# Social Studies of Mathematics onboarding talk

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Lecture notes edited a few weeks after the workshop, may not correspond exactly to the presentation.

## Aims

I work in the history and social/cultural studies of mathematics. This talk is meant to help set up the week's discussions at this workshop by introducing some of the most important and useful perspectives from the social studies of mathematics that will be relevant to the workshop themes. I will not lay out major theories or arguments from these fields, but rather introduce how researchers in social studies approach mathematics, what evidence we use, what we try to explain, and what facets of mathematics and its contexts matter to us.

There are five clusters of perspectives and methods I want to introduce:

1. social epistemology (social explanations for how one knows what one knows and shares that knowledge with others);
2. material culture (the 'stuff-ness' of mathematics);
3. political economy (how mathematics relates to interests and resources at different scales);
4. valuation (what about people and ideas is valued, to what effect, and how people agree or disagree about this); and
5. institutions (how to be a mathematician in a mathematical community).

Main aim of this talk is to give some frameworks and vocabulary for articulating a lot of things you already more or less know about mathematics. I.e. to help you all to feel more self-conscious all week (in a way that is perhaps uncomfortable but hopefully productive!).

## Aside: Chalk Talks

Who else is planning a chalk talk for this week? (Nobody.) Who does chalk talks in other mathematical settings? (Most.) This is an illustration of context-differentiation and context-specificity of mathematical communication. We're mostly familiar with at least some of the differences it makes to present with slides, chalk, or other media.

Teachable moment about the Lorentz Center meeting room and the initial chalk supply that was not legible for the blackboard-wall at the front of the room. Workshop organizers were initially asked not to use the much-hyped brand of chalk as it was difficult to clean up, but this proved the only legible option and had to be fetched from a local mathematician's office. Comment on different erasing cultures (dry block with different materials; towel; wet towel; wet sponge).

**Further Reading** (titles should be clickable with hyperlinks): Chalk: Materials and Contexts in Mathematics Research

Time-permitting, I would have talked about the longer history of blackboards in mathematics, as constitutively modern inscription media that were rapidly adopted in science and education in the early-to-mid 1800s, following centuries of primarily being used for church choirs. There are major social, cultural, political, and economic reasons for this shift and major implications for the history of science and mathematics!

## Two Basic Slogans

1. Mathematics is People

- we want to understand mathematical activities (understanding, learning, creating, surviving, etc) in terms of the people who do them: their needs, resources, constraints, values, etc.
- we want to ask how people and their contexts create the conditions for mathematics and its meanings and implications. Mathematics is not self-interpreting, self-sustaining, etc.: people do the work of creating and sharing and understanding and applying and evaluating mathematics. People have limitations and biases and needs and communities. This is our way into the social study of maths.

2. Mathematics is Hard

- Note: not saying that 'mathematics is hard' should be a public relations slogan for the discipline. This is about how to think about mathematics so as to enable social analyses of it.
- It takes work, resources, coordination, compromise.
- Yes, even basic or trivial mathematics is hard. It takes massive infrastructures of training and enculturation to give the impression that even some basic mathematics is obvious. Think about all the money and time and infrastructure that goes into mass primary education to achieve even somewhat reliable numeracy in a nontrivial fraction of a national population.
- It is hard to create mathematics and to understand it for yourself. As researchers, mathematicians attempt to come to new understandings of very difficult things, and these do not behave like the toy examples of mathematics that some people like to have in mind when talking about the nature of mathematics or what mathematics essentially is.
- It is hard to get someone else to understand mathematics and hard to convince each other that you understand this mathematics in more or less the same way.
- Social studies look to where this difficulty is apparent, how overcome or

managed or excused, what the difficulty requires in terms of social and material resources, what effects the difficulty has on how mathematicians approach their work, and so on.

## Remark: Endogenous social theory

I'm glad to be presenting right after Emily Riehl ('A new paradigm for mathematical proof?'), who included lots of sociological observations!

*Question*: who here identifies as a mathematician? When I ask this question to postgraduate students in mathematics PhD programmes, I pretty reliably get a half-half split. (We discussed this a bit during the presentation.)

This is a disciplinarily mixed workshop, most people here have substantial experience in one form or another with mathematics, including doing mathematics and being part of mathematics communities.

