STREAMER THEORY OF SPRITES

Victor Pasko and Ningyu Liu
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Victor Pasko and Ningyu Liu
Communications and Space Sciences Laboratory
The Pennsylvania State University
211B EE East, University Park, PA 16802, U.S.A.
E-mail: vpasko@psu.edu, nul105@psu.edu

Streamers, Sprites, Leaders, Lightning: from Micro- to Macroscales Workshop
Lorentz Center, Leiden University
Leiden, The Netherlands

October 8-12, 2007
• General Phenomenology of Streamers and Sprites
• Summary of Emissions Associated with Sprites
• Physical Mechanism of Sprites
• Motivation for Studies of Sprite Streamers in Weak Fields
• Modeling of Sprite Streamers and Comparison with Recent Observations
• Modeling Studies of NO-\(\gamma\) Emissions from Sprite Discharges
Electron Avalanche

Fig. 12.1. Shape and charge distribution of an electron avalanche at two consecutive moments of time. Arrows indicate directions of external field $E_0$ and velocity of motion of the avalanche head, $v_d$.

Fig. 12.3. Electric fields in a gap containing an electron avalanche. (a) Lines of force of the external field $E_0$ and of the field of space charge of the avalanche, $E'$, are shown separately. (b) Lines of force of the resulting field $E = E_0 + E'$. Circles mark the centers of space charges.

Concept of a Negative (Anode-directed) Streamer

- Streamers - filamentary plasmas driven by highly nonlinear space charge waves.

Fig. 12.6. Anode-directed streamer. (a) Photons and secondary avalanches in front of the streamer head at two consecutive moments of time. (b) Field in the vicinity of the head

Concept of a Positive (Cathode-directed) Streamer

Fig. 12.5. Cathode-directed streamer. (a) Streamer at two consecutive moments of time, with secondary avalanches moving towards the positive head of the streamer; wavy arrows are photons that generate seed electrons for avalanches. (b) Lines of force of the field near the streamer head

A summary of useful similarity relationships (\(N\) is air density, \(N_o=2.68 \times 10^{25} \text{ m}^{-3}\) is a reference value corresponding to ground pressure):

- **Length** (i.e., mean free path, streamer radius, etc):
  \[
  L = L_o \frac{N_o}{N}
  \]

- **Time** (i.e., between collisions, dielectric relaxation, 2-body attachment, etc):
  \[
  \tau = \tau_o \frac{N_o}{N}
  \]

- **Electric field** (in streamer head, in streamer body, etc):
  \[
  E = E_o \frac{N}{N_o}
  \]

- **Plasma and charge density** (i.e., electron and ion in streamer body, etc.):
  \[
  n = n_o \frac{N^2}{N_o^2}
  \]
• Streamers in a 80 mm gap imaged with optical gate $\sim$80 ns [Briels et al., 2006]
Recent Submillisecond Imaging of Sprite Development and Structure

Outline

• General Phenomenology of Streamers and Sprites
• Summary of Emissions Associated with Sprites
• Physical Mechanism of Sprites
• Motivation for Studies of Sprite Streamers in Weak Fields
• Modeling of Sprite Streamers and Comparison with Recent Observations
• Modeling Studies of NO-γ Emissions from Sprite Discharges
Overview of Emissions from Sprites

- The time averaged optical emissions in sprites are dominated by red emissions associated with the first positive band system of N$_2$ [Mende et al., 1995; Hampton et al., 1996; Morrill et al., 1998; Takahashi et al., 2000; Bucsela et al., 2003].

- The narrow band photometric and blue-light video observations of sprites [Armstrong et al., 1998, 2000; Suszcynsky et al., 1998; Morrill et al., 2002] indicate presence of short duration (∼ms) bursts of blue optical emissions associated with the second positive band system of N$_2$ and the first negative band system of N$_2^+$ appearing at the initial stage of sprite formation.

