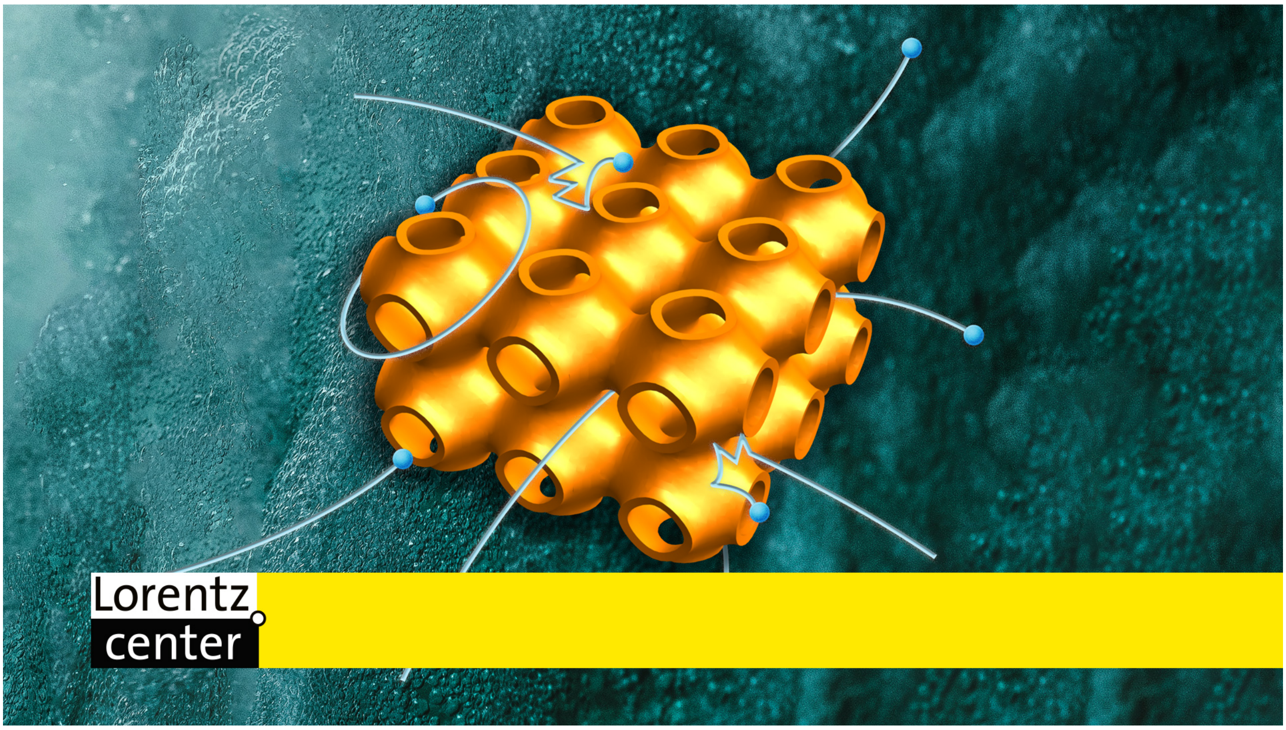
Learning from Insulators

New Trends in the Study of Conductivity of Metals



Aims

The purpose of this workshop is to promote a large-scale collaboration across different fields of mathematical physics in order to systematically investigate electronic transport in metals. To this end, we will draw inspiration on recent progress made in the area of topological insulators, interacting systems and disordered systems.

Since we aim to bring together a broad and diverse community of specialists in these fields, prior to the workshop we will make available four mini-courses to lay the ground and common language for the discussions that will take place during the week of the workshop (see next section).

During the workshop, we aim at addressing the following questions, inspired by Lifshitz's approach to the study of conductivity in metals using semiclassical equations of motion:

- A refinement of Lifshitz equations of motions as an approach to study conductivity and the study of fate of the Fermi surface in presence of perturbations;
- Derivation of a semiclassical theory that includes subleading corrections and topological terms and takes into account external fields;
- A fully quantum mechanical description that allows us to consider disorder and many-body effects.

