Confinement and screening in SU(N) and G(2) gauge theories

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The many faces...


- J. Greensite, K. Langfeld, H. Reinhardt, T. Tok, ŠO: *Color screening, Casimir scaling, and domain structure in G(2) and SU(N) gauge theories*, PR D75 (2007) 034501 [hep-lat/0609050].

- J. Greensite, ŠO: *Yang-Mills wave functional in (2+1) dimensions*, work in progress.
Pierre vs. Jeff and Casimir scaling

Subject: Lattice 97
Date: Mon, 28 Jul 1997 14:09:01 -0700 (PDT)
From: Jeff Greensite <greensit@stars.sfsu.edu>
To: faber@is1.kph.tuwien.ac.at, fyziolej@savba.sk

[...] In the afternoon, Pierre van Baal gave a one-hour, idiosyncratic plenary talk entitled “The QCD Vacuum.” … [There] He told the audience that the news here was that I had “changed my religion” to vortices, […], and [he asked], hey, what about all that Casimir scaling stuff?? […]
Outline

- **Introduction**
  - Roles of center symmetry
  - Center vortices and confinement in pure gauge theory
- **Question 1: What if center symmetry is broken by matter fields?**
  - Permanent vs. temporary confinement
  - SU(2) gauge field coupled to fundamental Higgs fields
- **Question 2: What if center is trivial?**
  - Temporary confinement in G(2) gauge theory
  - *Casimir scaling*
    - A simple (simplistic) model: *Casimir scaling* and color screening from domain structure of the QCD vacuum
- **Question 3: Can we derive (at least some) elements of the picture from first principles?**
  - In search of the approximate Yang-Mills vacuum wave functional in 2+1 dimensions
- **Conclusions and open questions**
Roles of center symmetry

- Additional symmetry of pure-gauge SU(N) YM theory:
  \[ U_0(\vec{x}, t_0) \rightarrow z U_0(\vec{x}, t_0) \quad z \in Z_N \]

- Polyakov loop not invariant:
  \[ P(\vec{x}) = \frac{1}{2} \text{Tr}[U_0(\vec{x}, 1)U_0(\vec{x}, 2) \ldots U_0(\vec{x}, L_t)] \]
  \[ \langle P(\vec{x}) \rangle = \begin{cases} 
  0 & \text{unbroken } Z_N \text{ symmetry phase} \\
  \text{non-zero} & \text{broken } Z_N \text{ symmetry phase}
  \end{cases} \]

- On a finite lattice, below or above the transition, \( \langle P(x) \rangle = 0 \), but:
  \[ \frac{1}{L^3} \sum_{\vec{x}} P(\vec{x}) = \begin{cases} 
  0 & \text{unbroken phase} \\
  z \exp(-TF_q) & \text{broken phase}
  \end{cases} \]

  Good order parameter:
  \[ \left\langle \left| \frac{1}{L^3} \sum_{\vec{x}} P(\vec{x}) \right| \right\rangle \]
• String tension depends on the representation class:
  • The asymptotic string tension depends only on the class or N-ality of the group representation to which the charge belongs
    • Non-zero N-ality color charges are confined.
    • Zero N-ality color charges are screened.
• Same N-ality means same transformation properties under the center subgroup $Z_N$.
• Particle language: A flux tube, e.g., between adjoint color sources can crack and break due to pair production of gluons.
• The string tension of a Wilson loop, evaluated in an ensemble of configurations from the pure YM action, depends on the N-ality of the loop representation.
• Large-scale vacuum fluctuations – occurring in the absence of any external source – must contrive to disorder only the center degrees of freedom of Wilson loop holonomies.

  Ambjørn, Greensite (1998)
Center vortices and confinement in pure gauge theory

- Reviewed in Maxim Chernodub’s talk on Tuesday

- The picture was proposed and elaborated at the end of 70’s and beginning of 80’s by ’t Hooft, Mack and Petkova, Ambjørn et al., Cornwall, Feynman and many others.

- Some people helped to “bury” the model (incl. Jeff Greensite).

- The model does not rely on any particular gauge, but ...

- ... how to identify center vortices in vacuum configurations?
  - Del Debbio, Faber, Greensite, ŠO (1997)
  - Del Debbio, Faber, Giedt, Greensite, ŠO (1998)
  - many other groups joined our efforts
• Center vortices are identified by fixing to an **adjoint gauge**, and then projecting link variables to the $Z_N$ subgroup of SU(N). The excitations of the projected theory are known as **P-vortices**.