- Possible features from the Meinel band system of N$_2^+$ have been discussed by Morrill et al. [1998] and Bucsela et al. [2003].
A Sprite Event Observed by the ISUAL on FORMOSAT-2

[2004-07-18/21:30:15.287]

[2004-07-18/21:30:15.297]

[2004-07-18/21:30:15.327]

[2004-07-18/21:30:15.337]

[2004-07-18/21:30:15.347]

[2004-07-18/21:30:15.357]

[2004-07-18/21:30:15.367]

[2004-07-18/21:30:15.377]

[2004-07-18/21:30:15.387]

[2004-07-18/21:30:15.397]

[2004-07-18/21:30:15.407]

[2004-07-18/21:30:15.417]

[Trigger 2004-07-18/21:30:15.316]

[Sprite]

[Lightning]

[Mende et al., 2006]
Summary of Emissions from Sprites

<table>
<thead>
<tr>
<th>Emission Band System</th>
<th>Transition</th>
<th>Excitation Energy Threshold (eV)</th>
<th>Lifetime at 70 km Alt. (µs)</th>
<th>Quenching Alt. (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1PN$_2$</td>
<td>N$_2$(B$^3\Pi_g$)$\rightarrow$N$_2$(A$^3\Sigma^+_u$)</td>
<td>$\sim$7.35</td>
<td>5.4</td>
<td>$\sim$53</td>
</tr>
<tr>
<td>2PN$_2$</td>
<td>N$_2$(C$^3\Pi_u$)$\rightarrow$N$_2$(B$^3\Pi_g$)</td>
<td>$\sim$11</td>
<td>50</td>
<td>$\sim$30</td>
</tr>
<tr>
<td>LBH N$_2$</td>
<td>N$_2$(a$^1\Pi_g$)$\rightarrow$N$_2$(X$^1\Sigma^+_g$)</td>
<td>$\sim$8.55</td>
<td>14</td>
<td>$\sim$77</td>
</tr>
<tr>
<td>1NN$_2^+$</td>
<td>N$_2^+$(B$^2\Sigma^+_u$)$\rightarrow$N$_2^+$(X$^2\Sigma^+_g$)</td>
<td>$\sim$18.8</td>
<td>69</td>
<td>$\sim$48</td>
</tr>
</tbody>
</table>

- The N$_2$(B$^3\Pi_g$) vibrational distribution obtained in [Bucselka et al., 2003] appeared to be consistent with those observed in laboratory afterglows, indicating an energy transfer process of the form:

$$N_2(A^3\Sigma^+_u, w) + N_2(X^1\Sigma^+_g, v \geq 5) \rightarrow N_2(B^3\Pi_g, w') + N_2(X^1\Sigma^+_g, v' \sim 0)$$  \hspace{1cm} (1)

- The spectroscopic features of sprites are consistent with emissions from pulsed corona discharges in laboratory experiments [Gallimberti et al., 1974; Teich, 1993; Simek et al., 1998, 2002; Kim et al., 2003].

- There is a need of further studies of processes related to vibrational excitation of ground state of N$_2$ molecules, and pooling and resonant energy transfer reactions involving N$_2$(A$^3\Sigma^+_u$) metastable species for understanding of emissions originating from B$^3\Pi_g$ and C$^3\Pi_u$ states of N$_2$, and NO $\gamma$-band emissions [e.g., Simek et al., 1998, and references therein], during both, initial and post-discharge stages of sprite discharge.
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"While the electric force due to the thundercloud falls off rapidly as \( r \) increase, the electric force required to cause sparking (which for a given composition of the air is proportional to its density) falls off still more rapidly. Thus, if the electric moment of a cloud is not too small, there will be a height above which the electric force due to the cloud exceeds the sparking limit."

The altitude distribution of different time scales characterizing the electrical breakdown associated with sprites [Pasko et al., GRL, 25, 2123, 1998]:

- DIFFUSE REGION
- TRANSITION REGION
- STREAMER REGION

Dielectric relaxation

Dissociative attachment

Streamer formation
Diffuse and Streamer Regions of Sprites

- The images illustrating the altitude transition between diffuse and streamer regions in sprites [Stenbaek-Nielsen et al., GRL, 27, 3827, 2000]:

![Altitude vs. Sprites Diagram]

Altitude (km)

40  50  60  70  80  90

40  50  60  70  80  90
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Minimum Fields for Propagation of Positive Streamers

- The minimum field required for the propagation of positive streamers at ground pressure [Allen and Ghaffar, 1995]:

