Assimilation of geomagnetic observations in dynamical models of the Earth’s core

Alexandre Fournier (with Julien Aubert and Erwan Thebault)

The convective motion of the liquid iron alloy flowing in the Earth’s outer core generates electric currents which sustain the geomagnetic field against Ohmic decay, in an inherently three-dimensional process known as the geodynamo. The poloidal component of the field is free to permeate the Earth’s mantle and reach its surface. Following Gauss and using a spherical harmonic analysis, one can show that the core field dominates the largest scales of the geomagnetic spectrum of internal origin, up to spherical harmonic degree 12-13. Smaller scales are concealed by the crustal field (created by remanent or induced magnetization). The variations of the geomagnetic field with time (the so-called geomagnetic secular variation), albeit limited to large scale events, open a unique window on the very rich and fascinating magnetohydrodynamics of the Earth’s core.

Assimilating geomagnetic observations in numerical models of Earth’s core dynamics is a task similar to the one undertaken in the fields of atmospheric or oceanic dynamics. Its primary goal is to combine in an optimal fashion the information contained in the database of geomagnetic observations and in the dynamical model, by adjusting the model trajectory in order to provide an adequate fit to the data. Compared to the previously mentioned parent fields where it has arguably entered a continuously improving operational stage, data assimilation still faces first order questions when considering its application to the core problem. The most obvious difficulty is related to the remoteness of the core, and the fact that we only have access to a tiny fraction of the state vector (the large scales of the poloidal field at the core surface). In a three-dimensional framework, this raises the question of the propagation of information from the surface of the core downwards, and puts even more weight on the definition of the covariance matrix chosen to that end. A second source of uncertainty lies in our actual difficulty to model numerically the entire catalog of physical phenomena responsible for the geomagnetic secular variation. Numerical limitations force three-dimensional simulations to operate in a region of parameter space away from the geophysical one, and make the use of scaling laws mandatory to extrapolate model outputs to geophysical values.

In this talk I will summarize some recent work devoted to the implementation of sequential data assimilation schemes to three-dimensional models of the geodynamo. I will try to put the emphasis and to trigger discussions on the two issues listed above: the definition of a sensible covariance matrix, and the uncertainties affecting the scaling laws used for extrapolation to the geophysical world.
Problems in Seismology and their Uncertainties

Heiner Igel

The understanding of our planet’s evolution, its current dynamics, the forecasting of strong ground motions after earthquakes and many other earthquake related issues rest to a large extent on the interpretation of information contained in ground motion observations (seismograms) recorded at the Earth’s surface. That means we are in the classical situation of solving an indirect problem (inferring physical parameters at locations we cannot access with few exceptions). The resulting images impact many fields beyond seismology (geodynamics, planetology, volcanology, geology, mineral physics, geodesy, etc.).

Seismology can be considered data rich. Ground motions are recorded permanently around the globe and the station network densities are steadily increasing. So what is the problem? Well, despite the relatively straightforward physics of wave propagation in a visco-elastic anisotropic body the computational solution of a realistic 3-D problem still is extremely challenging. What we learned about the Earth’s interior so far rests to a large extent on reducing seismograms to a few bytes of information (e.g., travel times) and reducing the physics to a ray theoretical problem. More recently, the increasing computational power allows the inverse problem to be based on the complete solution of the seismic wave propagation problem.

What about the uncertainties? Actually, despite the high relevance of the tomographic problem, the complete quantitative assessment of uncertainties must be considered an unsolved problem. Because of the number of degrees of freedom, the problem can in general not be solved using Monte Carlo type methods and probabilistic approaches. Adjoint methodologies allow the use of iterative approaches to minimize misfits between data and theory. The origin of uncertainties are manifold and will be discussed in the presentation.

Those include the uneven distribution of seismic sources and receivers around the globe, the trade-offs between various physical parameters (e.g., seismic velocities, density, anisotropic parameters) and their sensitivity w.r.t. observables and many others. Progress in the quantification of uncertainties in seismology might have a strong impact on better testing general hypotheses in Earth Sciences.
Variational data assimilation in geomagnetism: progress and perspectives

Andrew Jackson, Kuan Li and Philip Livermore

There has been tremendous activity in the last few years concerned with the applications of data assimilation to the problem of geomagnetic field evolution. Both variational and sequential data assimilation have been used to some extent in this arena; variational assimilation has been implemented on "reduced physics" models, whereas sequential data assimilation has been implemented using a fully self-consistent 3-D geodynamo model that solves the equations of momentum transfer, magnetic induction and heat transfer in the liquid core of the Earth. The latest application of sequential data assimilation has been to the prediction of the secular variation 5 years ahead, which is a necessity when the International Geomagnetic Reference Field is produced.

There are many challenges for geomagneticians, not least how to model the physics in the most realistic way possible. Thus the choice of dynamical model is open. On short timescales the use of quasi-geostrophic models seems promising; this has the advantage of leading to a 2-D problem. On longer timescales, one really wants to use a model in which the effects of viscosity are minuscule compared to other forces in the system. This creates numerical and algorithmic challenges which I will describe. The key observational database pertinent to this problem is that comprising direct observations of the magnetic field taken by humankind over the last 4 centuries. It turns out, from considerations of the time scales for overturn of the core fluid, that this is probably almost a long enough time window, being equivalent to several days of observation in meteorology.

In this talk we will review one methodology that is in the process of being implemented for variational data assimilation, based on a spectral formulation of the adjoint equations. The problem, akin to the weather prediction problem, is to find an optimal initial state of the system that evolves according to a prescribed dynamical law in agreement with the observations. We demonstrate good performance in a "proof-of-concept" problem (with noise-free data) when the gradients calculated using the adjoint technique are used with a quasi-Newton iterative update procedure.

We will illustrate the sensitivity to some of the control parameters that are assumed to be known in these toy problems. We will try to discuss open challenging questions such as how to implement these ideas within a Bayesian framework, making use of the constraints that lead to prior probability distributions on the parameters of the state vector.
Developments in High Performance Computing

Walter Lioen

In this talk we will give an account on the current and upcoming top-of-the-line supercomputers. Furthermore, we will have a look at (future) developments in (super)computer architecture and its potential consequences to parallel programming paradigms.

Many people most probably will be aware of Moore’s law: "The number of transistors that can be placed inexpensively on an integrated circuit doubles approximately every two years." Taking into account the increasing speed of transistors, integrated circuits doubled performance every 18 months. This trend has continued for more than half a century and is expected to continue until at least 2015 or 2020. Looking at the Top500 (which started in 1993) list of the most powerful supercomputer systems of the world, we observe the same performance trend.