- The mathematicians here already come with some highly developed social theories of mathematics
- These are essential to the conduct of mathematics (e.g. peer review - ER gave a great demonstration of this in discussing vibe proving and the apparently missing replication crisis in mathematics); you are familiar with preprint norms, institutional affiliation, etc, as proxies for trust and reliability; trust and credibility are generally major areas of focus for social studies of mathematics and often based on mathematicians' more or less formalised customs and understandings of who and what are reliable; most people here (some more than others!) have some idea of how to get a Fields Medal, including factors that you recognise as not strictly about picking some really good mathematics to figure out.
- Sometimes take the form of *working assumptions* (working myths?)
- Often quite different to social studies views of maths, but often social studies conclusions amount to formalising what is obvious/expected/well-known to mathematicians.

## Social epistemology

*Exercise*: do a simple calculation and a complex calculation; convince yourself you have done each correctly; how did your account differ between the two? convince your neighbour; how do you reach agreement (if you do)?

- what counts as basic or obvious or simple or clear or followable is highly context and person specific (cf. ER mentioned that LLMs struggle with basic numeracy); it is a matter of consensus among people doing mathematics in a particular setting.
- surveyable: you can take in all the relevant symbols or objects or steps of an argument at one go (this can mean lots of different things); this can be a form of simplicity.

- unsurveyable: can't be taken in at once, due to complexity or intricacy or sheer volume or opacity (as in a computer system perhaps); ER mentioned big/long/technical arguments as posing their own challenges for understanding and communication.
- trust: social studies of mathematics are really interested in trust, what trust looks like, why people trust or don't trust each other in specific circumstances; whence confidence; how people express consent; what forms disagreement may take and how people manage disagreements; and so on.
- there are many ways to engage with a mathematical narrative or argument that you are seeing or hearing, and these have different social conditions and expectations and consequences: validation (verifying, perhaps by hand or using a machine), following (going from one point to the next without necessarily validating each step or the underlying claims), believing, assenting (agreeing without necessarily believing),... can you think of others?
- Mathematical work involves not just expectations of what is true or plausible, but also expectations about how, when, and why arguments break, what the really hard parts or essential obstacles will be for an argument. This is a key problem for those thinking about mechanisation in interactional terms: checking or understanding a machine-produced mathematical text can be radically different to a human-produced one because machine shortcomings don't tend to conform to human expectations of where there will be gaps or errors or pitfalls.
- Self-evidence (or obviousness, etc) is a product of social consensus: from a social studies perspective, self-evidence is not an inherent quality of the mathematics but something that comes from collective decisions to treat things as obvious or not requiring further questions or justification.
- Mathematical communication provides evidence of the production and manifestation of (individual) conviction, (shared) agreement, management of disagreement, mobilisation of partial mutual understanding. Mathematicians do a lot of agreement-based things with only partial confidence that they are in agreement about the essential (or less essential) details of the concepts etc that they are working together with.
- ER talked about situations where errors are known or expected but specific errors have not been identified. This is a tremendously important phenomenon, and underscores the difficulties from the difference between line-by-line and other kinds of validation or understanding!
- Intuition and discernment are cultivated and socially situated facets of mathematical understanding: you learn and share how to recognize what is intiutive, insightful, and so on. This is something commonly recognized in mathematicians' endogenous social theories: as an illustration of this, ER talked about Terry Tao's view of developing intuitions.
- Images, conjectures, programs, intuitions, etc are important ways that mathematicians organize bodies of work and projects of collective understanding. These are both products and mechanisms of social organisation: mathematicians work together to develop images (etc) of their mathematics

of interest, and the images (etc) are tools for them to find and coordinate with other mathematicians. In my historical research, I have defined a theory as a social relationship of partial mutual understanding.

- **Key point**: In social studies of mathematics, we try to relocate where we look for evidence of *understanding* from individual minds to acts of communication.

**Further reading** (titles should be clickable with hyperlinks): The treachery of small numbers; On Remediation; Integration by parts

*Transition*: Our interest in communication means we can learn from the media of communication (paper, blackboards, computer screens, etc): this is the stuff of mathematics.

## Material culture: The Stuff-ness of Mathematics

We did not talk much about the key method for this, *critical inventory.* Basically, make a list of objects, connections, etc that are relevant to the specific context and phenomenon at hand. Ask how they fit together, what makes what possible, how they create or limit possibilities.

- *Key term*: **Infrastructure**: pervasive, taken-for-granted, enables the main activities, mostly invisible unless it breaks (like our assumption about chalk and blackboards in the Lorentz Center that did not hold up and required finding different chalk!.
- *More key terms*: **constraint**: how dependence on specific materials/etc limits what is possible (you can only write enough to fill all your available blackboards before you have to erase some of what you wrote). **affordance**: what specific materials/etc enable (on a blackboard you can annotate and gesticulate and smudge/replace quite freely).