<table>
<thead>
<tr>
<th>Reference</th>
<th>Propagation field (kV m(^{-1}))</th>
<th>Electrode gap details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phelps and Griffiths [3]</td>
<td>487</td>
<td>Plane-parallel, 90 mm</td>
</tr>
<tr>
<td>With dc source</td>
<td>510–542</td>
<td>Plane-parallel, 660, 470 and 210 mm</td>
</tr>
<tr>
<td>Present work</td>
<td>440</td>
<td>Plane-parallel, 180 mm</td>
</tr>
<tr>
<td>Acker and Penney [5]</td>
<td>460</td>
<td>Non-uniform, 31.7 mm</td>
</tr>
<tr>
<td>Bye et al [6]</td>
<td>470</td>
<td>Non-uniform, 450 mm</td>
</tr>
<tr>
<td>Allen and Dring [7]</td>
<td>414</td>
<td>Non-uniform, 600 mm</td>
</tr>
<tr>
<td>Geldenhuyss [8]</td>
<td>464,489</td>
<td>Non-uniform, 500 mm</td>
</tr>
</tbody>
</table>
Minimum Fields for Propagation of Negative Streamers

- The information about the absolute value of the similar field for the negative streamers at present is very limited. The existing sources indicate that this field is a factor of 2-3 higher than the corresponding field for the positive streamers [e.g., Raizer, 1991, p. 361; Babaeva and Naidis, 1997].
Double-headed Streamer at Ground Pressure

- Development of a double-headed streamer in uniform field $E_0 > E_k$ ($E_0=48$ kV/cm) at ground pressure [Bourdon et al., Plasma Sources Sci. Tech., 16, 656, 2007, http://www.iop.org/EJ/abstract/0963-0252/16/3/026]:

![Diagram showing electron density, electric field, and Sph at t = 3.5 ns](image-url)
Motivation for Studies of Sprite Streamers in Weak Fields

- The initiation mechanisms of sprites produced by lightning discharges associated with charge moment changes as small as 120 C km [Hu et al., GRL, 29, 1279, doi:10.1029/2001GL014593, 2002] are not understood at present.
Sprite Streamers Form on Time Scales $\ll 1$ ms

- The formation times of sprite streamers are $\ll 1$ ms and their lifetimes are on the order of $\sim 1$ ms [Marshall and Inan, Radio Science, 41, RS6S43, 2006]:

![Streamer diameter](image1)

![Streamer lifetime](image2)
Electric Field Driving Sprites Increases on Time Scales $\sim 1$ ms

- The event-level analysis reported by *Hu et al.* [J. Geophys. Res., 112, D13115, 2007] provides quantitative information on time variation of lightning current moment, charge moment and electric field driving sprite phenomena:

![Graph of electric field and current moment](image-url)
Positive Streamers in Sprites Are Initiated Before Negative Ones

- Recently reported observations of sprites at 10,000 frames per second:

[McHarg et al., GRL, 34, L06804, 2007; Stenbaek-Nielsen et al., GRL, 34, L11105, 2007]
Fractal Modeling of Sprites

- The observed initiation of positive streamers in sprites before the negative ones may be related to relatively slow variation of the driving electric field ($\sim 1 \text{ ms}$) [Hu et al., 2007] and the lower electric field threshold required for propagation of positive streamers [Pasko et al., Geophys. Res. Lett., 4, 497, 2000]:

![Diagram showing the initiation of positive streamers in sprites before the negative ones.](image)
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Simulation Setup to Initiate Streamers in Weak Fields

[Babaeva and Naidis, 1997]
Model Streamer in a Weak Electric Field at 70 km Altitude

- Modeling results on a streamer advancing in a weak electric field at 70 km altitude: (a) electron number density, (b) electric field, and (c) intensity (in Rayleighs) profiles of 1PN$_2$, 2PN$_2$, 1NN$_2^+$ and LBH N$_2$ emissions along the central axis of the modeled streamer [Liu et al., 2006].

![Graphs of electron density, electric field, and intensity profiles](image-url)
Comparison of ISUAL Observations with Streamer Modeling

• Comparison of results from sprite streamer modeling with spectrophotometric measurements by ISUAL instrument on FORMOSAT-2 satellite [Liu et al., Geophys. Res. Lett., 33, L01101, 2006].
Recent Advances in Remote Sensing of Electric Fields in Sprites

- *Morrill et al.* [Geophys. Res. Lett., 29, 1462, 2002] reported analysis of time integrated (33 ms) sprite emissions indicating that electric field in sprites closely followed $E_k$ up to 55 km altitude and dropped below $E_k$ above 55 km, where $E_k$ is the conventional breakdown threshold field.