From the application (software development) point of view, we are interested in the change of architecture. Coming from vector computers, we moved to parallel vector computers and evolved to massively parallel computers. This also meant changes in the programming paradigms. The current (June 2011) Top 1 system has well over half a million cores. What does this mean for high performance software development? In the current Top10 we find 4 systems with accelerators (3 GPU clusters and 1 PowerXCell). What will remain and how will the architecture of a future exascale system look like? An even more important question is what will this mean for the applications? The current de facto standard is using Fortran/C/C++ and MPI/OpenMP. Do we need new paradigms like PGAS languages and OpenCL and what is their current state?

There are a lot of uncertainties except for one: we have a lot of challenges ahead.

Modelling of forecast errors in Data Assimilation for Numerical Weather Prediction

Andrew C. Lorenc

The predictive skill of Numerical Weather Prediction (NWP) systems has improved steadily by a day per decade for the past three decades. This has been due to increases in computer power allowing higher-resolution forecast models, data assimilation (DA) systems which combine forecast and observed information allowing for the information content in each, advanced methods of predicting the flow dependence of forecast errors, and better observations.

Past improvements in DA have concentrated on the larger, better-observed scales, with errors that evolve approximately linearly for short periods. Practical methods are based on the extended Kalman filter: the most popular (incremental 4D-Var) uses a linear model (filtered to remove small unpredictable scales) to predict the evolution of errors over a relatively short assimilation window. However the longer-term growth, and hence the typical size and structure of error patterns, is governed by much more complex nonlinear effects. Fortunately, although
the smallest scales grow most rapidly, they tend to saturate at small amplitude, leave most error variance in the larger-scale “balanced” motions that 4D-Var can handle. This error spectrum can only be measured empirically; 4D-Var starts its time-window assuming that errors are drawn from a distribution described by a “background error covariance model”. Usually, for lack of better information, this is assumed to be constant, estimated from a large sample of past DA experiments. More recently, ensemble Kalman filers have been used to give some flow dependence.

The decades of development have given large complex systems: global models now have a billion degrees of freedom while regional models represent scales of a few kilometers. Improvements in resolution continue to demonstrate benefit, despite there being few observations at such fine scales. The information which makes NWP possible comes because the atmosphere exhibits a nonlinear “attractor” with much lower dimension. We do not have good practical methods for defining this attractor, other than by using the best available forecast model. (This has been called “initialization”, and “spin-up”.) We can describe the properties of balanced flow, fronts, cyclones, stratocumulus, ..., but mathematical rules and rules when to apply them, do not exist. The probability distribution describing this “information” is non-Gaussian, and not compatible with the linear methods which work well at larger scales. Our challenging goal is to continue the past rate of improvement, using increases in computer power, more observations and better mathematical techniques, into this regime.

Uncertainty Quantification in Earthquake Source Inversion

P. Martin Mai

Inversion and modeling studies using seismological and geodetic data to infer kinematic source parameters on extended faults have been carried out since the early 1980ties. Initially, linearized (multi time-window) methods have been widely used to estimate rupture speed and slip on the fault (parameterized as being released within one or more elementary slip functions). Subsequently, more refined non-linear source inversion strategies have been developed to estimate the space-time rupture evolution over the assumed fault plane. However, these works rarely have carried out a formal uncertainty analysis of the inferred parameters, owing to the fact that errors/uncertainties on Earth structure, detailed fault geometry, and site effects are difficult to quantify. More recently, Bayesian methods have been applied to source inversions, providing a formal a posteriori PDFs for rupture quantities in case of quantifiable data uncertainty; the extension to account for geometrical and geological uncertainties is work in progress.

In this presentation I will briefly review finite-fault earthquake source inversion methods and highlight some key results of these studies, addressing also their attempts for uncertainty quantification. Problems of data selection, model parameterization, inversion strategy and personal choices will be discussed, and how these lead to large model variability in inferred source models. Finally, I will present the Source Inversion Validation (SIV) project, a recently started initiative to conduct a large-scale, multi-year collaborative study on uncertainty quantification in earthquake source studies.
Uncertainty quantification in geophysical inversion: Challenges and the way forward

Max A. Meju

Geophysical inversion is a mathematical process that uses remote measurements of physical fields to predict the structure and state of subsurface materials relevant to operational decision making. These data are typically heterogeneous (multi-scale, band-limited and contain noise), and the mathematical representations of the complex subsurface in the inversion process are necessarily simple approximations of physical reality. Thus, the main sources of uncertainty in geophysical inversion are data inaccuracy and insufficiency, incomplete knowledge and information about the typically complex geological targets, and predictive modeling (theoretical and discretization) errors. A key challenge is how, for decision-making purposes, to quantify the uncertainty in the information from data inversion since all indirect inference of the parameters and states of the geophysical system being investigated is subject to uncertainty.

There are two essential components of geophysical inversion: model search and model appraisal. In model search, the various sources of uncertainty are treated using either deterministic or probabilistic approaches. In probabilistic geophysical inversion, an objective way is provided to weigh the influence of each data source. This is important given that in simulating a complex physical system such as the Earth based on limited information, uncertainty may arise in the selection of the relevant physics and associated models, approximations in the experiments used for validating these process simulation models, and the model resolutions achievable with the available computational resources. In deterministic inversion, we can also account for measurement errors, model prediction errors and lack of complete information about the subsurface by appropriately weighting our mathematical constructs. In model appraisal, understanding the range of possible solutions permitted by the inverted heterogeneous data and how to interpret this range pose major challenges. In particular, how to uniquely quantify the uncertainties associated with the parameter estimates from a given model search remains a subject of vigorous debate. The notable approaches include linear sensitivity analysis, Bayesian probability density function estimation, minimax, and construction of extremal solutions but these have been mostly applied to small-size inverse problems. For large data sets, there is an increasing interest in the use of Monte Carlo integration methods for approximating large error covariance matrices (especially in gravity field modeling) and facilitating the related error propagation computations.

In this paper, we discuss some key practical considerations for uncertainty analysis in geophysical inversion in a deterministic context. An important consideration is that model uncertainty can be reduced by combining measurements of fundamentally different physical attributes of the subsurface or by using reliable a priori information about the subsurface. It is shown that the non-linear regularised extremal bounds analysis (REBA) method allows for combining various data sources and their associated uncertainties in an objective manner to determine model bounds that are maximally consistent with data errors. Some emerging gradient-based metrics for simple assessment of interpretational uncertainty in small- to moderate-size multidimensional inversion models are also discussed.
Cost functionals for seismic inversion

W.A. Mulder

Subsurface imaging for hydrocarbon exploration and production is mainly based on seismic data. With the continuing increase of computing power, processing and inversion of seismic data with the full wave equation has become feasible, though often only in the acoustic approximation.

