An Expedition into Arithmetic Geometry

Tuesday 30 May (De Sitterzaal – Oort Building)

10:40–11:00 opening words by Lorentz Center
11:00–11:10 opening words by organizers
11:10–12:00 Lenny Taelman - Deformations of Calabi-Yau varieties in mixed characteristic
12:00–14:50 lunch and discussion time
14:50–15:40 Jan Vonk - Around the class number one problem
16:10–17:00 Jaclyn Lang - A structure result for vexing Hecke algebras
17:00–?? wine and cheese

Wednesday 31 May (De Sitterzaal – Oort Building)

9:50–10:40 Emmanuel Ullmo - On the distribution of the Hodge locus and applications
11:10–12:00 Anna Cadoret - On toric points of p-adic local systems arising from geometry
12:00–14:50 lunch and discussion time
14:50–15:40 Rachel Pries - Applications of special Shimura varieties to Newton polygons of curves
16:10–17:00 Kiran Kedlaya - Finite hypergeometric sums revisited

Thursday 1 June (De Sitterzaal – Oort Building)

09:50–10:40 Jennifer Park - Counting points on weighted projective spaces
11:10–12:00 Guido Lido – Geometric quadratic Chabauty method
12:00–14:50 lunch and discussion time
14:50–15:40 Pınar Kılıçer - Computing the primes of bad reduction for CM curves of genus 3
16:10–17:00 Jennifer Balakrishnan - Computing Selmer sets
17:45–18:15 bus to boat
18:30–21:30 boat trip Kaag lakes (including dinner)
21:30–22:00 bus back to train station, hotel, Lorentz Center

Friday 2 June – morning (De Sitterzaal – Oort Building)

09:50–10:40 Fabrizio Andreatta - Endoscopy for GSp(4) and rational points on elliptic curves
11:10–12:00 Ben Moonen - The Coleman-Oort conjecture - an update
12:00–12:10 closing
12:10–13:15 lunch

Friday 2 June – afternoon (C4/5 – Gorlaeus Lecture Hall)

13:15–13:50 doors symposium open (including coffee)
13:50–14:00 welcome
14:00–15:00 Hendrik Lenstra - Escher and the Droste effect
15:30–16:30 Jaap Top - Lines between Math and Art
16:30–16:45 closing
17:00–18:30 reception (foobar in the Snellius Building)

Abstracts

Fabrizio Andreatta (Università Statale di Milano)

Title: Endoscopy for GSp(4) and rational points on elliptic curves

Abstract: I report on a joint project with M. Bertolini, M.A. Seveso and R. Venerucci, aimed at studying the equivariant BSD conjecture for rational elliptic curves twisted by certain self-dual 4-dimensional Artin representations in situations of odd analytic rank. We use the endoscopy for GSp(4) to construct Selmer classes related to the relevant (complex and p-adic) L-values via explicit reciprocity laws.

Jennifer Balakrishnan (Boston University)

Title: Computing Selmer sets

Abstract: Kim's nonabelian Chabauty program for studying rational points on hyperbolic curves yields a series of refined Selmer sets at depth n, cut out by n-fold iterated p-adic integrals. We discuss computations that have been done of these sets in explicit cases, including punctured elliptic curves and smooth projective curves of genus 2 and 3.

Anna Cadoret (Sorbonne Université)

Title: On toric points of p-adic local systems arising from geometry.

Abstract: For a smooth variety over a number field and a p-adic local systems arising from geometry on it, classical conjectures on algebraic cycles predict that the toric points should fit with the CM points of the associated variation of Hodge structure; in particular they should have similar properties in terms of sparcity. I will discuss results in this direction. This is a joint work with Jakob Stix.