• **Direct maximal center (or adjoint Lorenz) gauge in SU(2):**

  One fixes to the maximum of

  \[ \mathcal{R}_{MCG} = \sum_{x,\mu} \left| \frac{1}{2} \text{Tr}[U_{\mu}(x)] \right|^2 \]

  and center projects

  \[ U_{\mu}(x) \longrightarrow Z_{\mu}(x) = \text{sign} \text{ Tr}[U_{\mu}(x)] \]

• Fit of a real configuration by thin-center-vortex configuration.
Numerical evidence

- Center dominance.
- Correlation with gauge-invariant information: vortex-limited Wilson loops.
- Vortex removal: confinement also removed.
- Scaling of the vortex density.
- Finite-temperature: deconfinement as vortex depercolation.
- Vortex removal: chiral condensate and topological charge vanish.

- Relation to other scenarios:
  - Monopole worldlines lie on vortex sheets.
  - Thin center vortices "live" on the Gribov horizon.
Question 1: What if center symmetry is broken by matter fields?
Permanent vs. temporary confinement

- **What is confinement?**
  The term is used, in the literature, in several inequivalent, related, ways:
  - **Electric flux-tube formation, and a linear static quark potential.**
  - Absence of color-electrically charged particle states in the spectrum.
  - Existence of a mass gap.

- **Permanent confinement:**
  - Global center symmetry.
  - Flux tube never breaks.
  - Pure gauge theories.

- **Temporary confinement:**
  - Asymptotic string tension is zero.
  - At large scales the vacuum state is similar to the Higgs phase of gauge-Higgs theory.
  - Static quark potential does rise linearly for some interval of color source separations.
• With temporary confinement:
  • The simple kinematical motivation for the center-vortex mechanism is lost.
  • Relevance (or irrelevance) of vortices is a dynamical issue, which can be investigated in numerical simulations.

• Real QCD with dynamical quarks.
• G(2) pure-gauge theory.
• SU(2)-gauge–fundamental-Higgs theory.
SU(2) gauge field coupled to fundamental Higgs fields

\[ S = S_W + \frac{g}{2} \sum_{x, \mu} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x + \hat{\mu})] \]

- Osterwalder, Seiler (1978); Fradkin, Shenker (1979); Lang, Rebbi, Virasoro (1981)
Center dominance

\[ \langle P \rangle \approx 0.120(4) \]
\begin{equation*}
f(R) = c_0 + c_1 \exp(-4\sigma R)
\end{equation*}

\begin{align*}
c_0 &= 0.0182 & \sigma &= 0.211
\end{align*}
Correlation with gauge-invariant information

- \( W_n(C) \) – a Wilson loop, computed from unprojected link variables, with the restriction that the minimal area of loop \( C \) is pierced by \( n \) P-vortices on the projected lattice.
Vortex removal

- **deForcrand-D’Elia procedure**: fix to maximal center gauge, and multiply each link variable by its center-projected value.

![Graph showing vortex removal](image)
Kertész line?

\[ \gamma \]

0 1 2 3 4

Higgs–like
Temporary Confinement
• **Osterwalder-Seiler–Fradkin-Shenker theorem**: no phase transition isolating the temporary-confinement region from a Higgs-like phase; at least no transition detected by any local order parameter.

• **Kertész line in the Ising model**: With small external magnetic field the global $Z_2$ symmetry of the zero-field model is explicitly broken, there is no thermodynamic transition between an ordered to a disordered state; still there is a sharp depercolation transition; the line of such transitions in the T-h plane is called Kertész line.
  - Kertész (1989); Chernodub, Gubarev, Ilgenfritz, Schiller (1998); Chernodub (2005)

• **Symmetry-breaking transition**: A local gauge symmetry can never be broken spontaneously (Elitzur). The local symmetry can be fixed by a gauge choice. Certain gauges, such as Coulomb or Lorenz gauge, leave unfixed a global remnant of the local symmetry, and this can be spontaneously broken.
Similar situation with other order parameters (v.e.v. of the Higgs field in Lorenz gauge).