The least-squares cost functional that measures the difference between observed and modelled data is an obvious candidate for inversion, but suffers from local minima caused by a lack of low-frequency energy in the data. Uncertainty estimates based on an approximation of the problem's hessian only give an indication of the size of the local basin of attraction, not of its distance to the global minimum.

There are several alternatives to least-squares fitting, all based on some kind of focusing of the image. Underlying these methods is the single scattering assumption, described by the Born approximation of the constant-density acoustic wave equation. Although removal of surface multiple is a common technique, interbed multiples may still lead the focusing algorithms astray.

Focusing in the data domain is a recent generalization that, in principle, should not suffer from the presence of surface and interbed multiples. Further development is still needed to mature the method.

Presently, the main source of uncertainty is the lack of robustness of the inversion methods.

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Uncertainty in Transdimensional inverse problems

Malcolm Sambridge

For many years seismologists have built models of the Earth's seismic structure over local, regional and global distance scales using derived quantities of a seismogram covering the frequency spectrum. A feature common to (almost) all cases is the objective of building a single `best' Earth model, in some sense. This is despite the fact that the data by themselves often do not require, and sometimes not even allow, a single best fit Earth model to exist. It is widely recognized that many seismic inverse problems are ill-posed and non-unique and hence require regularization or additional constraints to obtain a single structural model. Interpretation of optimal models can be fraught with difficulties, particularly when formal uncertainty estimates become heavily dependent on the regularization choices made.

An alternative approach is to embrace the non-uniqueness directly and employ an inference process based on parameter space sampling. Instead of seeking a best model within an optimization framework one seeks an ensemble of solutions and derives properties of that ensemble for inspection. While this approach is over 30 years old, it is only recently gained popularity in geophysics. Recent work has shown that trans-dimensional and hierarchical sampling methods have some considerable benefits for seismological problems involving
multiple parameter types, uncertain data errors and/or uncertain model parameterizations. In this formulation aspects of both the parametrization and the data noise become unknowns to be constrained by the data themselves, rather than being chosen in advance as is often the case in an optimization framework. Limitations exist with sampling based approaches in that computational cost is often considered to be high for large scale structural problems, i.e. many unknowns and data. Uncertainty estimation in a transdimensional inversion framework presents its own difficulties.

A tutorial on uncertainty quantification for inverse problems

Luis Tenorio

In this tutorial we will present some basic tools for uncertainty quantification for inverse problems with an emphasis on Tikhonov type estimators. It is intended to be accessible to a wide audience. To review the basic definitions, we start with a summary of penalized least squares and the Bayesian linear model. We then move to uncertainty assessment for Tikhonov regularization. In addition to a discussion on the formal statistical inference, the tutorial will present exploratory data analysis methods to assess inversion estimates using examples based on $l^2$- and $l^1$-regularization. These methods can be used to reveal the presence of systematic errors such as bias and discretization effects, or to validate assumptions made on the statistical model used in the analysis. The formal and exploratory methods include: confidence intervals and bounds for the bias, resampling methods for model validation, and construction of training sets of functions with controlled local regularity.

Poster Presenters

Towards a non-linear Backus-Gilbert theory

David Al-Attar

In linear inverse problems, each datum is related to the model by the action of a linear functional. From a finite set of such data one cannot determine the model uniquely. By forming suitable linear combinations of the data, Backus and Gilbert showed that it is, however, possible to calculate certain `averages' of the model of physical interest. Furthermore, if the data have known statistical properties, the uncertainty of these estimates can be determined in a simple manner. For non-linear inverse problems, Backus-Gilbert theory can be applied to the linearization of the problem about a given reference model. For many applications, however, linearization is not justified, and it is desirable to address the non-linear aspects of the problem in a direct manner.

In this poster, we describe preliminary work on an extension of Backus-Gilbert theory to non-linear inverse problems. For such problems, each datum is given by the action of a non-linear functional on the model. Consequently, each arbitrary combination of the data corresponds to
the action of a particular non-linear functional on the model. Supposing we wish to evaluate the action of a given functional on the model, we can then seek the combination of the given data which best estimates this quantity. We describe possible methods for addressing such approximations problems for non-linear functionals, and also consider methods for quantifying the uncertainty in these estimates.

Measurements of translation, rotation and strain: New approaches to seismic processing and inversion

Moritz Bernauer

We propose to include measurements of seismically induced rotation and strain in tomographic inversions for 3D Earth structure, thus going beyond standard seismic tomography that exclusively relies on recordings of translational ground motion. Exploiting more than 3 observables is expected to yield more detailed tomographic images.

The project is embedded in full waveform tomography concepts. Combining spectral element methods for accurate solutions of the elastic wave equation in laterally heterogeneous Earth models and adjoint techniques we can compute Fréchet kernels of translation, rotation and strain measurements with respect to arbitrary structural parameters. The challenge is to design alternative imaging schemes including these additional observables.

Our study is based on a systematic analysis of the sensitivity power of the complete set of translation, rotation and strain observations defined as the weighted sum of the corresponding Fréchet kernels. The strategy is to optimise the sensitivity power with respect to certain structural parameters. Using Lagrange functions and principal component analysis we try to maximise the sensitivity power for the structural parameter $p$, e.g. density, while simultaneously minimising the sensitivity power for the structural parameter set $m$, e.g. P and S wave velocities.

Andrea Colombi

The modern approach to seismic tomography can exploit all the information contained in seismograms, account for the finite-frequency character of the propagating waves and provides models that seem consistent with geodynamics or observations from mineral physics, geology and Earth magnetism. Yet, it fails to provide quantitative uncertainty estimations.

The rigorous mathematic approach of the adjoint method employed to calculate the model derivatives for a 3-D Earth already requires an immense computational power. An uncertainty analysis based on the exploration of the model-parameters space is therefore not yet feasible.

The adjoint technique, the core of finite-frequency tomography, relies on 1st order perturbation theory that might or might not be valid depending on how far the starting model is from the true one. Moreover, the performance of such an algorithm strongly depends on several parameters: data type and frequency, their error statistics, the chosen misfit, the sparsity of data coverage, the wavelength of the anomaly we wish to resolve, and inversion parameterization. Those
variables further widen the already huge parameter space where the tomographic model has to be solved.