- *Kuo et al.* [Geophys. Res. Lett., 32, L19103, 2005] used five selected sprite events recorded by ISUAL instrument to estimate the strength of the electric field in sprites to be $2.1-3.7E_k$.

- *Liu et al.* [Geophys. Res. Lett., 33, L01101, 2006] compared streamer modeling with ISUAL observations and concluded that in order to agree with observations during initial ($\sim0.5$ ms) stage of sprite development the maximum field driving emissions of an observed sprite event must be greater than $3E_k$.

- *Adachi et al.* [Geophys. Res. Lett., 33, L17803, 2006] analyzed twenty sprite events captured by ISUAL instrument and estimated that electric fields in upper/diffuse region of sprites do not exceed 0.5-0.7$E_k$ and in lower/streamer region are 1-2$E_k$, which are lower than those estimated by streamer theory presented in *[Liu et al., 2006]*.

Illustration of Time-Integrated Effects

- CCD photos of streamers in a 25 mm point-wire gap at ground pressure in air using an optical gate of (a) 0.8 ns and (b) 5 µs [van Veldhuizen and Rutgers, 2002].

- Modeling results of 1PN$_2$ streamer emissions at 70 km altitude at the moment of time 530 µs (c), and a series of moments of time between 0 µs and 530 µs (d) [Liu and Pasko, 2004].
A Positive Streamer in a Field of 25 N/N₀ kV/cm at 75 km Altitude

Electron Density

- Electron Density (m⁻³)
- Electric Field (V/m)
- Intensity (R)

Altitude = 75 km, Time = 0.39 ms, E₀ = 25 kV/cm N/N₀
Effects of Spatial Resolution of Imaging

![Image showing the effects of spatial resolution with different pixel resolutions: 5 m/pixel, 15 m/pixel, and 40 m/pixel. The images depict the spatial distribution at t = 0.39 ms with color-coded intensity values ranging from 5x10^6 to 3.9x10^7.]
Recent Observations ofSprites at 10,000 Frames per Second

[McHarg et al., GRL, 34, L06804, 2007; Stenbaek-Nielsen et al., GRL, 34, L11105, 2007]
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Observation of NO-γ Emissions in Laboratory Experiments

- Laboratory experiments at ground pressure suggest that NO-γ emissions can be generated during streamer discharges, which have a wavelength range overlapping with that of N₂ LBH emissions [e.g., Simek et al., J. Phys. D: Appl. Phys., 35, 1998; Tochikubo and Teich, Jpn. J. Appl. Phys., 39, 2000].

Figure 1: Time histories of 2PN₂, 1NN₂⁺ and NO-γ from repetitive positive streamer discharge (365 hPa N₂ + 10 hPa O₂) [Tochikubo and Teich, 2000].
NO Chemistry in Sprite Streamers

- Important species involved in NO chemistry: N$_2$, O$_2$, N$(^2D)$, O$(^3P)$, N$_2$(A$^3\Sigma_u^+$), NO($X^2\Pi_r$), and NO(A$^2\Sigma^+$)

