Abstracts

Nonlinear PDEs on Metric Graphs and Branched Networks

Sometimes the graphs are not what they seem

Pavel Exner

Spectra of periodic quantum systems are usually expected to be absolutely continuous, consisting of bands and gaps. Our aim is to show that if the systems in question are quantum graphs, some uncommon spectral features may arise. Using simple examples, we show that the spectrum may then have a pure point or a fractal character, and also that it may have only a finite but nonzero number of open gaps. Furthermore, motivated by recent attempts to model the anomalous Hall effect, we investigate a class of vertex couplings that violate the time reversal invariance. We will find spectra of lattice graphs with the simplest coupling of this type, the one with 'maximum' non-invariance, and demonstrate that it depends substantially on the lattice topology. Finally, we will discuss an interpolation between this 'maximal' coupling and the usual δ-type one.

Approximations of metric graphs and their Laplacians by thick graphs"

Olaf Post

We discuss the convergence of Neumann Laplacians on thin neighbourhoods of metric graphs (also called "thick graphs") towards Laplacians on metric graphs. We will explain the concept of norm resolvent convergence for operators acting in different Hilbert spaces. Finally, I will give some hints how to treat some mild non-linear operators.

Surgery Principles for the Spectral Analysis of Quantum Graphs

Gregory Berkolaiko

Bifurcations from the constant solution of nonlinear Schroedinger equation on a graph are governed by the eigenvalues of the linear Laplace equation. In a graph with a reflection symmetry, to understand whether the first bifurcation will produce a symmetric or anti-symmetric solution, it is necessary to compare the eigenvalues of subgraphs with Dirichlet and Neumann conditions. Indispensable tools for this analysis are the so-called "surgery principles": theorems describing the change in eigenvalues under various modifications of the graph's geometry and vertex conditions.

We will present a collection of surgery principles such as gluing vertices, inserting a subgraph at a given vertex, transplanting volume from one part of the graph to another, symmetrizing a part of the graph.
The effect of these operations on the spectrum (especially the first non-trivial eigenvalue) will be discussed, with a particular attention given to the cases of equality. Some applications will be given, both to estimating spectral gap of a graph in terms of its doubly connected component and to bifurcation analysis for NLS on a family of symmetric graphs.

This talk is based on a joint work with J.Kennedy, P.Kurasov and D.Mugnolo and inspired by joint work with J.Marzuola and D.Pelinovsky.

Effective Hamiltonian for a graph with a small compact core

Claudio Cacciapuoti

We consider a metric graph obtained by attaching several external edges to a compact core of size ε. On such a graph we consider an operator H_ε which is a selfadjoint realization of the Laplacian plus potential terms.

When ε goes to zero the compact core of the graph squeezes to one point, and the whole graph resembles a star-graph with one central vertex which we denote by v. Correspondingly, one expects the operator H_ε to converge to a limit operator H on the star-graph. The limit operator H should be selfadjoint and defined by suitable boundary conditions in the vertex v. We attempt to characterize the limit boundary conditions in terms of the asymptotic behavior (for $\varepsilon \to 0$) of the eigenvalues of an auxiliary operator defined only on the compact core of the graph.

On the Ground state of the NLS on a general graph
Dominico Finco

We discuss the existence of the ground state for the NLS on a general graph in presence of delta interactions in the vertx and in the small mass regime.

Nonlinear Dirac equation on graphs with localized nonlinearities: bound states and nonrelativistic limit
Lorenzo Tentarelli

I will present some recent results on the Nonlinear Dirac (NLD) equation on metric graphs with localized Kerr nonlinearities, in the case of Kirchhoff-type vertex conditions. Precisely, I will discuss existence and multiplicity of the bound states (arising as critical points of the NLD action functional) and I will show that, in the L^2-subcritical case, they converge to the bound states of the NLS equation in the nonrelativistic limit.
This is a joint work with W. Borrelli and R. Carlone.

Effective dynamics for Nonlinear PDEs on infinite periodic metric graphs
Guido Schneider

We discuss:
1) The validity of the NLS approximation for oscillating wave packets,
2) Diffusive stability of the trivial solution for nonlinear diffusion equations with irrelevant nonlinearity, and
3) Some remarks on the tight binding limit.

On the NLS equation with the logarithmic nonlinearity and point interactions on a star graph

Nataliia Goloshchapova (University of São Paulo)

Let $\mathcal{G}$ be a star graph, i.e. N half-lines attached to the common vertex $\nu = 0$. We consider the following NLS-log equation with δ- and δ'-interaction on $\mathcal{G}$

$$i\partial_t \mathbf{U} = -\Delta_{\text{int}}\mathbf{U} - \mathbf{U}\log|\mathbf{U}|^2, \qquad (*)$$

where $\mathbf{U}(t,x) = (u_j(t,x))_{j=1}^N : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{C}^N$. In the case of δ-interaction the operator Δ_{int} is defined by $\Delta_{\text{int}} = \Delta_\alpha$, where

$$\Delta_\alpha \mathbf{V}(x) = \left(v_j''(x)\right)_{j=1}^N, \quad D(\Delta_\alpha) = \left\{ \begin{array}{c} \mathbf{V} \in H^2(\mathcal{G}) : v_1(0) = \ldots = v_N(0), \\ \sum\limits_{j=1}^N v_j'(0) = \alpha v_1(0),\ \alpha \in \mathbb{R}\setminus\{0\} \end{array} \right\}.$$

For δ'-interaction one has $\Delta_{\text{int}} = \Delta_\lambda$, where

$$\Delta_\lambda \mathbf{V}(x) = \left(v_j''(x)\right)_{j=1}^N, \quad D(\Delta_\lambda) = \left\{ \begin{array}{c} \mathbf{V} \in H^2(\mathcal{G}) : v_1'(0) = \ldots = v_N'(0), \\ \sum\limits_{j=1}^N v_j(0) = \lambda v_1'(0),\ \lambda \in \mathbb{R}\setminus\{0\} \end{array} \right\}.$$

We will discuss briefly the well-posedness of equation $(*)$ in the energy domain. Furthermore the talk will be concerned with stability/instability of the standing wave solutions $e^{i\omega t}\mathbf{\Phi}(x)$ to equation $(*)$. Our method is based on the classical Grillakis-Shatah-Strauss method, the theory of extensions of symmetric operators, and the analytic perturbations theory.