We set up a synthetic experiment to investigate, quantify and potentially improve the resolving power of this tomographic technique.

The classical approach to seismic tomography investigates volumetric properties (velocity, density, impedance) of the Earth, disregarding other unknowns such as, for example, boundary perturbations. We include the effect of weak boundary perturbations with respect to the reference model in the synthetic experiment and inversion.

Such perturbations, besides their effects being not well known, have not been included as free parameter in a joint full-wave inversion (boundary + volumetric). We investigate the feasibility to resolve a trade off between these two contributions to the observations.

Starting from a known Earth model, slightly modified to embed a topography perturbation to the core mantle boundary, we are computing a collection of synthetic data varying the important parameters (frequency content, source-receiver geometry, structure dimensions…). The reference model being in our hands allows us to quickly evaluate at what extent unknown features has been recovered.

The short list of goals that this project set to itself is:

- To assess the validity of the finite frequency approach in reconstructing Earth structure with varying parameters.
- To quantify the trade off between boundary and volumetric perturbations both on seismograms and on inversion results.
- To extrapolate a series of best practices which will improve the usage of this technique and its accuracy.

While this approach does not quantify uncertainties rigorously, it should help designing inversions optimized to resolve certain features and give a sound intuition to the variability with respect to various parameter changes.
Uncertainty characterisation in atmospheric chemistry data assimilation and emission estimation

Henk Eskes

The use of data assimilation to systematically combine observations and prognostic model forecasts is still relatively new in the field of atmospheric chemistry. In Europe, the MACC project (www.gmes-atmosphere.eu) is a major effort to develop the infrastructure for the analysis of atmospheric composition, focusing on greenhouse gases, reactive gases like ozone, and aerosols. This analysis is done in combination with an analysis of the meteorological state of the atmosphere. A large variety of measurement datasets are available, including satellite sensors (EnviSat, EOS-Aura/Terra/Aqua, MetOp etc), national surface air pollution networks, international research networks (surface, balloon, aircraft, lidar measurements) and dedicated campaign data.

In my contribution I will introduce the models and observational data sets used in atmospheric chemistry. An overview of the data assimilation techniques and inverse modelling approaches used for atmospheric composition will be given, in particular the 4D-Var and Ensemble Kalman filter (EnKF) techniques. As example, the satellite observations, assimilation and inverse modelling of nitrogen dioxide (NO2, an important component of air pollution) will be discussed. An EnKF approach is used to infer the source strengths (e.g. traffic exhausts, industry, power generation) from the satellite NO2 data. The error characterisation of the observations and of the Kalman filter inversions will be treated in detail.

Fig 1a: Measurements of nitrogen dioxide (NO2) from space with the OMI satellite instrument, for 27 March 2007.
Resolution Analysis in Full Waveform Inversion

Andreas Fichtner & Jeannot Trampert

Full waveform inversion is a tomographic technique that is based on numerical wave propagation through complex media combined with adjoint or scattering integral methods for the computation of Fréchet kernels. While the tomographic method itself has advanced substantially, an essential aspect of the inverse problem has been ignored almost completely: The quantification of resolution and uncertainties.

We propose a new method for the quantitative resolution analysis in full seismic waveform inversion that overcomes the limitations of synthetic inversions while being computationally more efficient and applicable to any misfit measure. The method rests on (1) the local quadratic approximation of the misfit functional in the vicinity of an optimal Earth model, (2) the parametrisation of the Hessian in terms of a parent function, and (3) the computation of the space-dependent parameters via Fourier transforms of the Hessian, calculated with the help of adjoint techniques.

In the simplest case of a Gaussian approximation we can infer rigorously defined 3D distributions of direction-dependent resolution lengths and the image distortion introduced by the tomographic method. We illustrate these concepts with a realistic full waveform inversion for upper-mantle structure beneath Europe.

As a corollary to the method for resolution analysis we propose several improvements to full waveform inversion techniques. These include a pre-conditioner for optimisation schemes of the conjugate-gradient type, a new family of Newton-like methods, an approach to adaptive parametrisation independent from ray theory, and a strategy for objective functional design that aims at maximising resolution.

The computational requirements of our approach are less than for a typical synthetic inversion, but yield a much more complete picture of resolution and trade-offs. It allows for adaptations to
exploration scenarios and other wave equation based tomography techniques that employ, for instance, georadar or microwave data.

Network Design for Improved Co2 Flux Estimation over Temperate Asia

Vinod Gaur (P.S. Swathi and N.K. Indira)

CO2 flux estimates over temperate Asia obtained from inversion of atmospheric concentration data, continue to be plagued by large uncertainties owing, amongst several factors, to sparsity of adequately distributed data. We propose to discuss the problem of designing a minimum network that would help distil a fairly robust estimate of fluxes over this region, without overburdening the enterprise with logistics entailed in meeting the exacting requirements of data quality( 0.01 ppm)

A typical set of steps for estimation of fluxes involves: i) generation of Green's functions using a transport model, and ii) estimation of surface fluxes and their posterior uncertainties by minimizing a cost function which is a trade-off between closeness to a prior model and the differences between modeled and measured data. The GA-based algorithm, in turn, is tested in two way: first, we consider a simplified problem of locating only a small number of surface stations (1, 2 and 5) in a Transcom framework. We run the Transcom inversion for every point on the global grid and find the score. We then run the GA algorithm to see if it can pick up the correct points.

In the second test, we choose certain stations with very small data uncertainties (super stations) and test the expectation that the GA picks these out, usually about 80%.

In the complete network design problem, we start from a full 4D simulation of the forward transport problem so that we can build the model matrix for any number of stations. The number of these locations is quite large (128*64*28 =22376) and the GA problem is very computer intensive. We present the performance of the algorithm and possible networks for Temperate Asia.

Interannual Variability of CO2 Fluxes in Temperate Asia Using Data from Hanle, India

Vinod Gaur (N.K. Indira, P.S. Swathi, M. Ramonet, B.C. Bhatt)

The spatial and temporal estimations of the sources and sinks of CO2 have become increasingly important to gain an understanding of its future sequestration in the oceans and terrestrial biosphere as both global mean CO2 concentrations and the attendant temperatures rise.

One of the approaches to estimate these fluxes, is to invert atmospheric CO2 concentrations by deconvolving from it the atmospheric Green’s Functions, following an internationally agreed protocol amongst scientists - the Transcom. The Inversion for CO2 fluxes is accomplished by us using the atmospheric transport model MOZART in a Bayesian formulation. Uncertainties in estimates are analyzed by examining the misfits between prior and posterior models and
between modeled and observed data. The posteriori uncertainties, model correlation and resolution matrices are analyzed to examine the robustness of the inversion.