Kiran Kedlaya (UC San Diego)

Title: Finite hypergeometric sums revisited

Abstract: Building on work of Greene, Katz, Ono, and others, Beukers-Cohen-Mellit introduced a trace formula for arbitrary hypergeometric motives defined over Q based on finite hypergeometric sums. While one can give this formula a p-adic interpretation using the Gross-Koblitz formula, it has been an open problem to interpret this in the context of p-adic cohomological algorithms for computing zeta functions. We resolve this issue using "algebraic Frobenius structures", in a manner that suggests a possible generalization to other families of motives.

Pınar Kılıçer (University of Groningen)

Title: Computing the primes of bad reduction for CM curves of genus 3

Abstract: Goren and Lauter showed that a CM curve of genus 2 has (geometric) bad reduction at a given prime if and only if there is a solution to a certain embedding problem. In genus 3, it has been shown that if a CM curve has bad reduction at a given prime then there is a solution to an analogous embedding problem.

In this talk, we introduce a new embedding problem, called the Isogenous Embedding Problem (IEP) which also has a solution if the curve has bad reduction. We propose an algorithm that computes the solutions to the IEP, given a prime. Using this, we are able to compute a short list that contains the primes of bad reduction of any genus-3 curve that has CM by a given order.

This is a joint work with Ionica, Lauter, Lorenzo Garcia, Manzateanu, Vincent.

Jaclyn Lang (Temple University)

Title: A structure result for vexing Hecke algebras

Abstract: Fix a weight 2 CM modular form f with trivial character and level l^2 for some prime l. How many eigenforms of the same weight and level are congruent to f modulo a prime p? We will sketch a proof that when l is -1 mod p, this number is always divisible by p. Such an f is an example of a modular form that is “vexing at l mod p”, and the p-divisibility phenomenon is true for all such vexing forms (if p is at least 5). These vexing forms are so named due to the difficulties they pose in modularity lifting theorems. The divisibility result is deduced from a structure result on the corresponding Hecke algebra: we show that it is free over the group ring of a cyclic p-group. Our techniques use both modular representation theory as well as geometric/cohomological methods. This is joint work in progress with Robert Pollack and Preston Wake.

Hendrik Lenstra (Leiden University)

Title: Escher and the Droste effect

Abstract: In 1956, the Dutch graphic artist M.C. Escher made an unusual lithograph with the title `Print Gallery'. It shows a young man viewing a print in an exhibition gallery. Amongst the buildings depicted on the print, he sees paradoxically the very same gallery that he is standing in. A mathematical analysis of the studies used by Escher leads to a series of hallucinating computer animations, which show, among others, what happens inside the mysterious spot in the middle of the lithograph that Escher left blank.

Guido Lido (Università di Roma “Tor Vergata”)

Title: Geometric quadratic Chabauty method

Abstract: We present a generalization of Chabauty's method which started from Bas' idea to substitute the Jacobian with its Poincaré torsor: this method works when the Mordell-Weil rank is strictly smaller than g+s-1, where g is the genus of the curve and s is the rank of the Néron-Severi group of the Jacobian. Moreover we can see this method as a geometric version of the quadratic Chabauty method that uses p-adic heights.

Ben Moonen (Radboud University Nijmegen)
Title: The Coleman-Oort conjecture - an update
Abstract: A conjecture of Coleman says that if we fix a genus g (with g>7), there are only finitely many complex curves (smooth projective, taken up to isomorphism) of genus g for which the Jacobian is an abelian variety of CM type. Via the André-Oort conjecture (now a theorem of Tsimmerman, building upon work of many others), Coleman's conjecture can be reformulated as a problem about the non-existence of special subvarieties in the Torelli locus. In my talk, I will review this problem, and I will discuss recent progress.

Jennifer Park (Ohio State University)
Title: Counting points on weighted projective spaces
Abstract: Counting points on stacks by height is often translated into a statement in arithmetic statistics, yet the study of the general theory is only just beginning. I will report on the progress made on counting points of fixed degree on weighted projective spaces, one of the simplest kinds of stacks, with respect to various heights. This is joint work with Soumya Sankar.