Stationary States on Bounded and Unbounded Graphs in the Limit of Large Mass

Gregory Berkolaiko, Jeremy L. Marzuola, and Dmitry E. Pelinovsky

In this work, we elaborate on an asymptotic representation of the stationary states on a graph in the limit of large mass as shrinking NLS solitons. We show that this approach is applicable to rather general bounded and unbounded graphs. In particular, if the bounded graph has a center of symmetry, we study global minimizer of energy is selected at the symmetric or asymmetric state with respect to the central point of the graph. If the graph is unbounded, we would like to find a criterion for existence of global minimizers of energy and to compare it with the one obtained in the recent studies by Adami-Serra-Tilli. We will also illustrate numerically the validity of predictions of the asymptotic method.

Canonical perturbation approach to nonlinear quantum graphs

Sven Gnutzmann

We consider stationary solutions of the nonlinear Schroedinger equation on a metric graph with finitely may edges and vertices. For cubic nonlinearity it is well known that local solutions on the edges may be expressed in terms of Jacoby elliptic functions.

They uniquely express the wavefunction and its derivative at one end of an edge in terms of given values at the other end. Formally this reduces the problem of finding global solutions of the Nonlinear Schroedinger equation on the graph to a finite set of nonlinear algebraic equations that may be solved numerically. In the

asymptotic short-wave length limit canonical perturbation theory may be applied to simplify the solution process.
We will apply canonical perturbation theory to find global solutions on some basic graphs and identify various asymptotic regimes.

Wave dynamics on networks: sine-Gordon vs shallow water

Jean-Guy Caputo

Many fluid networks can be described by a graph, i.e. a set of vertices connected by links of a given extension. Waves can then propagate in such systems, an example is an inundation front in a river basin. For modelling purposes, it is desirable to study the system as a whole, to predict where flooding will occur.
The original system is typically a planar graph where the edges are two or three dimensional; the aim is to reduce this to a 1D effective model where the junctions are handled in an appropriate way.
We have been studying such network systems - assuming that they obey systems of conservation laws and have established the interface conditions at each node using a homothetic reduction. We present two main classes, first nonlinear wave equations including sine-Gordon and reaction-diffusion equations and then the shallow water equations. The former class yields simple coupling conditions while the latter has conditions that depend on the angle of the junction. We compare 2D and 1D results for both models on forks and show results for a full network for the sine-Gordon equation.

The Korteweg-de Vries equation on metric graphs

Christian Seifert

During the last two decades there has been a tremendous study of differential operators on metric graphs. However, most of the literature deals with first and second order linear differential operators. While first order operators are used for transport, second order operators can model diffusion and appear as Schrödinger operators in quantum mechanical models. In this talk we want study the Korteweg-de Vries equation on metric graphs, i.e. a third order non-linear differential equation of the form

$$\partial_t u = \alpha \partial_x^3 u + \beta \partial_x u + c u \partial_x u.$$

with constants α, β and c. We first focus on the linear part which yields the Airy equation, where we classify coupling conditions at the vertices yielding a well-posed Cauchy problem. In the second part we consider solitons for the KdV equation on a metric graph.
The talk is based on joint work with Delio Mugnolo and Diego Noja.

Quantum graphs: Ambartsumian-type theorems

Pavel Kurasov

The classical Ambartsumian theorem in inverse spectral theory states that the spectrum of a Schrödinger operator on an interval coincides with the spectrum of the Laplacian if and only if the potential is identically equal to zero (provided Neumann boundary conditions are assumed at the endpoints). The goal of this talk is to provide a comprehensive study how this theorem can be extended for the case of quantum graphs – Schrödinger operators on metric graphs. The geometry and topology of the metric graph comes into play in addition to the real-valued potential and boundary (or vertex) conditions.
We first prove a direct analog of the classical Ambartsumian theorem stating that a Schrödinger operator on a metric graph is isospectral to the Laplacian on an interval if and only if the metric graph coincides with the interval and the potential is identically zero, provided so-called standard vertex conditions are assumed at the

vertices. The assumption that the vertex conditions are standard cannot be removed in complete analogy with the classical variant of the theorem.
We continue by discussing isospectrality of Schrödinger operators to Laplacians on metric graphs. It is proven in particular that the spectrum of a Schrödinger operator coincides with the spectrum of a Laplacian (on a may be different metric graph) only if the potential is identically zero. Proving this result the theory of almost periodic functions is used.
This is a joint work with Jan Boman and Rune Suhr.

Classical and quantum wave dynamics for time-dependent metric graphs
Igor Popov

The report is devoted to non-stationary problems for metric graphs. Two types of metric graphs are considered: star-like graph and graph with a loop. The lengths of the graph edges vary in time. We deal with a few differential time-dependent operators on the graph edges: Schrodinger, Dirac, Schrodinger with a magnetic field (Landau), wave, Lame operators. Wave dynamics for each case are studied
The work was partly supported by RSF grant 16-11-10330. It is a joint work with D.S.Nikiforov and I.V.Blinova.