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Faculty of Humanities

JOINT EPISTEMIC ACTIONS IN MATHEMATICS AND MECHANIZATION

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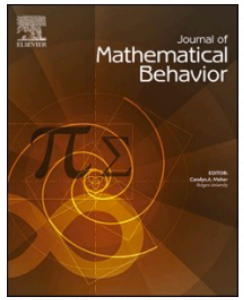
Journal of Mathematical Behavior 80 (2025) 101271

Contents lists available at ScienceDirect

Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb



Diagrams as joint epistemic actions: A dialogical account of diagrams in mathematical proofs

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Introduction

- How to make sense of the ubiquitous presence of diagrams in mathematical proofs?
- Adopting a dialogical account of mathematical proof, I discussed the cognitive/epistemic import of diagrams.
- I argued that diagrams are best seen as **joint epistemic actions** involving the producer and the receiver of a proof.
- How does the idea of joint epistemic action generalize to other external ‘devices’ used in proofs, in particular notations and computational tools?

Plan of the talk

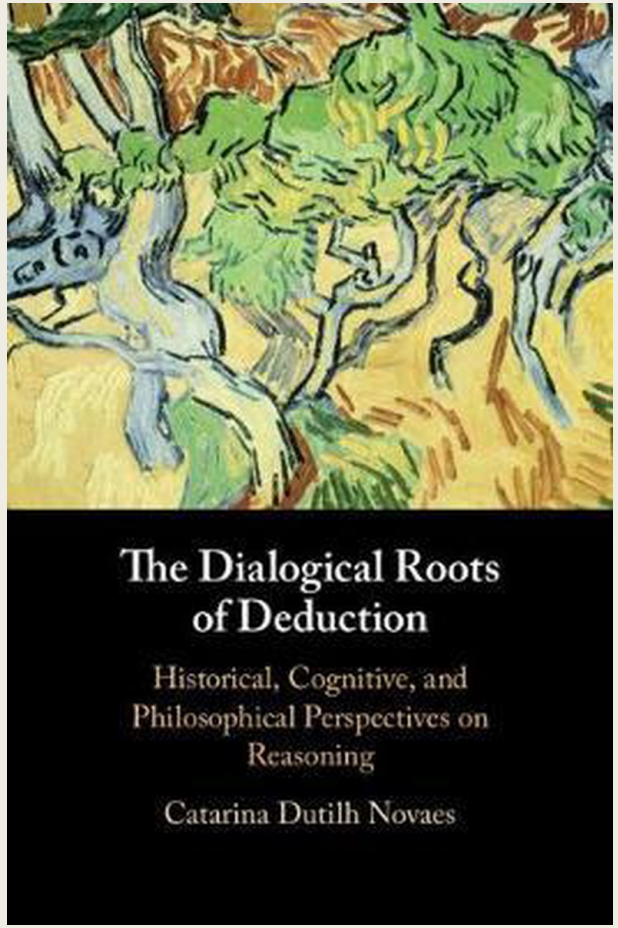
1. A dialogical account of mathematical proof
2. Dialogues and diagrams
3. Mechanization

Socrates and the boy doubling the area of a square

1. A DIALOGICAL ACCOUNT OF MATHEMATICAL PROOF

The dialogical hypothesis

- Deduction has dialogical roots, and these dialogical roots are still largely present both in theories and in practices of deduction.
- This approach highlights the deeply human and social nature of deduction, as embedded in actual human practices of special kinds of dialogues.