Rachel Pries (Colorado State University)
Title: Applications of special Shimura varieties to Newton polygons of curves
Abstract: Special subvarieties of Shimura varieties and Frobenius morphisms were some of Bas Edixhoven's areas of expertise. This talk is about Newton polygons of Jacobians of curves in certain unitary Shimura varieties. The curves we consider are cyclic covers of the projective line. The image of a family of these under the Torelli morphism lies inside a Deligne--Mostov unitary Shimura variety. By work of Kottwitz and Viemann/Wedhorn, there are restrictions on which Newton polygons can occur in this context. We studied the Newton polygons that occur for curves in Moonen's 20 special families. Using these as base cases, we developed an inductive approach to study the intersection of the Torelli locus with the non-ordinary locus in more general cases. As one application, we produce new examples of unusual Newton polygons which occur for Jacobians of smooth curves of arbitrarily large genus, providing evidence for a conjecture of Oort. As another application, we prove results about the density of primes of ordinary reduction of curves in Moonen's families; this proves a conjecture of Serre for certain abelian varieties of dimensions three to seven. This is joint work with Cantoral Farfan, Li, Mantovan, and Tang.

Lenny Taelman (University of Amsterdam)
Title: Deformations of Calabi-Yau varieties in mixed characteristic
Abstract: A smooth projective variety X is said to be Calabi-Yau if its canonical bundle is trivial. I will discuss joint work with Lukas Brantner, in which we use derived algebraic geometry to study deformations of Calabi-Yau varieties in characteristic p.

We prove a positive characteristic analogue of the Bogomolov-Tian-Todorov theorem (which states that deformations of Calabi-Yau varieties in characteristic 0 are unobstructed), and show that 'ordinary' Calabi-Yau varieties admit canonical lifts to characteristic zero (generalising earlier results of Serre-Tate for abelian varieties, and Deligne and Nygaard for K3 surfaces).

Jaap Top (University of Groningen)

Title: Lines between Math and Art

Abstract: Starting with a lecture entitled "Some elliptic curves from the real world" in 2014 during the annual Dutch Mathematical Congress, Bas Edixhoven presented several talks discussing the "torqued ellipses" made by the American sculptor Richard Serra. This resulted in a place for Bas among the artists with artworks on display in the "Virtual Museum Tesseract", see https://anton.shapespark.com/ars_et_mathesis_virtueel_deel2/ .

The present talk aims to recall and explain these contributions by Bas, and place it in a more general framework of inspirations from math to art and, as in this case, vice versa. In particular, it should become clear what the mentioned works of art have to do with the title of the 2014 lecture by Bas.

Emmanuel Ullmo (Institut des Hautes Études Scientifiques)

Title: On the distribution of the Hodge locus and applications.

Abstract: The lecture will discuss a joint work with Gregorio Baldi and Bruno Klingler. Given a polarized Z-variation of Hodge structures over a complex, smooth quasi-projective variety S, we describe some properties of the Hodge locus, a countable union of algebraic subvarieties of S where exceptional Hodge tensors appear, by a result of Cattani, Deligne and Kaplan. We prove the geometric part of the Zilber-Pink conjecture in this context: the maximal atypical part of the Hodge locus of postive period demension arise in a finite number of families. In level at least 3, we show that the typical Hodge is empty and therefore the positive dimensional part of the Hodge locus is algebraic. For instance the Hodge locus of positive period dimension of the universal family of degree d smooth hypersurfaces in the projective space of dimension n+1 is algebraic. We also prove that if the typical Hodge locus is not empty, then the Hodge locus is analytically dense in S. If times permits we will discuss some arithmetic consequences.

Jan Vonk (Leiden University)

Title: Around the class number one problem

Abstract: As part of a systematic computational study of equivalence classes of binary quadratic forms, Gauss stated several conjectures on their class numbers. In this talk, I will discuss some recent results related to these conjectures, with a special emphasis on those that were inspired by work of Bas Edixhoven.