Historical example: the blackboard is a specifically modern material context for mathematics. It involves particular ways of writing and techniques of communication, dependent on particular infrastructures, associated with the historical massification of engineering expertise and wider massification of 'everyday' numeracy in the 1800s.

We had a good discussion in the room of other examples and dimensions of material culture that went beyond my notes here.

**Further reading** (titles should be clickable with hyperlinks): Chalk: Materials and Contexts in Mathematics Research; Abstract relations

*Transition*: Looking for stuff opens up the question of where the stuff comes from, who makes it availability and with what resources and why. This is political economy.

## Political economy

This is a really big subject and it was good to see discussions and the terminology of 'political economy' extend throughout the week's workshop.

Fundamentally, political economy is about **interests** and **resources**, and even more fundamentally this makes it about **power** and specifically the *causes and consequences of uneven distributions of power*.

This is another area where critical inventory can be a useful method, especially critical inventories of funding (follow the money) and decision-making (follow the authorities).

- sponsors of mathematics have changed substantially across history and differ today substantially according to contexts. They have quite varied interests that are rarely the same as the interests of the mathematicians they are sponsoring. Examples:
  - philanthropy (often redirected corporate wealth)
  - governments (nation-states, militaries, civilian research funders)
  - corporations (but remember that large corporations' major clients are often nation states)
- often the things that most overtly interest mathematicians (e.g. theorems, theoretical programs) are essentially byproducts or waste products of the things mathematicians are being supported to produce (a component of systems of training numerate citizens or workers, for example)

We can look to 'hidden labor' which is often infrastructural labor as part of following money and operational resources and interests: secretaries, building maintenance, typesetters and postal systems, computer system administrators, and so on. These can give a different picture of what is at stake in mathematical work.

**Further Reading** (titles should be clickable with hyperlinks): Remunerative combinatorics; Fellow travellers and travelling fellows

## Valuation

A big part of social studies of mathematics is understanding what different people involved with mathematics (not just mathematicians!) value, and why, and how they come to agreement about this, and what consequences these valuations have.

- E.g. 'good, true, beautiful' are all valuations.
- Social studies emphasis: not what qualities make something or someone valued but rather how communities agree on evaluations and to what effects.
- How qualities are identified, characterised, and compared (e.g. measured). We talked later in the workshop about this with the question of benchmarking and what benchmarks capture and value.

- Disagreement and its consequences: there are many ways of containing and channeling disagreement over valuations in mathematics.
- Affect: significant role of emotional responses in mathematics. ER mentioned the thrill that comes from ‘computer says yes’ in a machine proof.
- Attention is a key limited resource managed through valuation, which gives a way of thinking sociologically about crank mathematics or LLM slop mathematics.

Example of investment logic and the evaluation of people in early 20th century: Rockefeller Foundation (e.g.) funded mathematical careers and institutions with the goal of producing investments of lasting impact, which meant investing in those who were already privileged and more likely to be able to capitalize on their privilege. This was shockingly explicit in some of their internal guidelines.

**Further Reading** (titles should be clickable with hyperlinks): The Fields Medal Should Return to its Roots; A postwar guide to winning a science grant; Practice makes Perfectoid

## Institutions

An institution is an organized social form associated with norms, values, roles, collective practices, and conditions of reproducing itself (a rough definition). Institutional analysis of mathematics looks at features such as careers, professional trajectories and standards and norms, professional organizations, research and teaching institutes, and so on.

- Communities: institutions create and sustain specific communities for specific purposes. Look for who is involved, how they are brought into the community, and how the community’s norms and forms produce the community’s goals.
- Institutional forms can be ways of managing scale, and many aspects of institutions revolve around gathering and managing and using information (verb: intelligencing) about people and subjects.
- Identify institutional customs, norms, expectations, ‘culture’
  - In training and practice, communities sustain themselves through cultural reproduction.
  - Inequality, bias, and abuse are common features of institutional organization.
- Personae, identities: these are ways of being in institutions.
  - Tom Lehrer, 1960: “Some of you may have had occasion to run into mathematicians, and to wonder, therefore, how they got that way”
  - Templates (role models, stereotypes, etc) and differentiated roles (e.g. ER cited Tim Gowers on problem solvers vs theory builders).

**Further Reading** (titles should be clickable with hyperlinks): “A Young Man’s Game”; Impersonation and Personification (Bourbaki and friends); The ‘In’ of the Association for Women in Mathematics