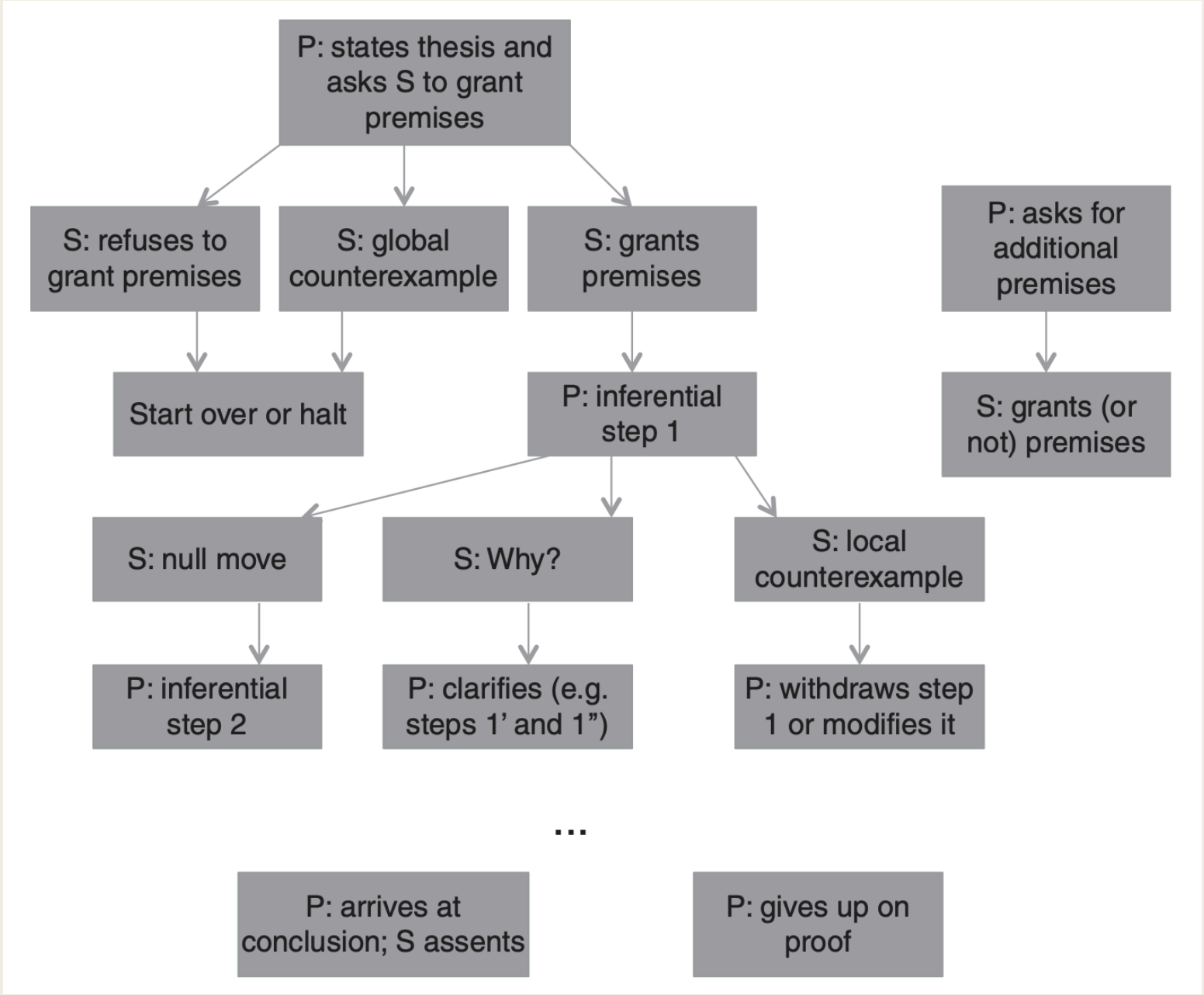
Greek mathematics

- “Greek mathematics reflects the importance of persuasion. It reflects the role of orality, in the use of formulae, in the structure of proofs … But this orality is regimented into a written form, where vocabulary is limited, presentations follow a relatively rigid pattern… It is at once oral and written…” (Netz, *The Shaping of Deduction*, 1999)

Reviel Netz

The Prover-Skeptic dialogues

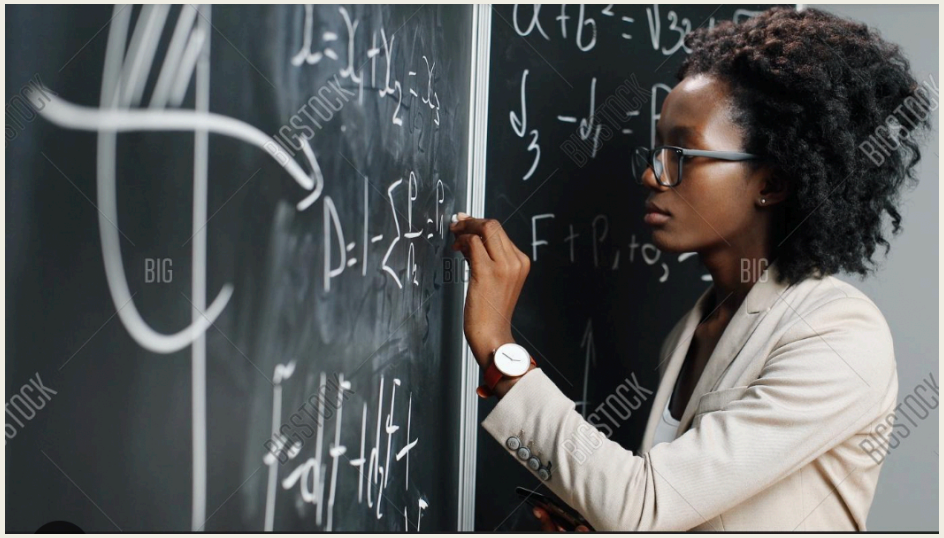
- Prover-Skeptic dialogues are a rational reconstruction of deductive arguments, such as mathematical proofs.
- Prover states the theorem to be proved.
- Prover asks Skeptic to grant certain premises, and then proceeds to establish the conclusion from what has been granted.
- Skeptic may come up with counterexamples, ask for further clarifications, grant or deny premises.
- Some amount of adversariality against a largely cooperative background: goal of *explanatory persuasion*.