<table>
<thead>
<tr>
<th>Reaction process or index</th>
<th>Reaction</th>
<th>Rate Coefficient [f(E/N) denotes function of reduced electric field]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electron collision reactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$e + N_2 \rightarrow e + N(^4S) + N(^2D)$</td>
<td>$f(E/N)$</td>
</tr>
<tr>
<td>2</td>
<td>$e + O_2 \rightarrow e + O(^3P) + O(^3P)$</td>
<td>$f(E/N)$</td>
</tr>
<tr>
<td>3</td>
<td>$e + N_2 \rightarrow e + N_2(A^3\Sigma_u^+)$</td>
<td>$f(E/N)$</td>
</tr>
<tr>
<td>4</td>
<td>$e + N_2 \rightarrow e + N_2(B^3\Pi_g)$</td>
<td>$f(E/N)$</td>
</tr>
<tr>
<td>5</td>
<td>$e + N_2 \rightarrow e + N_2(C^3\Pi_u)$</td>
<td>$f(E/N)$</td>
</tr>
<tr>
<td>6</td>
<td>$e + NO(X^2\Pi_r) \rightarrow e + NO(A^2\Sigma^+)$</td>
<td>$f(E/N)$</td>
</tr>
<tr>
<td><strong>Chemical reactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$N(^2D) + O_2 \rightarrow NO(X^2\Pi_r) + O(^3P)$</td>
<td>$5.20 \times 10^{-18} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td>8</td>
<td>$N_2(A^3\Sigma_u^+) + O(^3P) \rightarrow NO(X^2\Pi_r) + N(^2D)$</td>
<td>$7 \times 10^{-18} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td>9</td>
<td>$N(^2D) + NO \rightarrow N_2 + O(^3P)$</td>
<td>$6.0 \times 10^{-17} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td><strong>Excitation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$N_2(A^3\Sigma_u^+) + NO(X^2\Pi_r) \rightarrow NO(A^2\Sigma^+) + N_2(X^1\Sigma_g^+)$</td>
<td>$8.75 \times 10^{-17} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td><strong>Quenching</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>$N(^2D) + N_2 \rightarrow N(^4S) + N_2$</td>
<td>$1.70 \times 10^{-20} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td>12</td>
<td>$N_2(A^3\Sigma_u^+) + O_2 \rightarrow N_2 + O_2$</td>
<td>$8.75 \times 10^{-19} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td>13</td>
<td>$N_2(A^3\Sigma_u^+) + O_2 \rightarrow N_2 + 2O(^3P)$</td>
<td>$1.63 \times 10^{-18} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td>14</td>
<td>$NO(A^2\Sigma^+) + O_2 \rightarrow NO(X^2\Pi_r) + O_2$</td>
<td>$1.62 \times 10^{-16} \text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td><strong>Radiative transition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>$N_2(B^3\Pi_g) \rightarrow N_2(A^3\Sigma_u^+) + h\nu$</td>
<td>$1.7 \times 10^5 \text{s}^{-1}$</td>
</tr>
<tr>
<td>16</td>
<td>$N_2(C^3\Pi_u) \rightarrow N_2(B^3\Pi_g) + h\nu$</td>
<td>$2.0 \times 10^7 \text{s}^{-1}$</td>
</tr>
<tr>
<td>17</td>
<td>$NO(A^2\Sigma^+) \rightarrow NO(X^2\Pi_r) + h\nu$</td>
<td>$5 \times 10^6 \text{s}^{-1}$</td>
</tr>
</tbody>
</table>
Source and Loss Processes of $\text{N}_2(A^3\Sigma^+_u)$ and $\text{N}_2(a^1\Pi_g)$

- The $\text{N}_2(A^3\Sigma^+_u)$ state is responsible for NO-γ emissions; The $\text{N}_2(a^1\Pi_g)$ state is responsible for $\text{N}_2$ LBH emissions.

(a) $\text{N}_2(A^3\Sigma^+_u)$ source and loss processes.

(b) $\text{N}_2(a^1\Pi_g)$ source and loss processes.
Streamer Model

- Modeling results for a sprite streamer developing in an ambient field $E_0 = 5 \times N_{70}/N_0 \text{kV/cm}$, where $N_{70}$ and $N_0$ are air densities at 70 km and 0 km altitude, respectively.

![Diagram of electron density and electric field](image)
The initial density of NO is set to be $2 \times 10^{14} \ 1/m^3$ [Atreya, Adv. Space Res., 1, 127, 1981].
• $N_2$ LBH emissions dominate over NO-$\gamma$ emissions in the streamer head. The intensity ratio of NO-$\gamma$ to $N_2$ LBH emissions stays relatively constant in the streamer body.
Implications of This Work for Sprite Observations

- The NO-γ emissions from sprites are not observable for a wide bandwidth photometer.
- Strong bands of NO-gamma emissions are located in the wavelength range 240–260 nm in which N$_2$ LBH emissions are absent [Vallance-Jones, 1974, Tables 4.14 and 4.18, 1974].

- A dedicated narrow bandwidth photometer with the wavelength passband of 240–260 nm would be able to detect sprite NO-γ emissions from space.