td>
<td>155.1</td>
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<td>171.2</td>
<td>155.4</td>
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<tr>
<td>15625</td>
<td>116.5</td>
<td>119.4</td>
<td>118.6</td>
<td>134.6</td>
<td>117.8</td>
<td>136.6</td>
<td>119.6</td>
</tr>
</tbody>
</table>

- **Option prices VG setting:**
  - $C^{VG}$: vanilla call option price computed with Quanto futures-based calibration parameters
  - $QC^{VG} (\rho_{SX}^{i, VG})$: Quanto call option price computed with Quanto futures-based calibration parameters
  - $QC^{VG} (\rho_{SX}^{h, VG})$: idem with Historical correlation-based calibration parameters

- **Option prices BS setting:**
  - $QC^{BS}_{V1}$: Quanto call option price computed using ATM implied volatilities
  - $QC^{BS}_{V2}$: Quanto call option price computed using ATM USDJPY implied volatility and the Nikkei 225 implied volatility curve

$\rho_{SX}^{i, BS} = 88\%$, $\rho_{SX}^{h, BS} = \rho_{SX}^{h} = 28\%$. Maturity $T = 28$ days.
Quanto Option implied correlation

- **Left hand panel:**
  - $\rho_{i,BS}^K(K; v_1)$: Quanto call type implied correlation in function of strike $K$, extracted in a BS setting where the Nikkei 225 index and the USDJPY FX rate volatility are set at their at-the-money values.
  - $\rho_{i,BS}^K(K; v_2)$: strike corresponding Nikkei 225 index implied volatility instead

- **Right hand panel:**
  - $QC_{v_1}^{BS}$ and $QC_{v_2}^{BS}$, Quanto call on the Nikkei 225 index quotes in function of strike $K$ computed in both settings under BS model.
  - $QC^{VG}$, Quanto call on the Nikkei 225 index quotes in function of strike $K$ computed using Quanto-futures based calibration under VG model.
Conclusions

Conclusions:

- We have developed a multi-variate Lévy model for the joint dynamics of FX exchange rates and asset prices, based on the factor representation of Ballotta and Bonfiglioli (2014).

- Calculations in this model are easy by the characteristic functions (independence between the components); by propositions about the characteristic exponents under an Esscher change of measure and by the well-known FFT method which meanly depends upon the characteristic function (under the right probability measure).

- Implied correlation (by Quanto futures contracts and Quanto option contracts) can be quite different from the historical correlation, and can also behave differently in the time evolution.

- This has a particular impact on the indices of upper and lower tail dependence: Information based on historical correlation lead to underestimation of the probability of a joint downward movement.

- This has also a particular impact on the computation of risk measures related to portfolios containing these products: e.g. the VaR of short positions of a call.

- The quanto adjustment in the formula of Quanto futures equals a covariance term and terms driven by the jump part of the systematic risk factor.


References


References


Thank you for your attention