A Lagrangian model of Copepod dynamics in turbulent flows

Enrico Calzavarini
Laboratoire de Mécanique de Lille (LML)
Université de Lille
France

in collaboration with Hamidreza Ardeshiri
Francois G. Schmitt , Sami Soussi, Ibtissem Benkeddad
Laboratory of Oceanology and Geosciences (LOG) Wimereux, France
Federico Toschi TU/Eindhoven, The Netherlands

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• Single Copepod dynamics (the essential)
• Results from a Simple Experiment
• A Lagrangian Copepod Model
• Numerical experiment with Turbulence
• Analysis and Main lessons
• Perspectives
Copepods size and shape

- Group of crustaceans
- 9000 species
- **size 0.5mm to 15 mm**
- aspect ratio 3-5
Copepods locomotion

Main feature: **Swimming by jumps**
Copepods locomotion

Jump response to light and mechanical signals

**Acartia tonsa:**
- Stimulus (dashed line)
- Initiation of the escape response (3 ms delay for mechanical)
- Acceleration during initial escape ~10 g!

**Undulina vulgaris**, *Lenz et al.*, (1999)
Copepods react to deformation rate

*Copepods react to deformation rate*  

*)*  

A*cartia tonsa* estimated threshold $\sim 0.4 \text{ s}^{-1}$  

Threshold is copepod size dependent  

Sensitivity to strain rate $\sim 0.025 \text{ s}^{-1}$  

Woodson et al. (2005, 2007)
Copepods reaction to disturbances

Copepods comfort region

Copepods reaction to disturbances

Direction of Escape

Escapes often begin with rapid reorientation away from the source of the disturbance, with maximum turning rates of about 30° $ms^{-1}$
Simple experiment

Copepods cultures at LOG Lab in Wimereux (France)

Response to light stimuli of copepod “Eurytemora affinis” in still water
Jump Data Analysis (1)

Velocity track

Velocity |V| (m/s) vs. Time (ms)

0 0.05 0.1 0.15 0.2 0.25
0 5000 10000 15000 20000
Jump Data Analysis (2)

Zooming in...
Jump Data Analysis (3)
Jump Data Analysis (4)

\[ u_J e^{-t/\tau_J}, \quad u_J = 0.0939 \text{ m/s}, \quad \tau_J = 8.87 \text{ ms} \]

\[ u_J e^{-t/\tau_J} + \text{noise, noise} = 0.005 \text{ m/s} \]

**u_J \sim 10 \text{ cm/s}**

**\( \tau_J < 10 \text{ ms} \)**
Hydrodynamical forces on a particle in a flow

Point-like model

\[ \nabla \rho_p \ddot{X} = F_P + F_{AM} + F_D + F_H + F_L + F_B + F_{FX} \]

- **Pressure gradient**
- **Added mass**
- **Stokes drag**
- **History drag**
- **Lift**
- **Buoyancy**
- **Faxen**

\[ F_P = \nabla \rho_f \frac{DU}{DT} \]

\[ F_{AM} = \nabla \rho_f C_M \left( \frac{DU}{DT} - \ddot{X} \right) \]

\[ F_D = 6 \pi a \mu (U - \dot{X}) \]

\[ F_H = 6 a^2 \sqrt{\pi\nu} \int_0^T \frac{1}{\sqrt{T - \tau}} \frac{d(U - \dot{X})}{d\tau} d\tau \]

\[ F_L = \nabla \rho_f C_L (U - \dot{X}) \times \Omega \]

\[ F_B = \nabla (\rho_p - \rho_f) g \hat{e}_z \]

\[ F_{Fx} : \left( \frac{DU}{DT}, U \right) \rightarrow \left( \langle \frac{DU}{DT} \rangle_U, \langle U \rangle_s \right) \]

2a
Hydrodynamical forces on a particle in a flow (2)

Major approximations

\[ \nabla \rho_p \ddot{\mathbf{X}} = \mathbf{F}_P + \mathbf{F}_{AM} + \mathbf{F}_D + \mathbf{F}_H + \mathbf{F}_L + \mathbf{F}_B + \mathbf{F}_F \]

- Pressure gradient
- Added mass
- Stokes drag
- History drag
- Lift
- Buoyancy
- Faxen

Formal solution

\[ \dot{\mathbf{X}}(\tau) = e^{-\tau/\tau_p} \int_0^{\tau} \frac{e^{t/\tau_p}}{\tau_p} \mathbf{U}(t) dt + \dot{\mathbf{X}}(0) e^{-\tau/\tau_p} \]

Initial condition

\[ \dot{\mathbf{X}}(0) = \mathbf{U}(0) + \mathbf{U}_J \]

Slowly varying

\[ \dot{\mathbf{X}}(\tau) \approx \mathbf{U}(0) + \mathbf{U}_J e^{-\tau/\tau_p} \]

Our guess

\[ \tau_p \sim \tau_J \]
A minimal model

\[
\dot{x}(t) = u(x(t), t) + J(t, t_i, t_e, \dot{\gamma}, p)
\]