Caudy, Greensite, work in progress
Question 2:
What if center is trivial?
Temporary confinement in G(2) gauge theory

- Center-vortex confinement mechanism claims that the asymptotic string tension of a pure non-Abelian gauge theory results from random fluctuations in the number of center vortices.
- **No vortices implies no asymptotic string tension!**

- Is G(2) gauge theory a counterexample?
  - **No!**
  - The asymptotic string tension of G(2) gauge theory is zero, in perfect accord with the vortex proposal.
  - G(2) gauge theory however exhibits temporary confinement, i.e. the potential between fundamental charges rises linearly at intermediate distances. This can be qualitatively explained to be due to the group center, albeit trivial! A model will be presented.
  - Prediction: Casimir scaling.
Linear potential at intermediate distances

- Numerical simulations:
  - Metropolis algorithm with microcanonical reflections, real representation of G(2) matrices (Langfeld et al.).
  - Cabibbo-Marinari method, complex representation of G(2) matrices (Pepe et al.; Greensite, ŠO).
- Potential computed from expectation values of rectangular Wilson loops $W(r,t)$.
  - Smeared spacelike links.
  - Unmodified timelike links.
- Fit as usual: constant + Coulomb + linear terms.
- What should the linear rise be attributed to?
Casimir scaling

- At intermediate distances the string tension between charges in representation $r$ is proportional to $C_r$.
- Argument: Take a planar Wilson loop, integrate out fields out of plane, expand the resulting effective action:
  \[
  W_r(C) = \frac{1}{Z} \int D A_x(x, y) D A_y(x, y) \chi_r[U(C)] \exp(-S_{\text{eff}})
  \]
  \[
  S_{\text{eff}} = \int d^2 x \left[c_0 \text{Tr}(F^2) + c_1 \text{Tr}(D_\mu F D_\mu F) + \ldots\right]
  \]
- Truncation to the first term gives Casimir scaling automatically.
- A challenge is to explain both Casimir and N-ality dependence in terms of vacuum fluctuations which dominate the functional integral.

\[\text{Bali, 2000}\]
A simple (simplistic) model: Casimir scaling and color screening from domain structure of the QCD vacuum

- **Casimir scaling** results from uncorrelated (or short-range correlated) fluctuations on a surface slice.
- **Color screening** comes from center domain formation.
- **Idea:** On a surface slice, YM vacuum is dominated by overlapping center domains. Fluctuations within each domain are subject to the weak constraint that the total magnetic flux adds up to an element of the gauge-group center.

\[
G_r(\alpha^n) = \frac{1}{d_r} \chi_r \left[ \exp(i\alpha^n \cdot \vec{H}) \right]
\]

- Faber, Greensite, ŠO (1998)
- SU(2)
- G(2)
Consider a set of random numbers, whose probability distributions are indep’

\[
\langle \left( \sum_{n=1}^{M} x_i \right)^2 \rangle = \text{const} \left( \frac{M}{N} - \frac{M^2}{N^2} \right) + \left( \kappa \frac{M}{N} \right)^2 \quad M \leq N
\]

For nontrivial and trivial center domains, in SU(2):

\[
(\alpha^1_C(x))^2 = \text{const} \left( \frac{A}{A_v} - \frac{A^2}{A_v^2} \right) + \left( 2\pi \frac{A}{A_v} \right)^2
\]

\[
(\alpha^0_C(x))^2 = \text{const} \left( \frac{A}{A_v} - \frac{A^2}{A_v^2} \right)
\]

Leads to (approximate) Casimir scaling at intermediate distances and N-ality dependence at large distances.
The same would work for G(2) with only one type of “center domain”, but the string tension is always asymptotically zero.

- Prediction: Casimir scaling for potentials of various G(2)-representation charges – needs to be verified in simulations!
- How the domains arise and how to detect them? Why should the string tension in SU(N) be the same at intermediate and large R?
Question 3:
Can we derive (at least some) elements of the picture from first principles?
In search of the approximate Yang-Mills vacuum wave functional in 2+1 dimensions

- **Confinement** is the property of the vacuum of quantized non-abelian gauge theories. In the hamiltonian formulation in D=d+1 dimensions and temporal gauge:

\[
\mathcal{H} = \int d^d x \left[ -\frac{1}{2} \frac{\delta^2}{\delta A_k^a(x)^2} + \frac{1}{4} F_{ij}^a(x)^2 \right]
\]

\[
(\delta^{ac} \partial_k + g\epsilon^{abc} A_k^b) \frac{\delta}{\delta A_k^c} \Psi = 0
\]