Format

To structure the discussion, we will ask some of the attendees to take notes during the working groups; these (with the permission of the participants) will eventually become part of a summary document containing interesting questions and ideas for the future. Among the participants of the working groups, one of them will be chosen to serve as moderator. Their task will be to take initiative, promote and moderate discussions, and report back to the whole group. We hope this will help foster lively and productive discussions and motivate everybody to participate.

All the material will be summarised alongside a summary of the discussions in a document that we will share with all of you once the workshop is over, and that we hope will lay the basis for a fruitful program of collaborations and cross-pollination of ideas.

We hope that these activities, together with the broad expertises of the participants will allow for a fruitful exchange of ideas and for a fresh take at an old topic, bringing recent advances in the study of insulators to the study of metals.

Since participants to the workshop are affected by rather different time zones, the workshop has been split in three parts:

1. (pre-recorded) **mini courses**, which will be made available *before the workshop*
2. **core program**, taking place on a time that allows **all** participants to attend
3. **encore program**, which allows participants from different time zones to actively take part

In all the timetables below, the times are shown with respect to the CEST (Europe/Amsterdam) time zone.

Mini courses

The following mini courses (titles are indicative) will be published by the week before the workshop, you can find them in Microsoft Teams and on the workshop page (after logging in):

1. *Classical mechanics and transport* (Andreas Knauf)
2. *Semiclassical methods for electronic transport [YouTube Playlist]* (Max Lein)
3. *Algebraic methods for interacting systems [Adiabatic quantum transport]* (Sven Bachmann)
4. *Disordered quantum systems [YouTube Playlist]* (Christian Sadel)

The aim of the mini courses is to level the knowledge of the participant and provide the necessary background to understand each other's language prior to the beginning of the workshop. Depending on the availability of the speakers these may be in the form of pre-recorded lectures, lecture notes or curated reading lists.

Core program

Time	Monday *overview*	Tuesday *non-commutative*	Wednesday *random*	Thursday *semiclassical*	Friday *classical*
Review **13.00–13.10**	*Intro by Lorenz Center*		*Recap*	*Recap*	*Recap*
Panorama talk **13:10–13:40**	*Intro to the workshop*		Alex Elgart	Peter Kuchment	Eva Miranda
Q&A **13:40–13:50**	*Q&A*		*Q&A*	*Q&A*	*Q&A*
Break		*13:55 Recap*			
Expert-level talk **13:55–14:35**	Andrei Ya. Maltsev	14:05 Panorama Emil Prodan	Jean-Bernard Bru	Martin Vogel	Sonja Hohloch
Q&A **14:35–14:45**	*Q&A*	*Q&A*	*Q&A*	*Q&A*	*Q&A*
Break					
Discussion panels **14:50–15:20**	*Discussion*	*Discussion*	*Discussion*	*Discussion*	*Final discussion and recap*

Titles and abstracts

Jean-Bernard Bru, *Large deviations for weakly interacting Fermions at equilibrium: an approach to macroscopic behaviour at nanoscales.*

In 2012, experimental measurements of electric resistance of nanowires in Si doped with phosphorus atoms demonstrate that quantum effects on charge transport almost disappear for nanowires of lengths larger than a few nanometers, even at very low temperature (4.2K). Such experiments suggest an exponentially fast convergence of microscopic quantities towards macroscopic ones in real fermion systems. In Mathematics, it corresponds to the existence of so-called large deviations for the corresponding quantities. We will discuss this mathematical issue for fermion systems on the lattice.

Alexander Elgart, *A panoramic view on Anderson localization*

We will review different aspects of Anderson localization, including the spectral and dynamical properties of underlying random systems, spectral statistics, and transport. Our main focus will be on non-interacting models, but we will briefly describe known results for many body systems as well. Our presentation will include some open problems in the field.

Andrei Ya. Maltsev, Sergei P. Novikov, *Properties of dynamical systems on complex Fermi surfaces and electronic transport phenomena in normal metals*

In this presentation, we will try to highlight the most basic pictures arising from the study of electron dynamics at the Fermi surface in the presence of strong magnetic fields. The presented results are based on deep topological studies of the dynamical system describing such dynamics on Fermi surfaces of arbitrary complexity.
The main consequences of the presented results are connected with electronic transport phenomena in strong magnetic fields.

