Lorentz Center Workshop “Nonlinear diffusion models: analytical & numerical challenges”

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Contents

Schedule

Porous-media flows I, degenerate (cross-diffusion) systems I		Higher-order degenerate parabolic problems I, long-time dynamics & traveling waves I	
Time, day 1	**Activity**	**Time, day 2**	**Activity**
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10:15–10:30	opening by the organizers	10:25–10:55	talk 6: R. Bailo
10:30–11:20	talk 1: S. Pop	10:20–10:50	discussion & coffee
11:25–11:55	talk 2: A. Zurek	11:00–11:55	discussion & coffee
12:00–13:30	lunch break	12:00–13:30	lunch break
13:30–14:20	talk 3: M. Ptashnyk	13:30–14:20	talk 7: V. Ehrlacher
14:25–14:55	talk 4: K. Hopf	14:25–14:55	talk 8: H. J. Hupkes
15:00–17:00	discussions, teambuilding & coffee	15:10–17:30	problem session
17:00–18:30	forum "wine and cheese" reception	17:30–18:30	forum

Free-boundary problems, higher-order degenerate parabolic problems II		Long-time dynamics & traveling waves II, degenerate (cross-diffusion) systems II	
Time, day 3	**Activity**	**Time, day 4**	**Activity**
09:30–10:20	talk 9: I. Tice	09:30–10:20	talk 14: J. Velázquez
10:20–10:50	discussion & coffee	10:20–10:50	discussion & coffee
10:55–11:25	talk 10: H. Saito	10:55–11:25	talk 15: J. Joussen
11:25–11:55	talk 11: K. Nik	11:25–11:55	talk 16: G. Brüll
12:00–13:30	lunch break	12:00–13:30	lunch break
13:30–14:00	talk 12: E. Demattè	13:30–14:00	talk 17: A. Holzinger
14:05–14:35	talk 13: R. Schubert	14:05–14:35	talk 18: M. Schmidtchen
14:40–15:10	problem session	14:40–17:30	problem session
15:10–18:00	social event	17:00–18:30	forum
18:30-22:00	workshop dinner		

Time, day 5	Porous-media flows II
09:30–10:20	talk 19: G. Vallet
10:25–10:55	talk 20: N. David
11:00–11:55	discussion & coffee
12:00–13:30	lunch break
13:30–14:20	Wrap up forum & coffee; departure

Blue cells: opening; green cells: long talks; yellow cells: short talks.

Venue: Lorentz Center @Lambda, 2nd floor Snellius building, Niels Bohrweg 1, 2333 CA Leiden, The Netherlands

Abstracts

Monday

Robust, fast, and adaptive splitting schemes for nonlinear doubly-degenerate diffusion equations.

9:30-10:20

Sorin Pop

Hasselt University

We consider linear iterative schemes for the time-discrete equations stemming from a class of nonlinear, doubly-degenerate parabolic equations. More precisely, the diffusion is nonlinear and may vanish or become multivalued for certain values of the unknown, so the parabolic equation becomes hyperbolic or elliptic, respectively. After performing an Euler implicit time-stepping, a splitting strategy is applied to the time-discrete equations. This leads to a formulation that is more suitable for dealing with the degeneracies. Based on this splitting, different iterative linearisation strategies are considered, namely the Newton scheme, the L-scheme, and the modified L-scheme. We prove the convergence of the latter two schemes even for the double-degenerate case. In the non-degenerate case, we prove that the scheme is contractive, and the contraction rate is proportional to a non-negative exponent of the time-step size. Moreover, an a posteriori estimator-based adaptive algorithm is developed to select the optimal parameters for the M-scheme, which accelerates its convergence. Numerical results are presented, showing that the M- and the M-adaptive schemes are more stable than the Newton scheme, as they converge irrespective of the mesh. Moreover, the adaptive M-scheme consistently out-competes not only the M/L-schemes, but also the Newton scheme showing quadratic convergence behavior.

This work is done jointly with Ayesha Javed and Koondanibha Mitra.