- Strong-coupling lattice-gauge theory – systematic expansion:

\[
\Psi_0[U] = \exp \left[ -\sum_{n=1}^{\infty} \beta^{2n} R_n[U] \right], \quad \beta = \frac{4}{g^2}
\]

Greensite (1980)
- At large distance scales one expects:

\[
\psi_{0}^{\text{eff}}[A] \approx \exp \left[ -\mu \int d^d x \ F_{ij}^a(x) F_{ij}^a(x) \right]
\]

- Property of **dimensional reduction**: Computation of a spacelike loop in \(d+1\) dimensions reduces to the calculation of a Wilson loop in Yang-Mills theory in \(d\) Euclidean dimensions.

\[
W(C) = \langle \text{Tr}[U(C)] \rangle_{D=3+1} = \langle \psi_0^{(3)} | \text{Tr}[U(C)] | \psi_0^{(3)} \rangle \\
\sim \langle \text{Tr}[U(C)] \rangle_{D=2+1} = \langle \psi_0^{(2)} | \text{Tr}[U(C)] | \psi_0^{(2)} \rangle \\
\sim \langle \text{Tr}[U(C)] \rangle_{D=1+1} \ldots \text{area law! (+Casimir scaling)}
\]
- At weak couplings, one would like to similarly expand:

$$\psi_0[A] = \exp \left[ - \sum_{n=0}^{\infty} g^{2n} Q_n[A] \right]$$

- For $g \to 0$ one has simply:

$$\psi_0[A] \xrightarrow{g \to 0} \exp \left[ - \frac{1}{2} \int d^d x d^d y \, F_{ij}^a(x) \left( \frac{1}{\sqrt{-\nabla^2}} \right)_{xy} F_{ij}^a(y) \right]$$

- Wheeler (1962)
A possibility to enforce gauge invariance:

\[ \Psi_0[A] = \exp \left[ -\frac{1}{2} \int d^d x d^d y \ F^a_{ij}(x) \left( \frac{V^{ab}(x, y)}{\sqrt{-\nabla^2}} \right)_{xy} F^b_{ij}(y) \right] \]

\[ V(x, y) = \sum_{C_{xy}} f(C_{xy}) \exp \left[ ig \int_{C_{xy}} dx^\mu A_\mu \right] \]

\[ \lim_{g \rightarrow 0} \sum_{C_{xy}} f(C_{xy}) = 1 \]

No handle on how to choose f's.
Our suggestion for the YM vacuum wave-functional in $D=2+1$

\[
\psi_0[A] = \exp \left[ -\frac{1}{2} \int d^2x d^2y \ B^a(x) \left( \frac{1}{\sqrt{-D^2 - \langle \lambda_0 \rangle + m^2}} \right)^{ab} B^b(y) \right]
\]

\[D^2 = D_k \cdot D_k\]

$D_k$ ... covariant derivative  \hspace{1cm} $\lambda_0$ ... lowest eigenvalue of $(-D^2)$

$m$ ... constant proportional to $g^2$

\[
(-D^2)^{ab}_{xy} = \sum_{k=1}^{d} \left[ 2 \delta^{ab}_{xy} - U^{ab}_k(x) \delta_{y,x+k} - U^{ba}_k(x-k) \delta_{y,x-k} \right]
\]

$U^{ab}_k(x) = \frac{1}{2} \text{Tr} \left[ \sigma^a U_k(x) \sigma^b U^\dagger_k(x) \right]$ ... links in adjoint rep'n

- Samuel (1996)
- Diakonov (unpublished)
Zero-mode, strong-field limit

- Let's assume we keep only the zero-mode of the A-field, i.e. fields constant in space, varying in time. The lagrangian is

\[ \mathcal{L} = \frac{1}{2} V \left[ \partial_t \vec{A}_k \cdot \partial_t \vec{A}_k - g^2 (\vec{A}_1 \times \vec{A}_2) \cdot (\vec{A}_1 \times \vec{A}_2) \right] \]

and the hamiltonian operator

\[ \hat{\mathcal{H}} = -\frac{1}{2V} \frac{\partial^2}{\partial \vec{A}_k \cdot \partial \vec{A}_k} + \frac{1}{2} g^2 V (\vec{A}_1 \times \vec{A}_2) \cdot (\vec{A}_1 \times \vec{A}_2) \]