Sonja Hohloch, *From Hamiltonian $\mathbb{S}^1$-actions to integrable systems with mild degeneracies*

Let (M,ω) be a 4-dimensional compact symplectic manifold and assume that the Hamiltonian $L: M \to \mathbb{R}$ induces an effective Hamiltonian $\mathbb{S}^1$-action on (M,ω). Such a triple (M,ω,L) was called a {\em Hamiltonian $\mathbb{S}^1$-space} by Karshon who classified these spaces by means of labeled directed graphs. Karshon also showed that some $\mathbb{S}^1$-actions can be extended to a Hamiltonian $\mathbb{S}^1 \times \mathbb{S}^1$-action on (M,ω) induced by a smooth integrable `toric' system $(L,T):(M,\omega) \to \mathbb{R}^2$ and she gave the precise conditions under which this is possible.
Thus, if one wants to extend a larger class of Hamiltonian $\mathbb{S}^1$-spaces to an integrable system one needs to admit a larger class than toric systems as target.
Hohloch $\&$ Sabatini $\&$ Sepe $\&$ Symington's work in progress removes part of Karshon's conditions and then they can extend to so-called {\em semitoric integrable systems} $(L,S):(M,\omega) \to \mathbb{R}^2$ which are, roughly, integrable systems that only allow elliptic-regular, elliptic-elliptic, and/or focus-focus points as singularities. These systems induce in fact an $\mathbb{S}^1 \times \mathbb{R}$-action on M, thus the name.
The aim of this talk is the class of so-called {\em hypersemitoric integrable systems} (see the joint work Hohloch $\&$ Palmer available at {\em arXiv:2105.00523}), which is the `easiest, smallest, and most reasonable' set of smooth integrable systems $(L,H):(M,\omega) \to \mathbb{R}^2$ to which {\em all} Hamiltonian $\mathbb{S}^1$-spaces (M,ω,L) can be extended. These systems induce an $\mathbb{S}^1 \times \mathbb{R}$-action on M and the {\em nondegenerate} singularities of (L,H) may have components of regular, elliptic, focus-focus, and/or hyperbolic type, thus the name. The occurrence of (finitely many) degenerate singular points cannot always be avoided, but there is a certain choice of what kind of degeneracies one wants to admit in the extension. Since parabolic (`cuspidal') points are a very natural class of degenerate points we opted to admit this type of degeneracies.
Eventually, we will outline the construction of extending a Hamiltonian $\mathbb{S}^1$-space to a hypersemitoric system by means of an example.

Peter Kuchment, *Analytic geometry of dispersion relation (Bloch variety) and Fermi surface*

This is a brief survey of known (and conjectured) properties of the Bloch and Fermi varieties as analytic sets and their relations to spectral/physics properties of the periodic operator in question. Time permitting, both continuous and discrete (tight binding) models will be addressed.

Eva Miranda, *From the electrons of the Helium atom to (singular) non-compact Symplectic and contact Topology.*

In this talk we take as starting point the article by Urs Frauenfelder "Helium and Hamiltonian delay equation" arXiv:2002.00598 where a particular case of the 3-body problem is discussed. In the Helium atom two electrons which are mutually repulsing are attracted by a positive nucleus. The problem is highly non-integrable and the system shows chaotic features. However, it is possible to compute a periodic orbit in this set-up using variational techniques. Even though this variational approach works, as the energy surfaces are not compact it is not possible to view this result in terms of classical Floer theory. This example will be used as an excuse to make an excursion through new challenges in the current Symplectic and contact Topology scenario to extend Floer's techniques further. This includes the study of tentacular Hamiltonians (Federica Pasquotto-Jagna Wiśniewska), convex symplectic manifolds (Misha Gromov-Urs Frauenfelder-Felix Schlenk) and the case of Hamiltonian/Reeb on non-compact manifolds which can be compactified as a singular geometric structure (b-symplectic structures). This last scheme appears naturally in several problems in celestial mechanics such as the restricted three-body problem where the existence of singular periodic orbits is related to the existence of escape orbits. We will discuss open problems and new avenues in the theory.

