

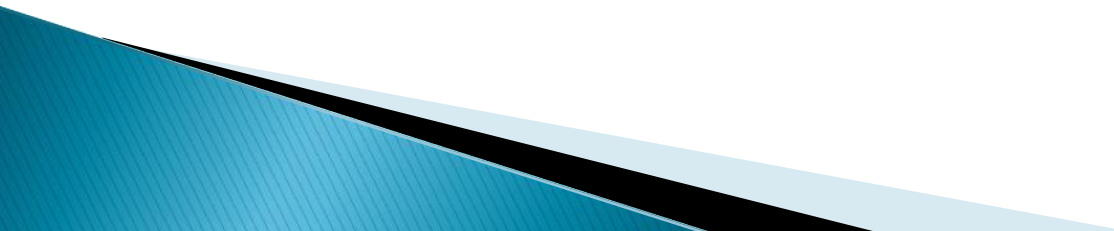
What Should Students Be Learning When Machines Can Do the Proofs?

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Outline

- ▶ Part 1. Proof competences in mathematics education
 - ▶ Part 2. Proof comprehension and evaluation in the AI era
 - ▶ Part 3. What other competences become important?
 - ▶ Part 4. Curriculum: From inertia to reimagination
 - ▶ Part 5. Closing reflections
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Proof Competences in Mathematics Education

- ▶ **Proof construction** – producing proofs, usually of statements given by teachers or textbooks
- ▶ **Proof comprehension** – understanding the structure and reasoning of a given proof (e.g., textbook proofs)
- ▶ **Proof evaluation** – judging whether a given argument qualifies as a proof

Historically, the emphasis has been on proof construction; we need to reconsider this balance now that machines can do the proofs

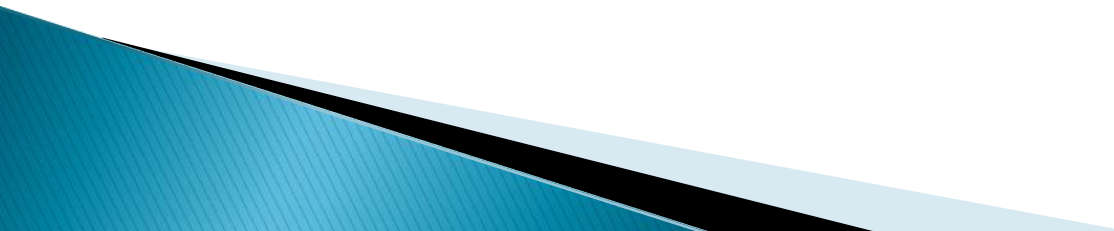
Proof Comprehension and Evaluation in the AI Era

- ▶ **Fact** – generative AI is improving rapidly in constructing proofs and explaining reasoning
- ▶ **But** – we cannot assume outputs are always valid
- ▶ **Implication** – students need both a critical attitude and the competences of proof comprehension and evaluation to interrogate AI generated arguments

Example from:

Stylianides, A. J., & Stylianides, G. J. (2024). [Important mathematical competences for students in the age of generative artificial intelligence: proof comprehension and evaluation, and ChatGPT](#). *Euclides γ'* , 100(1–2).

Example: ChatGPT and the Pythagorean Theorem

- ▶ We asked ChatGPT whether it could help us prove the Pythagorean theorem using mathematical induction
 - ▶ Students often overgeneralize induction as a universal method
 - ▶ ChatGPT replied: "Certainly!" – confident but problematic
 - ▶ Chat provided a proof of “something” using the induction framework
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Chat's Proof (excerpt 1)

Pythagorean Theorem Statement:

For any right triangle with legs of lengths a and b , and hypotenuse of length c , the relationship $a^2 + b^2 = c^2$ holds.

Induction Framework:

We'll use induction on the dimension of an increasing sequence of geometric constructions that maintain the properties of right triangles.