- The ground-state solution of the YM Schrödinger equation, up to 1/V corrections:

\[ \psi_0 = e^{-VR_0} = \exp \left[ -\frac{1}{2} g V \frac{(\vec{A}_1 \times \vec{A}_2) \cdot (\vec{A}_1 \times \vec{A}_2)}{\sqrt{\vec{A}_1 \cdot \vec{A}_1 + \vec{A}_2 \cdot \vec{A}_2}} \right] \]
- Now the proposed vacuum state coincides with this solution in the strong-field limit, assuming

\[ |g\vec{A}_{1,2}| \gg m, \sqrt{\langle \lambda_0 \rangle} \]

- The covariant laplacian is then

\[ (-D^2)^{ab}_{xy} \approx g^2 \delta(x - y) \left[ (A_1^2 + A_2^2) \delta^{ab} - A_1^a A_1^b - A_2^a A_2^b \right] \]

and it can be shown easily

\[
\psi_0 = \exp \left[ -\frac{1}{2} \int d^2x d^2y \ B^a(x) \left( \frac{1}{\sqrt{-D^2}} \right)^{ab}_{xy} B^b(y) \right]
\]

\[
\rightarrow \exp \left[ -\frac{1}{2} gV \frac{(\vec{A}_1 \times \vec{A}_2) \cdot (\vec{A}_1 \times \vec{A}_2)}{\sqrt{\vec{A}_1 \cdot \vec{A}_1 + \vec{A}_2 \cdot \vec{A}_2}} \right]
\]
Dimensional reduction

- It was Samuel who suggested that the Ansatz

\[
\psi_0 = \exp \left[ -\frac{1}{2} \int d^2x d^2y \ B^a(x) \left( \frac{1}{\sqrt{-D^2 + m_0^2}} \right)_{xy}^{ab} B^b(y) \right]
\]

interpolates between perturbative vacuum at short wavelengths, and a dimensional reduction form at large wavelengths.

- However, what is short and long wavelength is ambiguous and gauge-dependent. To make things better defined, we decompose:

\[
(-D^2)^{ab} \phi_n^b(x) = \lambda_n \phi_n^a(x)
\]

\[
B^a(x) = \sum_{n=0}^{\infty} b_n \phi_n^a(x) \quad B^{a,\text{slow}}(x) = \sum_{n=0}^{n_{\text{max}}} b_n \phi_n^a(x)
\]
If we keep $n_{\max}$ fixed for $V \to \infty$, the eigenvalues can be approximated by $\lambda_0$ and

$$
\int d^2x d^2y \ B^{a,\text{slow}}(x) \left( \frac{1}{\sqrt{-D^2 - \langle \lambda_0 \rangle + m^2}} \right)_{xy}^{ab} B^{b,\text{slow}}(y)
$$

$$
\approx \frac{1}{m} \int d^2x \ B^{a,\text{slow}}(x) B^{a,\text{slow}}(x)
$$

So the part of the wave-functional that depends on “slowly-varying” $B$ has the dimensional reduction form, i.e. the probability distribution of the D=2 YM theory. One can then compute the string tension analytically and gets

$$
\sigma = \frac{3m}{4\beta}
$$

An experiment: take $m = (4/3)\beta \sigma$ and compute the mass gap with the full proposed wave-functional.
**Numerical simulation of $|\Psi_0|^2$**

- To extract the mass gap, one would like to compute

$$G(x - y) = \langle (B^a B^a)_x (B^b B^b)_y \rangle - \langle (B^a B^a)_x \rangle^2$$

in the probability distribution:

$$P[A] = |\Psi_0[A]|^2 = \mathcal{N} \exp \left[- \int d^2 x d^2 y \ B^a(x) K_{xy}^{ab}[A] B^b(y) \right]$$

$$K_{xy}^{ab}[A] = \left( \frac{1}{\sqrt{-D^2 - \langle \lambda_0 \rangle + m^2}} \right)^{ab}_{xy}$$

- Looks hopeless, $K[A]$ is highly non-local, not even known for arbitrary fields.
- But if - after choosing a gauge - $K[A]$ does not vary a lot among thermalized configurations... then something can be done.
Define:

$$\mathcal{P}[A; A'] = N \exp \left[ - \int d^2x d^2y \, B^a(x; A) K_{xy}^{ab}[A'] B^b(y; A) \right]$$