**Modified Chlamydomonas Model**

\[
J(t, t_i, t_e, \dot{\gamma}, p) = H[\dot{\gamma}(t_i) - \dot{\gamma}_T] H[t_e - t] u_J e^{\frac{t_i - t}{\tau_J}} p(t_i)
\]

**Terms:**
- **Copepod velocity**
- **Fluid velocity**
- **Jump velocity term**
- **Shear-rate trigger**
- **Inhibition time**
- **Exponential decay**
- **Orientation**
- **Jump initial time**
- \( t_i = t \) if \((\dot{\gamma}(t) > \dot{\gamma}_T) \cap (t > t_e)\)
- **Jump end time**
- \( t_e = t_i + c \tau_J = t_i + \log(10^2) \tau_J \)
- **Upper shear-rate threshold value**
- \( \dot{\gamma} = \sqrt{2 S : S} \)
A minimal model (orientation)

Sphere

\[ \dot{p}(t) = \Omega \cdot p(t) \quad \Omega_{ij} = \frac{1}{2}(\partial_i u_j - \partial_j u_i) \]
antisymmetric gradient tensor

Ellipsoid: Jeffery equation

\[ \dot{p}(t) = \left( \Omega + \frac{\alpha^2 - 1}{\alpha^2 + 1} (S - p^T(t) \cdot S \cdot p(t)) \right) \cdot p(t) \]

\[ S_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) \quad \text{symmetric gradient tensor} \]

\[ \alpha \equiv l/d \quad \text{Aspect ratio} \]
A minimal model (summary)

Model assumptions:

- Rigid particle (spherical or ellipsoidal)
- neutral, homogeneous density
- Hydrodynamical forces: only Stokes Drag
- passive orientation

- Fixed response to external flow disturbances
- React to high shear-rate intensity (i.e. scalar single threshold)
- no-memory of previous jumps

- Model parameters: $u_j, \tau_j, \gamma_T, (t_e-t_i)$
Lagrangian Copepods (LC) in a turbulent flow

In turbulent ocean flows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Range</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v)</td>
<td>(m^2s^{-1})</td>
<td>(\sim 10^{-6})</td>
<td>(10^{-6})</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>(m^2s^{-3})</td>
<td>(10^{-8} - 10^{-4})</td>
<td>(10^{-6})</td>
</tr>
<tr>
<td>(\eta)</td>
<td>(m)</td>
<td>(3 \times 10^{-3} - 3 \times 10^{-4})</td>
<td>(10^{-3})</td>
</tr>
<tr>
<td>(\tau_\eta)</td>
<td>(s)</td>
<td>10 - 0.1</td>
<td>1</td>
</tr>
<tr>
<td>(u_\eta)</td>
<td>(ms^{-1})</td>
<td>(3 \times 10^{-4} - 3 \times 10^{-3})</td>
<td>(10^{-3})</td>
</tr>
<tr>
<td>(Re_\lambda)</td>
<td>(-)</td>
<td>(O(10^2))</td>
<td></td>
</tr>
</tbody>
</table>

\[ u_J/u_\eta \sim O(10^2) \]
\[ \tau_J/\tau_\eta \sim O(10^{-2}) \]
\[ \tau_\eta \dot{\gamma} T \sim O(1) \]
Numerical Experiment

Lagrangian — Eulerian

LC model
\[ \dot{x}(t) = u(x(t), t) + J(t, t_i, t_e, \dot{\gamma}, \rho) \]

Tracking of \( \sim 10^6 \) LC families with different
u_j
\( \dot{\gamma}_T \)
\( \tau_j \sim 10^{-2} \tau_\eta \)

Homogeneous Isotropic Turbulence
\[ \partial_t u + u \cdot \nabla u = -\nabla p/\rho + \nu \Delta u + f \]
\[ \nabla \cdot u = 0 \]

pseudo-spectral algorithm
\( \text{Re}_\lambda \sim 80 \)
N = 128^3
3-periodic cube

Will we see clusters?
Numerical Experiment

Lagrangian — Eulerian

LC model

\[ \dot{x}(t) = u(x(t), t) + J(t, t_i, t_e, \gamma, p) \]

Tracking of \(~10^6\) LC families with different

\[ u_j \]

\[ \dot{\gamma}_T \]

\[ \tau_j \sim 10^{-2} \tau_\eta \]

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Numerical Experiment

Lagrangian — Eulerian

LC model
\[ \dot{x}(t) = u(x(t), t) + J(t, t_i, t_e, \dot{\gamma}, p) \]

Tracking of \(~10^6\) LC families with different

\[ \begin{align*}
  u_j \\
  \dot{\gamma}_T \\
  \tau_j \sim 10^{-2} \tau_\eta
\end{align*} \]

Homogeneous Isotropic Turbulence
\[ \begin{align*}
  \partial_t u + u \cdot \nabla u &= -\nabla p / \rho + \nu \Delta u + f \\
  \nabla \cdot u &= 0
\end{align*} \]

pseudo-spectral algorithm
\[ \text{Re}_\lambda \sim 80 \]
\[ N = 128^3 \]
3-periodic cube
**Numerical Experiment**