A finite volume scheme for the local sensing chemotaxis model

11:25-
11:55

Antoine Zurek

Université de Technologie de Compiègne, LMAC, 60200 Compiègne, France

In this talk I will present a finite volume scheme for a cross-diffusion system which models chemotaxis with local sensing. This system has the same gradient flow structure as the celebrated minimal Keller-Segel system, but unlike the latter, its solutions are known to exist globally in 2D. The long-time behavior of solutions is only partially understood which motivates numerical exploration with a reliable numerical method. For this purpose, I will introduce a linearly implicit, two-point flux finite volume approximation of the system. I will explain why this scheme preserves, at the discrete level, the main features of the continuous system, namely mass, non-negativity of solution, entropy, and duality estimates. These properties will be crucial to prove the well-posedness, unconditional stability and convergence of the scheme. Moreover, this scheme possesses an asymptotic preserving (AP) property in the quasi-stationary limit. Finally, I will present some numerical experiments illustrating the convergence and AP properties of the scheme as well as its reliability with respect to stability properties of steady solutions. This work is in collaboration with M. Herda (INRIA Lille) and A. Trescase (Université de Mathématiques de Toulouse).

Multiscale modelling, analysis and simulation of mechanical properties of biological tissues

13:30-
14:20

Mariya Ptashnyk

Heriot-Watt University, Edinburgh, Scotland, UK

A formal derivation of a continuous model from an individual-based description of mechanical interactions between two contiguous cell populations with different proliferative and mechanical characteristics leads to a free-boundary problem for the cell densities. The well-posedness results and the construction of travelling-wave solutions for the one-dimensional free-boundary problem with nonlinear transmission conditions are presented, and numerical simulations illustrate the consistency between the individual-based model and the corresponding free-boundary formulation.

The interplay between the complex microscopic structures and microscopic (cellular) processes enables many biological tissues to combine the ability to resist high pressure and mechanical forces with the flexibility required for large expansions and growth. To analyse the interactions between the mechanics, microstructure, and the chemistry we derive microscopic models for plant biomechanics, assuming that the elastic/viscoelastic properties of plant cell walls depend on the chemical processes and chemical reactions depend on the mechanical stresses. To analyse and simulate the complex multiscale models, the macroscopic effective equations are derived using periodic or stochastic homogenization techniques. Numerical solutions for the macroscopic models demonstrate the impact of the microscopic structure and tissue heterogeneity on deformation and growth.

On the Cauchy problem for rank-deficient cross-diffusion: Normal form and measure-valued solutions

14:25-
14:55

Katharina Hopf

WIAS Berlin

We discuss a second-order quasilinear cross-diffusion system arising in population biology with a full but rank-deficient diffusion matrix. Owing to the degeneracy of the diffusion matrix and the nonlinear coupling, both local strong and global generalised solvability have remained largely open for this system. In our contribution, we show that the PDE system possesses a symmetric hyperbolic–parabolic structure that ensures the local well-posedness of the Cauchy problem for positive initial densities in $H^s(\mathbb{T}^d), s > \frac{d}{2} + 1$. To address the question of global-in-time solvability, we exploit underlying nonlinear entropic structures based on Rao and Shannon–Boltzmann functionals. We then introduce a weak formulation based on Young measures, for which we establish a weak–strong stability principle that implies consistency with the strong theory. Finally, we briefly illustrate generation results for such Young measure solutions by means of suitable structure-preserving finite-volume schemes.

Based on joint work with P.-E. Druet and A. Jüngel.

Tuesday

A mediocre two-component variant of the famous result on equilibration in scalar Fokker–Planck equations

9:30-10:20

Daniel Matthes

Technical University of Munich, School of CIT / Department of Mathematics

One of the most striking by-now-classical results on Wasserstein gradient flows is the one about exponential equilibration in nonlinear Fokker–Planck equations: if the nonlinearity satisfies McCann's condition, and the potential is λ-uniformly convex, then any solution tends to the steady state at exponential rate $\lambda > 0$. The proof a la Felix Otto is a remarkable application of uniform displacement convexity. Despite lots of efforts and some progress, no result of comparable generality and simplicity has ever been derived for a system of two such Fokker–Planck equations that are coupled by means of cross diffusion.

In this talk, we indicate the main difficulty, which is the total break-down of the displacement convexity as soon as coupling is introduced, no matter how tame. Our main result is that for sufficiently weak coupling, equilibration still happens exponentially fast, with a rate $\lambda_\epsilon = \lambda - O(\epsilon)$, where $\epsilon > 0$ measures the coupling strength. The proof is based on the λ-uniform displacement convexity of the decoupled system, and treats the non-convexity as deformation, using non-standard nonlinear functional inequalities. A central challenge is that the steady state is non-explicit and compactly supported. Some adaptations of the idea are discussed as well: to systems of non-local aggregation equations, to a Keller–Segel system from chemotaxis, and to the second-order destabilized fourth order thin film equation.