The uncertainties in these estimates are particularly large in Asia, Africa and South America, especially due to paucity of stations. We propose to discuss the case where uncertainties in the flux estimates over temperate Asia registered a significant improvement after data from just one station at Hanle (35N,79E and 4500m), operated by us since 2005, were added to the global data, as well as explore other means of improving the estimates.

Stochastic inversion of dynamic geophysical and hydrological data for estimating subsurface hydrological properties: insights and observations

James Irving

Traditional hydrological measurements for estimating subsurface properties controlling groundwater flow and contaminant transport are limited by their support volume and expense. A considerable benefit of geophysical measurements is that they provide a degree of spatial coverage and resolution that are unattainable with such traditional methods, and the data can be acquired in a cost-effective manner. In particular, dynamic geophysical data allow us to indirectly observe changes in hydrological state variables as flow and transport processes occur, and can thus provide a link to hydrological properties when coupled with a process-based model. Stochastic inversion of these two data types offers the potential to provide not only estimates of subsurface hydrological properties, but also a quantification of their uncertainty. This information is critical when considering the end use of the data, which may be for groundwater remediation and management decision making.

Here, we examine a number of key issues in the stochastic inversion of dynamic hydrogeophysical data. We focus our attention on two problems. The first involves the use of time-lapse crosshole electrical resistivity measurements and saline tracer-test concentration data to estimate the spatial distribution of hydraulic conductivity (K) below the water table. The second involves the use of dynamic crosshole ground-penetrating radar measurements, acquired during an infiltration experiment, to estimate van Genuchten - Mualem (VGM) parameters in the vadose zone. In both cases, the available measurements are assimilated in a stochastic manner using a Bayesian Markov-chain- Monte-Carlo (McMC) approach. This provides multiple realizations of subsurface properties that are consistent with the measured data and assumptions regarding model structure and data errors. To make the spatially distributed, stochastic inverse problems computationally tractable, we take the important step of considerably narrowing the space of acceptable solutions by considering only a limited number of facies having constant hydrological properties, and by enforcing prior correlation between model cells.

The Bayesian-McMC inversion methodology importantly shows that the introduction of dynamic geophysical data into the subsurface characterization problem allows us to better identify subsurface hydrological parameters, and more importantly to reduce hydrological prediction uncertainty. Our work, however, also demonstrates that proper model and data error
quantification are crucial for reliable posterior uncertainty estimates, and it shows the importance of the issue of model versus measurement scale.

Object-Based Probabilistic Full Waveform Tomography - Methodology and Application to the Australian Continental Lithosphere

Paul Kaeufl

We present a probabilistic full waveform inversion, based on spectral-element simulations of seismic wave propagation on a continental scale and a regionalised parameterization of the upper mantle. The classical approach to seismic tomography consists of finding one single velocity model of the Earth’s interior that minimises the misfit between simulated and observed seismograms. This deterministic approach does not account for the possible existence of multiple solutions that explain the data equally well, and it does not provide information on the reliability and the resolution of a model. Furthermore, the full elastic wave equation is often replaced by a ray-theoretical approximation, thus reducing the exploitable data to the arrival-times of well-defined seismic phases. In this project we circumvent the above issues by making use of a Monte Carlo optimisation method based on a Bayesian statistical framework. This leads to an ensemble of models and to an error estimate for each model parameter. We use the spectral-element method for the simulation of 3D wave propagation through heterogeneous Earth models. An object-based parametrisation of the Earth, motivated by the geological structure of western and central Australia, enables us to limit the dimensionality of the model space and to test hypothesis efficiently. The appraisal of the synthetic seismograms is taking into account the full waveform data available. We apply our methodology to the Australian continental lithosphere. This is intended to answer the following questions: (1) How good is the resolution of the tomographic models? (2) Is there reliable information on density variations in the upper mantle contained in the waveforms? (3) How robust is the frequently inferred low-velocity layer around 150 km depth beneath Proterozoic Australia?

Continental lithospheric strength: controversy fuelled by complex, and often ignored, uncertainty analysis

Lara M. Kalnins, Frederik J. Simons, Sofia C. Olhede, and Dong V. Wang

The mechanical strength of the lithosphere is a fundamental geophysical parameter that modulates geophysical processes ranging from plate tectonics to sea-level change. As geophysical modeling becomes increasing complex and seeks to incorporate an increasingly realistic lithosphere, the values and variations of the effective elastic thickness ($T_e$), a proxy for the long-term (> 1 Ma) strength of the lithosphere, become increasing important. However, continental $T_e$ remains a topic of considerable controversy, with some groups reporting maximum $T_e$ values in the cratons in excess of 100km (e.g. Peréz-Gussinyé et al., 2009), whilst others prefer a maximum of 25-30km (e.g. McKenzie,2003) for the same regions.
These apparently contradictory results arise from a variety of methods analyzing gravity anomaly and topography data, including techniques based on free-air admittance, Bouguer coherence, fan wavelets and forward modeling, each of which brings its own set of uncertainties, often complicated and poorly quantified | yet if correctly assessed, results from all methods should be within error of each other. Results from all methods are commonly published with limited information about the associated uncertainties. In this paper, we explore the sources and complications associated with uncertainty for each of the above methods, from the strongly non-Gaussian distributions of admittance and coherence to the difficulties in identifying the geographical source of observed results using the wavelet method. We then consider the uncertainties associated with a new conservative Bouguer coherence analysis of continental Te and introduce a new method based on maximum-likelihood estimation theory.

The maximum-likelihood estimator has a Gaussian error distribution, even when the original data were not Gaussian, allowing uncertainties to be easily quantified, and by working with the full information available from the cross- and auto-power spectral densities of gravity and topography avoids the trade-off between Te and the distribution of loading between the surface and subsurface, another parameter of geological interest. However, despite the potential of this new technique to address some of the fundamental limitations of earlier methods, a convincing end to the controversy still requires demonstration that all methods produce consistent results | within error.

References


A Bayesian approach to inverse problems

Bartek Knapik, Aad van der Vaart, and Harry van Zanten

In this poster we present a Bayesian approach to estimating a parameter μ from an observation Y following the model

\[ Y = K\mu + (1/\sqrt{n}) Z. \]

The unknown parameter μ is an element of a separable Hilbert space H1, and is mapped into another Hilbert space H2 by a known, compact, injective, linear operator K : H1 → H2. The image Kμ is perturbed by unobserved, scaled Gaussian white noise Z.