Emil Prodan, *A groupoid approach to interacting fermions*

We consider the algebra of inner-limit derivations over the GICAR algebra. Under standard assumptions such as Galilean invariance and finite interaction range, we demonstrate that this algebra is a groupoid-solvable pro-C*-algebra. This generalizes a result by Bellissard and Kellendonk to arbitrary N-fermion sectors and provides new tools to investigate the thermodynamics limit. This is work done in collaboration with Bram Mesland.

Martin Vogel, *Spectral asymptotics of noisy non-selfadjoint operators*

The spectral theory of non-selfadjoint operators is an old and highly developed subject. Yet it still poses many new challenges crucial for the understanding of modern problems such as scattering systems, open or damped quantum systems, the analysis of the stability of solutions to non-linear PDEs, and many more. The lack of powerful tools readily available for their selfadjoint counterparts, such a general spectral theorem or variational methods, makes the analysis of the spectra of non-selfadjoint operators a subtle and highly varied subject.
One fundamental issue of non-selfadjoint operators is their intrinsic sensitivity to perturbations, indeed even small perturbations can change the spectrum dramatically. This spectral instability, also called pseudospectral effect, was initially considered a drawback as it can be at the origin of severe numerical errors. However, recent works in semiclassical analysis and random matrix theory have shown that this pseudospectral effect also leads to new and beautiful results concerning the spectral distribution and eigenvector localization of non-selfadjoint operators with small random perturbations.
In this talk, I will discuss recent results and some fundamental techniques involved in the analysis.
The talk is partly based on joint work with Anirban Basak, Stéphane Nonnenmacher, Johannes Sjöstrand and Ofer Zeitouni.

Encore program

Time	Monday	Tuesday	Wednesday	Thursday	Friday
10:00–10:30		Guo Chuan Thiang	Giovanna Marcelli	Francesca Arici	André Belotto
10:30–10:40		*Q&A*	*Q&A*	*Q&A*	*Q&A*
Break					
10:50–11:40		Discussion in breakout rooms	Discussion in breakout rooms	Discussion in breakout rooms	Discussion in breakout rooms
11:40–12:00		*Summary discussion*	*Summary discussion*	*Summary discussion*	*Summary discussion*
12:00–13:00		Lunch	Lunch	Lunch	Lunch
13:00–15:20	**Core Program**				
Break					***Continuation of the final discussion and recap***
15:50–16:20	Jean Lagacé	Bruno Nachtergaele	Simone Warzel	Gihyun Lee	
16:20–16:30	*Q&A*	*Q&A*	*Q&A*	*Q&A*	
Break					
16:40–17:30	Discussion in breakout rooms	Discussion in breakout rooms	Discussion in breakout rooms	Discussion in breakout rooms	
17:30–18:00	*Summary discussion*	*Summary discussion*	*Summary discussion*	*Summary discussion*	

Titles and abstracts

Francesca Arici, *Toeplitz extensions in NCG and mathematical physics*

André Belotto, *Three dimensional Strong Sard Conjecture in sub-Riemannian geometry*

Given a totally nonholonomic distribution of rank two Δ on a three-dimensional manifold M, it is natural to investigate the size of the set of points $\mathcal{X}^x$ that can be reached by singular horizontal paths starting from a same point $x \in M$. In this setting, the Sard conjecture states that $\mathcal{X}^x$ should be a subset of the so-called Martinet surface of 2-dimensional Hausdorff measure zero.
I will present a reformulation of the conjecture in terms of the behavior of a singular foliation. By exploring this geometrical framework, in a recent work in collaboration with A. Figalli, L. Rifford and A. Parusinski, we show that the strong version of the conjecture holds for three dimensional analytic varieties, that is, the set $\mathcal{X}^x$ is a countable union of semi-analytic curves. Next, by studying the regularity of the solutions of the set $\mathcal{X}^x$, we show that sub-Riemannian geodesics are all C^1. Our methods rely on resolution of singularities of surfaces, vector-fields and metrics; regularity analysis of Poincaré transition maps; and on a symplectic argument, concerning a transversal metric of an isotropic singular foliation.