This is joint work with Lisa Beck (Augsburg), Christian Parsch (TUM), and Martina Zizza (SISSA).

Pedestrian models with congestion effects

10:25-
10:55

Rafael Bailo

Eindhoven University of Technology

We study the validity of the dissipative Aw-Rascle system as a macroscopic model for pedestrian dynamics. The model uses a congestion term (a singular diffusion term) to enforce capacity constraints in the crowd density while inducing a steering behaviour. Furthermore, we introduce a semi-implicit, structure-preserving, and asymptotic-preserving numerical scheme which can handle the numerical solution of the model efficiently. We perform the first numerical simulations of the dissipative Aw-Rascle system in one and two dimensions. We demonstrate the efficiency of the scheme in solving an array of numerical experiments, and we validate the model, ultimately showing that it correctly captures the fundamental diagram of pedestrian flow.

This is work in collaboration with Pedro Aceves-Sanchez, Pierre Degond, and Zoe Mercier.

Nonlinear hybrid finite volume scheme for anisotropic cross-diffusion systems

13:30-
14:20

Virginie Ehrlacher

Ecole Nationale des Ponts et Chaussées, IPP & INRIA

In this talk, we present recent results about a hybrid finite volume method on general polygonal/polyhedral meshes for modelling an anisotropic cross-diffusion system, with size exclusion. Indeed, anisotropic cross-diffusion systems naturally occur as thermodynamic limits of multi-species stochastic exclusion processes defined on a lattice, to model diffusion in solid mixtures. The systems of interest here exhibit an entropic structure and the latter is exploited to define the numerical scheme in terms of the (discrete) entropy variables. Therefore, additionally to the existence of bounded discrete solutions and the convergence of the scheme under mesh refinements, we also prove that it preserves the entropic structure of the continuous model on the discrete level. One significant advantage of the scheme is that it enables to consider general meshes which do not have to satisfy the orthogonality property required for standard Two-Point Flux Approximation finite volume schemes.

(joint work with Annamaria Massimini and Julien Moatti)

Scaling relations for auxin waves

14:25-
14:55

Hermen Jan Hupkes
Leiden University

We analyze an 'up-the-gradient' model for the formation of transport channels of the phytohormone auxin, through auxin-mediated polarization of the PIN1 auxin transporter. We show that this model admits a family of travelling wave solutions that is parameterized by the height of the auxin-pulse. We uncover scaling relations for the speed and width of these waves and verify these rigorous results with numerical computations. In addition, we provide explicit expressions for the leading-order wave profiles, which allows the influence of the biological parameters in the problem to be readily identified. Our proofs are based on a generalization of the scaling principle developed by Friesecke and Pego to construct pulse solutions to the classic Fermi–Pasta–Ulam–Tsingou model, which describes a one-dimensional chain of coupled nonlinear springs.

Joint work with: Bente Hilde Bakker, Timothy E. Faver, Roeland M. H. Merks and Jelle van der Voort

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Wednesday

Traveling bore wave solutions to the free boundary incompressible Navier-Stokes equations

9:30-10:20

Ian Tice
Carnegie Mellon University

In this talk we will discuss the construction of two-dimensional traveling bore wave solutions to the free boundary incompressible Navier-Stokes equations for a single finite depth layer of constant density fluid. Our construction is based on a rigorous justification of the formal shallow water limit, which postulates that in a certain scaling regime the full free boundary traveling Navier-Stokes system of PDEs reduces to a governing system of ODEs. We find heteroclinic orbits solving these ODEs and, through a delicate fixed point argument employing the Stokes problem in thin domains and a nonautonomous orbital perturbation theory, use these ODE solutions as the germs from which we build bore PDE solutions for sufficiently shallow layers. This is joint work with Noah Stevenson.

Asymptotic stability for the Rayleigh–Taylor problem in the whole space with a horizontal interface

10:55-11:25

Hirokazu Saito
The University of Electro-Communications

In this talk, we consider the motion of two immiscible incompressible viscous fluids separated by a horizontal interface in the N-dimensional Euclidean space for $N \geq 3$ under a uniform gravitational field acting vertically downward. Surface tension is also taken into account on the interface. This motion is governed by the two-phase Navier–Stokes equations, which admit a trivial steady state, i.e., the motionless state with a flat interface and constant pressure. The fluid above the interface, occupying an upper half-space-like domain, is called the upper fluid, while the fluid below the interface, occupying a lower half-space-like domain, is called the lower fluid. It is well-known that the trivial steady state is unstable if the upper fluid is heavier than the lower fluid due to gravity. This instability is called the Rayleigh–Taylor instability. On the other hand, stability can be expected if the upper fluid is lighter than the lower fluid. In this stable case, we prove asymptotic stability of the trivial steady state based on decay properties of the two-phase Stokes semigroup and maximal regularity in an L_p-in-time and L_q-in-space setting with a suitable choice of p and q.