In order to make inference about μ one can put a Gaussian process prior on μ. The poster is focused on two aspects of inverse problems - estimation of the full parameter μ and linear functionals of μ.
Both in nonparametric and linear functional case, the rate of the contraction of the posterior distribution around the truth is shown. The additional result is Bernstein-von Mises phenomenon for linear functionals of $\mu$, which under suitable conditions on the linear functional and the prior shows that the posterior for the linear functional of the truth approaches a normal distribution centered at an efficient estimator of the parameter and with variance equal to its asymptotic variance.

In particular, the behavior of the posterior depends on the regularity of the element $\mu$, the regularity of the prior, and the ill-posedness of the operator $K$, which is defined by its spectral properties. Correct combinations of these characteristics lead to the minimax rate. The results are numerically illustrated by the problem of recovering a function from observation of a noisy version of its primitive.

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The maximum entropy approach for a posteriori estimation of model and data errors

Svetlana Losa

When dealing with inverse modelling (Data Assimilation, DA) using variational techniques or statistical Monte Carlo based methods (at least for oceanographic applications) we pretty often know little about model uncertainties and data error statistics. The inverse solution, however, crucially depends on our prior assumptions on the error statistics. Kivman et al. (2001) suggested the entropy approach (PME) to tune the weights of model and data costs in the generalized inversion. Implementation of such an approach for a problem of state and parameter estimation in biogeochemical modelling allowed us to obtain reliable estimates of physiological parameters and, moreover, to inference about data quality and model uncertainties (Losa et al., 2004). Here we investigate a possibility of the PME application for calibrating an ensemble based DA system for an operational circulation model of the North and Baltic Seas.

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Seismology of sea ice surface in context of inverse problem - Sea ice thickness determination

Andrey Nagurny

The theory of sea-ice vibration was developed the resonance of elastic-gravity waves moves in sea ice and upper layer of ocean induced by atmosphere processes and ice deformation. Sea ice thickness was calculated with help of sea-ice surface vibrations data by tiltmeters and seismometers on the "North Pole" Russian drift stations during 1970-1990, 2003-2005 yy. Ice thickness was determined for all Arctic Basin and for every months of all seasons. The
interannual trend of a more rapid decrease of ice thickness in general in the entire Arctic Basin, which began in first half 1990th and comprised about 1 m for the last decade, is preserved.

On the footprint of subjective choices in mapping seismic data to model

Tarje Nissen-Meyer and Alexandre Fournier

The partial derivatives of seismic data with respect to model parameters ("sensitivity kernels") form the basis for linearized and nonlinear iterative inversions. Although much tomographic progress has been achieved on the grounds of theoretical foundations, computational resources, and high-quality data, it is clear that the generally non-unique inversion of seismic data still poses a formidable validity challenge on all ends. Here, we are concerned with posing questions (rather than providing definite answers) in the realm of the inevitable subjective choices one is mandated to undertake before solving this inverse problem, and their effect on sensitivity kernels in the context of variability and uncertainty:

1) Does the inevitable forward modeling (dispersion) error significantly contribute to a bias in the computation of sensitivity kernels and synthetic seismograms in practice?

2) How do measurement errors and uncertainties in the modeling assumptions (e.g., false source properties) propagate into the sensitivity kernel?

3) How can we assess the variability of sensitivity kernels due to subjective choices in the high-dimensional parameter space effectively?

4) What is an appropriate approach to sub-sample the vast data and model spaces without losing valuable information or introducing a bias?

5) How do we quantify the validity realm for a certain scattering and inverse theory (e.g., Born, gradient) depending on frequency and heterogeneity scale given a certain data set and model space a priori?

To understand the nature and significance of some of these questions, we compute sensitivity kernels across a multi-dimensional parameter space spanned by model dependencies (background model, parameterization for the inverse problem), seismic source dependencies (location, time function, frequency, radiation pattern), data dependencies (components, azimuth, epicentral distance, time and frequency windows, misfit functions). We discuss this in the context of global seismic tomography, in which data distributions are highly irregular, forward computations expensive, models elusively unverifiable and as such these uncertainties need to be evaluated with care.

A first step toward uncertainty analysis is sensitivity analysis

Maelle Nodet
Global stochastic methods of sensitivity analysis require many runs of the numerical model we are interested in. For large and computationally demanding models, it is impossible to perform a large number of runs, it is therefore relevant to replace the full numerical model by a surrogate model, which should be a good approximation of the full model while being much faster to run. We will present a reduced basis method, allowing to build efficient and accurate surrogate models, while keeping track at cheap numerical cost of the error with respect to the full model. We will present a small application to a toy model and Sobol sensitivity indices computation.

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**From data to model: how should we handle uncertainties in a chain mixing model and data uncertainty?**

**Helle A. Pedersen** (preference oral presentation – Monday or Tuesday)

The road from raw data to final model is often composed of multiple steps which may each be complex, and involving some type of inversion. As an example, in teleseismic surface wave tomography, where the source is located outside the study area, uncertainties in source parameters are replaced by uncertainties on the propagation effects outside the seismic array. On top of this problem, the full waveform is not inverted for, the local sensitivity kernels between stations at a regional scale quite particular, and station amplitudes are associated with large uncertainties. Finally, the raw data must be transferred into some observable (often travel times in some form) which can serve as input data for subsequent inversion. The question of model uncertainty therefore transforms into a problem of successive steps of data quality control and uncertainty in the first part of the analysis, and more ‘inversion technique’ related issues in the second part of the analysis. Most present-day tomographies tend to underestimate (or at least unsufficiently present) the first element, thereby rendering the second part relatively meaningless. On the other hand, the seismological community has no standard tools or best practice at hand. Such guidelines could be a very valuable (admittedly very pragmatic) contribution to the community to help ensure better quality of tomographic models.

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**On Instability in Data Assimilation**

**Roland Potthast**

*Instability is one of the key features of many inverse problems in geophysics. Instability is a property which can arise from the underlying dynamics systems, in particular when they are chaotic or effectively have branching points. Instabilities of data assimilation algorithms can also arise from compact or ill-posed observation operators. Often, data assimilation algorithms regularize the inversion for every time-step or cycle of the assimilation method, such that each individual reconstruction is stable.