Hypothesis:

$$P[A] = \mathcal{P}[A; A] \approx \int dA' \, \mathcal{P}[A; A'] P[A']$$

Iterative procedure:

$$P^{(1)}[A] = \mathcal{P}[A; 0]$$

$$P^{(k+1)}[A] = \int dA' \, \mathcal{P}[A; A'] P^{(k)}[A']$$
Practical implementation:
choose e.g. axial $A_1=0$ gauge, change variables from $A_2$ to $B$. Then

1. given $A_2$, set $A_2'=A_2$,
2. $P[A;A']$ is gaussian in $B$, diagonalize $K[A']$ and generate new $B$-field (set of $B$s) stochastically;
3. from $B$, calculate $A_2$ in axial gauge, and compute everything of interest;
4. go back to the first step, repeat as many times as necessary.

All this is done on a lattice.

Of interest:
- Eigenspectrum of the adjoint covariant laplacian.
- Connected field-strength correlator, to get the mass gap:

\[ G(x - y) = \langle (K^{-1})^{ab}_{xy}(K^{-1})^{ba}_{yx} \rangle \]

For comparison the same computed on 2D slices of 3D lattices generated by Monte Carlo.
Eigenspectrum ($\beta=18$, $40^2$ lattice)

- Very close to the spectrum of the free laplacian.
- Very little variation among lattices.
Mass gap

\[ \langle B^2 B^2 \rangle \text{ Correlator, } \beta = 18, \ 40^2 \text{ lattice} \]
\[ G_0(x) = \delta^{ab} \delta^{ba} \left[ \left( \sqrt{-\nabla^2 + \mu^2} \right)_{xy} \right]^2 = \frac{3}{4\pi^2} \left( 1 + \frac{1}{2} MR \right)^2 \frac{e^{-MR}}{R^6} \]
• M(fit) is our result, obtained as described earlier.
• M(MT) is result of the computation of the \( O^+ \) glueball mass by Meyer and Teper.
• M=2m is the naïve estimate using the mass parameter entering the approximate vacuum wave-functional.
• Optimistic message: The glueball mass comes out quite accurately.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>M (fit)</th>
<th>M (MT)</th>
<th>2m</th>
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<td>18</td>
<td>0.404</td>
<td>0.397</td>
<td>0.349</td>
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</table>
Why should $m$ be different from 0?

\[
\langle \hat{H} \rangle = \frac{1}{2} \left( \text{Tr} \left[ \sqrt{-D^2 - \langle \lambda_0 \rangle + m^2} \right] + \frac{1}{2} \text{Tr} \left[ \frac{\langle \lambda_0 \rangle - m^2}{\sqrt{-D^2 - \langle \lambda_0 \rangle + m^2}} \right] \right)
\]
What about N-ality?

- **This cannot be the whole story** – with the simple wave-functional one would expect Casimir scaling even of higher-representation asymptotic string tensions, while asymptotic string tensions should in fact depend only on the N-ality of representations. The simple zeroth-order state is clearly missing the center-vortex or domain structure which has to dominate the vacuum at sufficiently large scales.

- Add a gluon mass term. Then $|\Psi_0|^2$ has local maxima at center vortex configurations (seems a bit ad hoc, though).

- Another possibility: include the leading correction to dimensional reduction.

Cornwall (2007)
Conclusions and open questions

- **Part I:**
  - The vortex mechanism for producing linear potential can work even when the gauge action does not possess global center symmetry; global center symmetry is not necessarily essential to the vortex mechanism.
  - No single Kertész line separating the “confinement phase” and “Higgs phase” in gauge-Higgs theory.
  - Do vortices have a branch polymer structure at large scales?
  - What is the situation in QCD with dynamical quarks?

- **Part II:**
  - G(2) gauge theory is not an exception to the claim that there is no asymptotic string tension without center vortices.
  - Temporary confinement and screening may be explained by domain structure of the vacuum.
  - It is crucial to verify the prediction of Casimir scaling for G(2) gauge theory.
Part III:

- The proposed approximate vacuum wave-functional
  - is a solution of the YM Schrödinger equation in the $g\to 0$ limit;
  - solves the YM Schrödinger equation in the strong field, zero-mode limit;
  - confines if $m>0$, and $m>0$ seems energetically preferred;
  - results in the numerically correct relationship between the string tension and mass gap.

- But: Does the variational solution satisfy $m \sim 1/\beta$?
- Validity of approximations?
- Will corrections to dimensional reduction give the right $N$-ality dependence?
- $D=3+1$?