A mathematical approach to epidermal wound healing

11:25-
11:55

Katerina Nik
KAUST

A mathematical model for epidermal wound closure is considered. The model is based on a moving boundary problem for the wound edge, in which the edge moves if a generic epidermal growth factor exceeds a given threshold value. Existence of a waiting time before boundary motion sets in is demonstrated. We discuss the natural emergence of delay in the solution when the diffusion of the growth factor concentration fails to balance the rapid motion of the interface. We then allow the thickness of the active layer to vary in time, leading to a progressive shrinkage of the region where growth factor is produced, with the analysis focusing on the characterization of the conditions under which complete healing occurs.

This talk is based on joint work with Giovanni Testa (KAUST) and Christoph Walker (Leibniz University Hannover).

A two-phase Stefan problem with radiation: Well-posedness and traveling wave solutions

13:30-
14:00

Elena Demattè
Max Planck Institute for Mathematics in the Science

In this talk we consider a one-dimensional two-phase free boundary problem which models the melting of ice assuming transport of heat via conduction in both phases of the material and via radiation in the solid phase only. The problem reduces to the study of a Stefan-like problem which contains a non-local non-linear integral term describing the radiative heat transfer. We will first prove the well-posedness of the free boundary problem. We will also study the existence of traveling wave solutions. In particular, we will show that there exists traveling wave solutions for which the solid expands. This is joint work with Juan J. L. Velázquez.

Convergence to the flat interface for unbounded solutions of the Mullins–Sekerka evolution

14:05-
14:35

Richard Schubert
University of Bonn

The Mullins–Sekerka (MS) evolution is a non-local third-order geometric evolution rooted in materials science that is characterized by preservation of mass and reduction of surface area. In this talk we consider the evolution of fairly wild perturbations of the plane by the MS law in three ambient space dimensions. Only assuming initial finiteness of the excess mass and the excess surface energy, and using the nonlinear structure of the evolution, we argue that the surface eventually becomes a graph, even going through singular geometric events, and that the energy converges with an optimal algebraic rate towards the flat ground state. We will discuss how the energy dissipation structure and the L1-method for conservation laws enter the proof. The evolution problem for the interface curve in two ambient dimensions is critical and we will discuss progress if time allows. Based on joint work with Felix Otto, Elena Salguero and Maria Westdickenberg.

Thursday

Oscillatory behaviour in a bi-monomeric polymerization model

9:30-10:20

Juan Velázquez
University of Bonn

To provide a mechanistic explanation of sustained then damped oscillations observed in a depolymerisation experiment, a bi-monomeric variant of the seminal Becker-Döring system has been proposed. We study in detail the mechanisms leading to such oscillations and characterize the different phases of the dynamics, from the initial high-amplitude oscillations to the progressive damping leading to the convergence towards the unique positive stationary solution. We give quantitative approximations for the main quantities of interest: period of the oscillations, size of the damping (corresponding to a loss of energy), number of oscillations characterizing each phase. We illustrate these results by numerical simulations in line with the theoretical results.

(Joint work with Marie Doumic (Polytechnique, Paris), Klemens Fellner (Graz) and Mathieu Mezache (INRAE, Paris)).

Dynamics of a rimming flow in a cylinder of finite length

10:55-11:25

Juri Joussen
University of Stuttgart

A rimming flow is the flow of a fluid partially filling a rotating cylinder. We assume that the fluid forms a capillary-driven thin film coating the inner wall. We formally derive a closed fourth-order degenerate-parabolic evolution equation for the height of the liquid film h:

$$h_t + h_\theta + \gamma \operatorname{div}\left(h^3 \nabla(\Delta h + h)\right) = \delta(h^3 \cos(\theta))_\theta$$

with time $t > 0$, cylindrical coordinates $(\theta, z) \in \mathbb{T} \times (0, L)$ and Neumann-type boundary conditions on the cylinder covers. Competing effects relevant to this equation are the cylinder rotation, surface tension $\gamma > 0$, viscosity and gravity $\delta \geq 0$. We investigate the stability of positive steady states. For small gravity $0 < \delta \ll 1$ and short cylinders $(L < \pi)$ we show that certain steady states are stable and while they are unstable for long cylinders ($L > \pi$). For cylinders of critical length $L = \pi$ we investigate solutions of the form