But over time instabilities are likely to occur in basically all practically applicable realizations of such systems. We will describe a mathematical framework which is suitable to study and analyse such instabilities. In particular, we work out the behaviour of cycled data assimilation systems such as 3dVar or 4dVar explicitly for simple linear systems. Also, we show how assimilation...*
systems can be stabilized, a general theory for self-adjoint linear system dynamics is presented. Numerical examples are provided.
Uncertainty Visualization

Tobias Pfaffelmoser (preference oral presentation)

Numerical data that is produced by measurements, simulations and inverse computations is always affected by a degree of uncertainty due to intrinsic, random, or systematic errors in the acquisition process, the underlying physical model, or the used computational methods and numerical precision. Analyzing the data while ignoring local and global uncertainties can result in misinterpretations and false assumptions. Although uncertainty cannot be eliminated, it can be provided to the user by adequate visualization approaches.

Although uncertainty visualization is regarded one of the biggest challenges in visual data exploration, standardized procedures for modeling and visualization the effect of uncertainty on certain features in multidimensional data sets are poorly researched. In this talk an overview on current approaches for visualizing parameterized uncertainty and uncertainty given by stochastic simulation results will be given. Thereby a strong focus will be put on integrating uncertainty effects in interactive 3D data representations by direct and indirect volume rendering techniques. In several examples the effectiveness of uncertainty visualization will be illustrated: e.g. positional and geometric variability analysis of surfaces in 3D, local and global correlation visualization for structural uncertainty analysis, etc.

Furthermore future visualization challenges, e.g. visual representation of multimodal probability distributions, visual hypothesis tests, sensitivity analysis, combined prior and posterior covariance visualization, etc. are planned to be discussed with the audience and feedback on possible requirements for inverse problems are more than welcomed.
Imaging mantle plumes with instantaneous phase measurements of scattered waves

Florian Rickers, Andreas Fichtner and Jeannot Trampert

The imaging of mantle plumes remains a challenge in seismic tomography. Despite numerous attempts, undisputed tomographic proof for the existence of lower mantle plumes is still missing. Finite frequency effects such as diffraction and wave front healing conceal travel time delays and pose problems to current tomographic methods.

We conduct two full 3D iterative synthetic inversions based on adjoint methods, which differ only in the choice of the misfit function. The aim is to recover the idealized mantle plume that has been used to generate the data. The diameter of the plume is of the order of the dominant wavelength.

In the first inversion, using travel time misfits based on cross-correlation measurements of the P wave, we only succeed to recover the upper mantle part of the plume. The lower mantle part remains largely unconstrained, despite the use of finite frequency kernels.

In the second inversion, we use misfits based on the instantaneous phase difference, a time-domain, amplitude-independent continuous measure of phase differences between two time series. This enables us to extend the data window to include the P wave coda which contains a larger fraction of the scattered wave field. Because the measurement is independent of amplitude, the scattered waves obtain a similar weight as the main phase in the tomographic inversion.

We succeed to recover the plume considerably better in shape as well as in amplitude throughout the mantle.

We conclude that the scattered wave field provides the necessary information to put strong constraints on lower mantle plumes and that the instantaneous phase difference is a suitable misfit function to take advantage of these small phases in tomographic inversions.

Cross-correlation measurements do no simply allow us to use the scattered signal.

Assimilation of oceanic angular momentum observations - implementations to annual ocean mass change.

Jan Saynisch

The oceanic contribution to Earth rotation anomalies can be manifold. Possible causes are a change of total ocean mass, changes in current speed or location and changes in mass distribution. To derive the governing physical mechanisms of oceanic Earth rotation excitation we assimilate Earth rotation observations with a global circulation ocean model. Before
assimilation, observations of length of day and polar motion were transformed into estimates of ocean angular momentum.

We study the oceanic excitations by Singular Evolutive Interpolated Kalman-Filtering of ocean angular momentum. Here the models error-covariance is propagated in time by an ensemble of parallel model simulations. Eigen-value decomposition of the error-covariance is the tool to generate an sufficient but still efficiently small ensemble.

We succeeded in the reproduction of the observation based oceanic angular momentum and report on the methods sensitivity with respect to error budgets and ensemble initialization strategies. Sufficient and insufficient mechanisms of oceanic angular momentum generation are discussed. The results show the need to adapt the models atmospheric forcing. This produces an enhanced annual mass cycle than in model runs without angular momentum assimilation.
Bayesian Source Inversion for seismic body-wave tomography

Simon Stähler, Karin Sigloch and Heiner Igel

Seismic tomography tries to infer earth structure from seismograms measured at the surface. These seismograms contain information on the seismic source and the three-dimensional structure of the earth. Therefore on some level every tomography is a joint inversion for structure and the source. Classically, source inversion was omitted in tomography and source parameters were taken from catalogues. These catalogues usually give the following 10-dimensional parametrization of the source: moment tensor (6 parameters), location (3 parameters), time.

However, comparison between different catalogues show that results, especially for hypocenter depth, vary significantly more than the provided error estimates would allow for.

Furthermore, the implicit assumption of Gaussian distributed uncertainties is highly implausible in the context of source inversion, e.g. the hypocenter depth cannot be negative. Also the model parameters are usually highly correlated. Uncertainties should therefore be expressed as probability density functions of marginals.

Previous work using the Neighbourhood algorithm showed that global search approaches can provide interesting insights into the parameter dependencies (Sambridge & Kennett, 2001) (Kennett, Marson-Pidgeon, & Sambridge, 2000).

Recently, adjoint strategies for source inversion have been proposed (Hingee, Tkalčič, Fichtner, & Sambridge, 2011), as well as combined inversion for source and structure (Valentine & Woodhouse, 2010). However, these strategies do not include an unbiased uncertainty estimation in the Bayesian sense, since they again assume Gaussian distributed parameter uncertainties.

Finite-frequency tomography is a waveform inversion technique that models body waves across the entire observed frequency range. Hence, the moment rate, termed “source time function” (STF) cannot simply be approximated as a delta-pulse. To fit the observed waveforms, an estimate of the STF is necessary, especially for earthquakes with magnitudes larger than 6.0 (Sigloch & Nolet, 2006).

Experience shows that strong interdependencies exist between the shape of the STF and the estimated depth of the earthquake. So far, no efforts have been made to invert for the STF in a Bayesian way, particularly due to the difficulty in describing the STF with a small number of parameters.

We show results from a linearized STF inversion for different depths, which demonstrate the effects of depth on the source mechanism. We compare them with results from a Global search scheme and discuss possible parametrizations of the STF using Gaussian base functions, Empirical orthogonal functions and wavelets.
Convective dynamo scaling laws derived from numerical models

Zacharias Stelzer & Andrew Jackson

Scaling laws derived from numerical dynamo models are beginning to play a part in geomagnetic data assimilation efforts. This originates in the fact that numerical dynamo models are currently run in the wrong parameter regime compared to the Earth. Scaling laws link quantities such as saturation field strength, heat flux and velocities to control parameters such as the Rayleigh, Ekman, Prandtl and magnetic Prandtl numbers. A large library of results has been amassed by Christensen, Aubert and co-workers, from which scaling laws have been derived. Favoured scaling laws are suggested to be independent of diffusivities.

