Scientific Summary

In the last few decades the focus of the mathematical physics community lied with insulators, and we would like to change that. There were two major driving forces for that: the discovery of *topological insulators* with the Quantum Hall Effect uncovered the surprising role that topology and symmetries can play. The second revelation was that disorder can suppress conductivity, a phenomenon known as *Anderson localization*. These two lines of investigation have stimulated the development of new mathematical approaches that draw upon an incredible number of disciplines, particularly topology, functional analysis, semiclassics, probability theory, and noncommutative geometry. Our idea is to take a fresh look at an old topic and ***bring recent advances in e.g. semiclassics and algebraic topology from topological insulators to metals***.

1. Refinements of Lifshitz's classical approach

The link of conduction properties to **topology, geometry and conduction properties in metals** can be traced back to the 1950s and is still being investigated today by the likes of Novikov and Maltsev. Lifshitz appealed to physical intuition when he replaced quantum dynamics by simpler by *semiclassical equations of motion*. Due to energy conservation the classical trajectories are constrained to the **Fermi surface**, the energy level set for the Fermi energy. The approach has two major shortcomings we wish to address: first, it only works for perfectly periodic systems — as soon as *perturbations* are present, **even the *very definition* of the Fermi surface has eluded mathematicians**. And secondly, the semiclassical equations are *only correct to leading order*. To explore this purely classical approach, we welcome researchers on the global geometry of integrable systems, both from the point of view of differential geometry and algebraic topology.

2. A critical look at Lifshitz's semiclassical approximation with modern tools

Lifshitz appealed to physical intuition when he replaced quantum dynamics by simpler classical dynamics. Thanks to advances in semiclassics we should be able to make Lifshitz's arguments mathematically rigorous and **derive semiclassical equations** that include **subleading corrections**. These equations include **external fields** that drive currents. For insulators, the subleading terms have been essential to correctly predict currents from the semiclassical equations of motion. It stands to reason they will be important to describe conductivity in metals, too. Consequently, we welcome researchers in semiclassical theory both, from the point of view of integrable systems and magnetic pseudodifferential theory.

3. Fully quantum mechanical approach to deal with disorder and many-body effects

The semiclassical definition of the Fermi surface rests on a single-particle quantum model where on a microscopic scale the system is, to leading order, periodic. As soon as we want a more realistic model that includes *disorder* or *electron-electron interactions*, semiclassical techniques break down, and we have to find a **“quantum native” approach**. Operator-algebraic methods from **noncommutative geometry** have been used to great success to justify *Kubo-type formulas* for conductivity coefficients. In this setting, the Fermi surface can be associated to the notion of “eigenstate” of an operator in a certain noncommutative operator algebra. The question now is whether the notion of eigenstate is *still valid in the presence of perturbations that break the periodicity* of the system, including **random** perturbations of the lattice and *many-body interactions*. The assumption that the material is metallic means the states at the Fermi energy are delocalized. While the phenomenon of Anderson localization, associated to insulators, is by now well-known in the literature, the *presence* of electron transport, or delocalization, remains one of the biggest open problems in the context of **random Schrödinger operators**. To this end we welcome researchers in disordered and many-body quantum systems, working in both the noncommutative operator algebraic setting and on random Schrödinger operators.