P: states thesis and asks S to grant premises
S: refuses to grant premises
S: global counterexample
S: grants premises
P: asks for additional premises
Start over or halt
P: inferential step 1
S: grants (or not) premises
S: null move
S: Why?
S: local counterexample
P: inferential step 2
P: clarifies (e.g. steps 1' and 1")
P: withdraws step 1 or modifies it
...
P: arrives at conclusion; S assents
P: gives up on proof

3. DIALOGUES AND DIAGRAMS

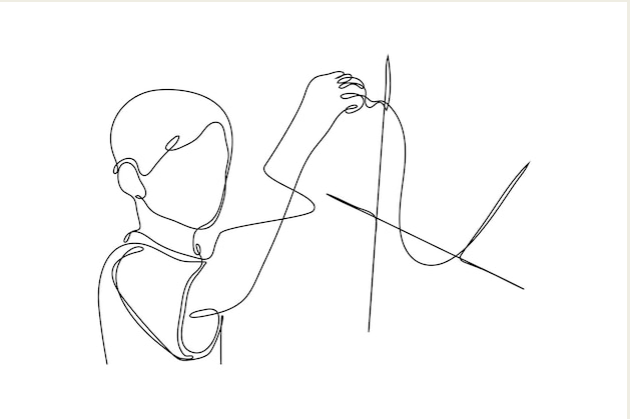
Introducing diagrams in Prover-Skeptic dialogues

- As it stands, the Prover-Skeptic model does not explicitly deal with diagrams in mathematical proof.
- The ‘moves’ available to Prover and Skeptic do not include drawing diagrams.
- More generally, the model does not offer a more extensive discussion of different kinds of *speech acts* in these dialogues, such as *imperatives*.



Instructions for diagram construction

- Many of the imperatives occurring in mathematical proofs are instructions for specific mathematical actions to be carried out, such as *constructing diagrams*.
- Back to Prover-Skeptic dialogues:
  - ➢ *Prover may use imperatives to invite Skeptic to draw diagrams.*
  - ➢ *Skeptic may comply with the instructions, reject them, or ask for further clarification.*



Diagrams as joint epistemic actions

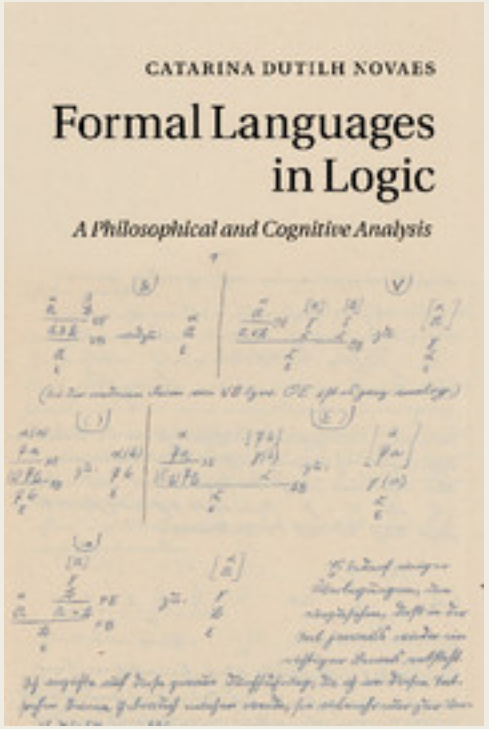
- **Epistemic action**: concept introduced in (Kirsh & Maglio 1994) to describe actions by Tetris players inside the game (such as rotations) to improve performance.
- "Actions performed to uncover information that is hidden or hard to compute mentally."
- **Joint action**: actions in which two or more people coordinate in space and time in order to bring about a change in the environment (Sebanz, Bekkering & Knoblich, 2006).
- **Joint epistemic action**: joint action that aim at **epistemic** (rather than practical) aims.

➢ **Constructing a diagram is a joint epistemic action involving Prover and Skeptic.**

3. MECHANIZATION

The cognitive impact of notations for reasoning

- Formalisms and notations as *technologies* to aid and support reasoning.
- Rather than mere expressions or representations of ('internal') cognitive processes, notations and formalisms are *constitutive* of these very processes.
- Thinking 'on the paper' (Feynman).



Mechanization in proofs

- Can the use of computational devices and techniques, including AI, be understood in terms of joint epistemic actions?
- Perhaps as a **joint epistemic action** between the mathematician and the LLM?
- Or do the use of these tools in fact jeopardize the social (joint) component of mathematical practice?
- Do these tools interfere with the epistemic dimension of mathematical understanding?

Conclusions

- Constructing a diagram is an epistemic action jointly performed by the participants in these dialogues (Prover and Skeptics), aiming at *explanatory persuasion.*
- Is the use of computational devices (LLMs and other forms of mechanization) a natural extension of mathematical reasoning, such as notations and diagrams?
- Or does mechanization in fact threatens the social and the epistemic dimensions of mathematical reasoning?

Thanks!