We have begun to study the statistical basis for these laws. Using a subset of the available data, we examine which control parameters appear to be required to account for the variability in the data. We have adopted a Bayesian approach to the model selection problem, which naturally favours simple theories and conversely penalises elaborate theories which require more parameters for their description. Our results are in accord with more classical statistical tests of parameter significance.

Computing in the x-k domain for wave equation imaging

Chris Stolk (preference oral presentation)

In wave equation seismic imaging there is the problem of obtaining images with correct amplitudes, i.e. such that the the inhomogeneities that are being imaged are not only at the correct position, but also of the correct size. The correct amplitudes are obtained by applying a pseudodifferential operator to an initial image. Such an operator can be seen as something between a pointwise multiplication and a convolution: The operator is not local in the space domain (like the multiplication by a function would be), nor is it local in the Fourier domain (like a convolution operator would be), but it is approximately local in both domains. When we restrict to a small set, that is still larger than the resolution of the image, the operator becomes approximately a convolution with constant coefficients. We discuss various techniques to compute and apply these amplitude factors in the single and multi-source case.

Large uncertainties, limited information - is there still hope for seismic tomography?

Andrew P. Valentine

Seismic tomography depends on two linked inverse problems: one to determine the details of seismic sources (subject to best estimates of earth structure), and the other to determine earth structure (subject to best estimates of the seismic sources). In each case, we attempt to adjust model parameters to bring recorded and synthetic waveforms into better agreement. Traditionally, alternate inversions for source and structure are performed.
Recent work (Valentine, O'Toole, Woodhouse & Trampert, in prep.) suggests that the uncertainties associated with a seismic source inversion may be significantly larger than is usually assumed. What is the impact of this for structure determination? Valentine & Woodhouse (2010) showed that combining source and structure determination into a single inverse problem appears to allow improved accuracy in structure recovery at the expense of increased uncertainty in sources - how should this effect be explained? Can it be mitigated by use of more data, or is this symptomatic of a deeper issue? It appears that seismic waveforms may contain much less information than previously thought (Valentine & Trampert, in review); does this suggest that there is a fundamental limit to our ability to image the Earth? How should we make best use of the information that is available?


Towards quantifying uncertainty in travel time tomography using the null space shuttle

Ralph de Wit

The solution of large linear tomographic inverse problems is fundamentally non-unique. We suggest to explore the non-uniqueness explicitly by examining the null space of the forward operator. We show that with the null space shuttle [Deal and Nolet, 1996] it is possible to assess uncertainty in tomographic models and illustrate the concepts for the global P-wave model MIT-P08 by Li et al. [2008]. A broad range of acceptable solutions was found for the case of the inversion of the travel time data. The root mean square (RMS) velocity perturbations vary from 0.2 to 0.6 % in the lowermost mantle and from 0.3 to 1.3 % in the upper mantle. Such large variations in average amplitudes prohibit robust inferences on thermochemical variations in the Earth solely from tomographic models (based on currently available travel time residuals). On a global scale much short wave-length structure resides in the null space of the forward operator, suggesting that the data do not everywhere resolve structure on the smallest length scale allowed by the (block) parameterization used in MIT-P08 and similar models. This does not mean that such structures are not constrained in selected, well sampled areas, but it indicates that great care should be taken when interpreting such structure. High velocity anomalies in the lower mantle are required by the seismic data. By contrast, low-velocity anomalies that are commonly interpreted as (upwellings of) mantle plumes, for instance underneath Hawaii, Iceland and Africa, show varying degrees of robustness.
Travel time tomography of the western US upper mantle - A Bayesian approach

Ran Zhang

Tomographic inversion using full 3D finite-frequency sensitivity kernels and least-square formulation for the parameter estimation of the Earth has been a great success in recent years. Because of the large amount travel time data provided from the dense seismic station network USArray, structure of western US upper mantle has been studied extensively. (see Sigloch, McQuarrie and Nolet, 2008, Nature Geoscience; Tian, Sigloch, and Nolet, 2009, GJI, 178(3):1384-1402). However, uncertainty of the optimization solutions has not been well quantified. Using classical Markov chain Monte Carlo sampling methods such as Metropolis Hastings and Gibbs samplers we estimate the P-wave velocity based on 3D finite-frequency sensitivity kernels on irregular grids generated for the western US upper mantle. Non-intrinsic prior for neighborhood information of the grid points is applied. (see Pettitt, Weir and Hart, 2002, Statistics and Computing, 12:353-367; Rue and Held, 2005) With efficient algorithms we sampled posterior distributions over 11000 parameters and analyzed the uncertainty of the parameter estimation for the upper mantle structure as well as the origin time and source locations parameters.

Reduced-order minimax state estimation

Sergiy Zhuk

We introduce a new filtering approach for high-dimensional numerical systems.

It is based on a reduction of the high-dimensional system to some low-dimensional Differential-Algebraic Equation (DAE), and on the application of linear minimax filtering to the resulting DAE. In the minimax approach, the model error and the observational error can be deterministic or stochastic, and of any shape provided they have bounded energy. In practice, it is assumed that the errors belong to some ellipsoid. Based on this information, the algorithm describes a reachability set that contains all states consistent with the model, the observations and the assumptions on the errors. In the non-reduced version of the filter, the estimator of the state is taken as Chebyshev center of the reachability set. Note that the non-reduced version of the filter coincides with the Kalman filter provided there is no systematic error and the description of the ellipsoid is reinterpreted in terms of variances.

The non-reduced filter is intractable as it involves solving a high-dimensional matrix Riccati equation. The formulation of the filter for DAE allows to estimate only a part of the state or a projection of the state onto some subspace (e.g., computed from a proper orthogonal decomposition). The reduced-model error is decomposed into the projection of the model error onto the subspace and a commutation error between the projection operator and the model. The reachability set is provided in the subspace. Computing the reachability set and the estimator for the reduced state is tractable whenever the dimension N of the reduced space is low enough—the algorithm involves N calls to the tangent linear model. The performance of the
filter will be illustrated in the poster, with a 2D Navier-Stokes equation in vorticity-velocity formulation or with full air quality simulations if available at the time.