Jean Lagacé, *Homogenisation, minimal surfaces, and large eigenvalues*

Gihyun Lee, *Magnetic Pseudodifferential Super Operators*

In quantum mechanics, the time evolution of a state is determined by the Liouville equation $d\rho dt = -\frac{i}{\hbar}(H\rho - \rho H)$, where ρ is the density operator describing the given state and H is the Hamiltonian of a given system. Here we can observe that the Liouville operator $\rho \mapsto L_H\rho := -\frac{i}{\hbar}(H\rho - \rho H)$ is a super operator, i.e., it assigns operators to operators. The resolvent $(L_H - \lambda)^{-1}$ of the Liouville operator L_H plays a central role in the study of quantum dynamics. Meanwhile, The main idea of pseudodifferential calculi is to construct systematic ways of assigning operators to symbol functions, which enable us to translate properties of symbols to properties of associated pseudodifferential operators. In this talk, I will introduce a novel pseudodifferential calculus of super operators, and explain that the Liouville operator L_H and its resolvent $(L_H - \lambda)^{-1}$ can be incorporated into this new pseudodifferential theory. The asymptotic expansion of the product of symbols in small parameters will also be discussed.
This talk is based on the joint work with M. Lein.

Giovanna Marcelli, *A new approach to transport coefficients in the quantum spin Hall effect*

We investigate some foundational issues in the quantum theory of spin transport, in the general case when the unperturbed Hamiltonian operator does not commute with the spin operator in view of Rashba interactions. A gapped periodic one-particle Hamiltonian is perturbed by adding a constant electric field of small intensity and the linear response, with respect to the strength of the electric field, in terms of a spin current is computed. We derive a general formula for the spin conductivity that covers both the choice of the conventional and of the proper spin current operator. We study the independence of the spin conductivity from the choice of the fundamental cell, and we isolate a subclass of discrete periodic models (including the Kane-Mele model) where the conventional and the proper spin conductivity agree. As a consequence of the general theory, we obtain that whenever the spin is (almost) conserved, the spin conductivity is (approximately) equal to the spin-Chern number. The method relies on the characterization of a non-equilibrium almost-stationary state, which well approximates the physical state of the system. This seminar is based on joint work with G. Panati and S. Teufel.

Bruno Nachtergaele, *The stability of gapped phases and automorphic equivalence*

I discuss the stability of gapped ground state phases of quantum many-body systems under small perturbations of the interactions defining the Hamiltonian. This includes symmetry protected topological phases as well as situations with discrete symmetry breaking. I will review the current status of the problem and then discuss three recent results: 1) the stability of the bulk gap in the presence of gapless edge excitations; 2) stability of the spectral gap in the presence of discrete symmetry breaking: 3) stability of a dimerized phase in a family of quantum spin chains with O(n) symmetry. The first two results apply to a general class of frustration-free models on arbitrary lattices. The third is more specialized and deals with a class of one-dimensional systems that are not frustration-free. (Includes joint work with Robert Sims and Amanda Young and with Jakob Bjoernberg, Peter Muehlbacher, and Daniel Ueltschi).

Guo Chuan Thiang, *Mathematical tools for anomalous boundary metals*

The boundary of a topological insulator or semimetal is understood by physicists to be metallic in some “anomalous” sense. I will introduce the mathematical techniques of spectral flow and gerbes, and discuss how they allow for a rigorous understanding of the topological protection of the anomalous metallic boundary. These ideas are applied to experimentally verified examples such as Chern insulators, Weyl semimetals, and quantum Hall Hamiltonians.

Simone Warzel, *Spectra & gaps of Haldane pseudopotentials*

Haldane pseudopotentials are short-range interactions projected onto the lowest Landau level. They are tailored to model properties of fractional Hall fluids and their excitations, and in particular reproduce a Laughlin wave function in its ground state. Their bosonic versions effectively describe rotating Bose gases. In this talk I will give an overview of recent progress regarding mathematical concepts and proofs of the associated spectral properties in a truncated system which describes a thin-cylinder limit. This will be done by focusing on the half-filled bosonic system, which from a technical aspect poses additional challenges to existing techniques for proofs of spectral gaps. (Based on works with B. Nachtergaele and A. Young.)