**Lagrangian — Eulerian**

**LC model**

\[
\dot{x}(t) = u(x(t), t) + J(t, t_i, t_e, \dot{\gamma}, p)
\]

**Tracking of \(\sim 10^6\) LC**

families with different

\[
\begin{align*}
\dot{u}_j \\
\dot{\gamma}_T \\
\tau_j \sim 10^{-2} \tau_\eta
\end{align*}
\]

**Homogeneous Isotropic Turbulence**

\[
\begin{align*}
\partial_t u + u \cdot \nabla u &= -\nabla p / \rho + \nu \Delta u + f \\
\nabla \cdot u &= 0
\end{align*}
\]

**pseudo-spectral algorithm**

\[
\begin{align*}
\text{Re}_\lambda &\sim 80 \\
N &= 128^3 \\
\text{3-periodic cube}
\end{align*}
\]
Spatial distribution

2D slice of thickness $\eta$, Jump intensity $\frac{u_J}{u_\eta} = 250$, $\tau_J = 10^{-2} \tau_\eta$

Jump shear-rate threshold

1) **Low**, many jumps, few dense islands

\[ \tau_\eta \dot{\gamma}_T = 0.35 \]
2D slice of thickness $\eta$, Jump intensity $u_J/u_\eta = 250$, $\tau_J = 10^{-2} \tau_\eta$

Jump shear-rate threshold

1) **Low**, many jumps, few dense islands

2) **Intermediate**, efficient escape, sheet-like clusters

$\tau_\eta \dot{\gamma}_T = 0.92$
Spatial distribution

2D slice of thickness $\eta$, Jump intensity $u_J/u_\eta = 250$, $\tau_J = 10^{-2} \tau_\eta$

Jump shear-rate threshold

1) **Low**, many jumps, few dense islands

2) **Intermediate**, efficient escape, sheet-like clusters

3) **High**, efficient avoiding of extreme events, fading clusters

$\tau_\eta \dot{\gamma}_T = 1.77$
Movie

$t/\tau_\eta = 0$

$\tau_\eta \dot{\gamma}_T = 0.92$
Grassberger-Procaccia $D_2$

$$C(r) = \frac{2}{N(N-1)} \sum_{i<j} H(r - |X_i - X_j|)$$

$$D_2 = \lim_{r \to 0} \frac{\log C(r)}{\log r}$$

Correlation Dimension

- **Low mobility**
- **Low reactivity**

- **High mobility**
- **High reactivity**

- Peak velocity intensity
- Shear rate threshold
Correlation Dimension

$$D_2 \quad \text{vs.} \quad \text{peak velocity intensity}$$

$$D_2 \quad \text{vs.} \quad \text{shear rate threshold}$$

Saturation for huge values of peak jump intensity

$$u_j > u_{rms} \sim 30 \, u_\eta$$

Optimal threshold value why?
Clustering mechanism (1)

1) Probability of a successful jump
\[ \sim \mathcal{V} \dot{\gamma} < \dot{\gamma}_T \]

2) Rate of jumps
\[ \sim \mathcal{V} \dot{\gamma} > \dot{\gamma}_T \]

3) Clustering
\[ \sim \mathcal{V} \dot{\gamma} < \dot{\gamma}_T \cdot \mathcal{V} \dot{\gamma} > \dot{\gamma}_T \]
\[ = \mathcal{V} \dot{\gamma} > \dot{\gamma}_T \cdot (\mathcal{V}_{tot} - \mathcal{V} \dot{\gamma} > \dot{\gamma}_T) \]

Maximum for
\[ \mathcal{V} \dot{\gamma} > \dot{\gamma}_T = \mathcal{V}_{tot} / 2 \]
Clustering mechanism (2)

Fraction of time Time spent in alert regions \( \dot{\gamma} > \dot{\gamma}_T \)

\[
\frac{\langle T_\gamma \rangle_{\dot{\gamma}_T}}{T_{tot}}
\]

\( \nu_{\dot{\gamma} > \dot{\gamma}_T} = \nu_{tot}/2 \)
PDF of Velocity

\[ \frac{\dot{x}}{u_\eta} \]

\[ \tau_\eta \dot{\gamma}_T = \begin{cases} 0.21, & \alpha = 0.5 \\ 0.5, & \alpha = 0.7 \\ 0.7, & \alpha = 1.2 \\ 1.2, & \alpha = 1.77 \end{cases} \]

\[ \dot{x} = U + \alpha p u J e^{-t/\tau_J} \]

Gaussian random variable
jumping fraction?
random vector
flat random [0, te]
Summary

- LC model exhibits clustering in turbulence $D_2 \sim 2.3$
- Necessary conditions for Clustering:
  1) high jump speed $u_J > u_{rms}$
  2) sharp sensitivity to shear rate $O(\tau^{-1})$
- Clustering comes from inhomogeneity rather than anisotropy of the model
  (Excluded volume mechanism)
- Different mechanism from the one identified for motile algae (gyrotaxis induced).

LC Model v1.0
Many possible extensions...
Effect of Aspect ratio

Particle orientation dynamics
no impact on clustering
or velocity distribution