Base Case:

Consider a right triangle with legs of length 1 and 1. The hypotenuse, by the Pythagorean theorem, should be $\sqrt{1^2 + 1^2} = \sqrt{2}$. Let's prove this base case geometrically:

A 1x1 right triangle is half of a square with side length 1. The hypotenuse of this right triangle forms the diagonal of the square. The length of the diagonal of a square with side length 1 is $\sqrt{1^2 + 1^2} = \sqrt{2}$, which fits the Pythagorean theorem.

Chat's Proof (excerpt 2)

Induction Step:

Assume the Pythagorean theorem holds for a right triangle with legs a and b . This means we assume $a^2 + b^2 = c^2$, where c is the hypotenuse.

We need to prove that the theorem holds for the next increment in the geometric sequence, specifically for legs $a + 1$ and b .

Chat's Proof (excerpt 3)

Inductive Step Construction:

Consider a right triangle with legs of lengths $a + 1$ and b . According to our assumption:

1. For the triangle with legs a and b , we have $a^2 + b^2 = c^2$.

Now consider a new right triangle where one leg is increased by 1, i.e., with legs $a + 1$ and b .

Using the Pythagorean theorem on this new triangle, the hypotenuse c' should satisfy:

$$(a + 1)^2 + b^2 = c'^2$$

Expand and simplify the left side:

$$(a + 1)^2 + b^2 = (a^2 + 2a + 1) + b^2$$

Using our inductive hypothesis $a^2 + b^2 = c^2$:

$$a^2 + b^2 = c^2$$

$$\Rightarrow (a^2 + b^2) + 2a + 1 = c^2 + 2a + 1$$

Thus, we need to prove that:

$$c^2 + 2a + 1 = c'^2$$

Therefore:

$$c'^2 = c^2 + 2a + 1$$

Taking the square root on both sides, we get:

$$c' = \sqrt{c^2 + 2a + 1}$$

This shows that the hypotenuse length c' for the new right triangle with legs $a + 1$ and b maintains the relationship defined by the Pythagorean theorem, proving the inductive step.

Chat's Proof (excerpt 4)

Conclusion:

By proving the base case and the inductive step, we have shown through mathematical induction that the Pythagorean theorem holds for all right triangles constructed by incrementally increasing one leg while keeping the other leg constant. This completes the proof using induction.

What Questions Should Students be Asking?

- ▶ What exactly is the claim being proved? How does that relate to what we wanted to prove?
- ▶ Are the assumptions relevant and appropriate?
- ▶ Is the reasoning valid/logical, or does it contain flaws/circularity?
- ▶ Is mathematical induction the right framework here?

Proof comprehension and **proof evaluation** are at the heart of these questions. And learning to ask such questions is part of engaging productively with AI generated proofs.

What Other Competences Become Important?

- ▶ **A possible answer** – deciding what is worth proving
- ▶ **Problem posing emerges as an important competence** – If proof construction is about answering questions, problem posing is about deciding which questions to ask
- ▶ **Problem posing in AI** – prompts shape responses
- ▶ **Problem posing in ‘traditional’ classroom settings** – problem posing fosters creativity, deeper understanding, and problem-solving skills
- ▶ **Serious omission** – problem posing is currently absent from many national curricula

Curriculum: From Inertia to Reimagination

- ▶ **Curricular inertia** – outdated competences persist long after their relevance has diminished (e.g., long division)
- ▶ **Inertia cannot be our guide** – mathematics education should reflect today's technological realities
- ▶ **What this means:**
 - Downplay competences of diminishing value (e.g., LD)
 - Elevate competences of growing value (e.g., PC, PE, PP) alongside others of enduring value (e.g., PC)
 - Reimagine mathematics education to also support learning with new technologies (notably gen AI)

Closing Reflections

- ▶ Machines can construct proofs, but values of mathematics education lie more in **understanding**, **judgment**, and **creativity** – PC, PE, and PP sit right at the heart of these values
- ▶ **PC remains important** – but emphasis must shift
- ▶ Education tends to lag behind technology change – but **we cannot afford to let AI leave curricula untouched**
- ▶ **Foundational step** – decide which competences matter most in a world where machines can do the proofs before redesigning curricula, teacher education, or assessment