$$h(t, \theta, z) = m + a_1(\delta^2 t)e^{i(\theta - t)} + a_{-1}(\delta^2 t)e^{-i(\theta - t)} + b(\delta^2 t)\cos(z)$$

and formally derive an ODE for the (slow) coefficients $a_1, a_{-1} \in \mathbb{C}, b \in \mathbb{R}$, thus describing the dynamics of such solutions on the slow time scale $\tau = \delta^2 t$.

This talk is based on joint work with Janne Laudien (Stuttgart), Christina Lienstromberg (Stuttgart) and Juan J. L. Velázquez (Bonn).

Thin fluid films heated from below

11:25-
11:55

Gabriele Brüll
Lund University

Thin liquid films find various applications in industrial processes. These films, ranging from nanometers to micrometers in thickness, are often exposed to temperature gradients, which compete with stabilizing surface tension effects. A mathematical model describing the evolution of thin fluid film's height resting on a horizontal, heated plate can be derived via lubrication approximation, constituting a quasilinear, degenerate, fourth-order partial differential equations. When a critical temperature disparity between the heated plate and the surrounding environment is reached, the flat steady state becomes spectrally unstable and periodic stationary solutions bifurcate. Using a Hamiltonian formulation of the stationary problem and analytic bifurcation theory, we prove the existence of a global bifurcation curve converging to a weak stationary periodic solution exhibiting film rupture.

Based on a joint work with B. Hilder and J. Jansen.

Mean-field convergence in relative entropy for multi-species models and fluctuations in the moderate regime

13:30-
14:00

Alexandra Holzinger
Utrecht University

In this talk, I will show how to prove a mean-field limit in relative entropy for a multi-species model of moderately interacting particles. In our setting, the inter- and intra-species interaction kernels approximate attractive/repulsive singular potentials up to Newtonian/Coulomb singularities. A crucial argument in the proof is to show a convergence in probability result between the particle level and the corresponding smooth approximation. Moreover, I will explain how this crucial convergence in probability can be used as a new technique to extend Oelschläger's well-known result for fluctuations around the porous media equation to the case of aggregating particles. This talk contains joint work with José A. Carrillo and Shuchen Guo as well as Li Chen and Ansgar Jüngel.

Nonlocal approximation of an anisotropic cross-diffusion system

14:05-
14:35

Markus Schmidtchen
TU Dresden

Localisation limits and nonlocal approximations of degenerate parabolic systems have experienced a renaissance in recent years. However, only few results cover anisotropic systems. This work addresses this gap by establishing the nonlocal-to-limit for a specific anisotropic cross-diffusion system encountered in population dynamics featuring phase-separation phenomena, i.e., internal layers between different species. A critical element of the proof is an entropy dissipation identity, which we show to hold for any weak solution.

This is joint work with Tomasz Dębiec.

Friday

Obstacle problems and Lewy-Stampacchia's inequalities

10:30-
11:20

Guy Vallet

LMAP - UMR CNRS 5142, Pau, France

The presentation will focus on obstacle problems for nonlinear PDEs and the associated Lewy-Stampacchia inequalities. The first part will deal with deterministic elliptic and parabolic cases, while the second part will deal with stochastic parabolic and first order hyperbolic cases.

The technique is based mainly on passing to the limit in a penalization procedure of the constraint, emphasizing compactness techniques.

Lewy-Stampacchia inequalities are then stability estimates of the Lagrange multiplier associated with the constraint. This requires an assumption called the dual order hypothesis.

Emphasis will be placed on how these inequalities are also a tool for obtaining existence results.

On the singular limit of Brinkman's law to Darcy's law

10:20-
10:50

Noemi David

CNRS, LMRS, Université de Rouen

In this paper we study singular limits of congestion-averse growth models, connecting different models describing the effect of congestion. These models arise in particular in the context of tissue growth. The main ingredient of our analysis is a family of energy evolution equations and their dissipation structures, which are novel and of independent interest. This strategy allows us to consider a larger family of pressure laws as well as proving the joint limit, from a compressible Brinkman's model to the incompressible Darcy's law, where the latter is a Hele-Shaw type free boundary problem.

This is joint work with Matt Jacobs and Inwon Kim.
