Bayesian Magnetic Field Reconstruction using MSE, Polarimetry and Magnetic coils.

Jakob Svensson, Oliver Ford, Mathias Brix, Andreas Werner
Bayesian Probability

Prior

Likelihood

Model Evidence

Posterior

\[ p(T|d) = \frac{p(d|T)p(T)}{p(d)} \]
Bayesian Intuition Pump: Interferometry inversion +

Profile: \( n_e(r_{\text{eff}}) = n_0 * (1 - r_{\text{eff}}^2)^q \)
Bayesian Interferometry Inversion

Prior

1 channel

2 channels

5 channels
Bayesian Interferometry Inversion

5 interferometer channels, no YAG

5 interferometer channels, 1 YAG channel
Diagnostic Analysis

Single Diagnostic Analysis

Observable a(A,C) → Diagnostic A(a,C?) → Measured A
Observable b(A,B,C) → Diagnostic for C(b,B?,A?) → Measured C

Integrated Diagnostic & Plasma Model

Likelihood of joint a,b,c as a function of joint A,B,C.

Inversion {A,B,C} | (a,b,c)
Dependencies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Diagnostic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$</td>
<td>LIDAR</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Interferometry</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Electron Cyclotron Emission</td>
</tr>
<tr>
<td>${n_Z}$</td>
<td>Polarimetry</td>
</tr>
<tr>
<td>$j_{tor}$</td>
<td>Motional Stark Effect</td>
</tr>
<tr>
<td>...</td>
<td>Bremsstrahlung</td>
</tr>
<tr>
<td></td>
<td>Pickup Coils, Flux Loops, Saddles</td>
</tr>
<tr>
<td></td>
<td>CX Spectroscopy</td>
</tr>
<tr>
<td></td>
<td>...</td>
</tr>
</tbody>
</table>

Interferometry
Electron Cyclotron Emission
Polarimetry
Motional Stark Effect
Bremsstrahlung
Pickup Coils, Flux Loops, Saddles
CX Spectroscopy
One plasma. One model.
W7-AS Model

**Pressure**

Electron pressure, shot 54285, t=0.3282s

**Pressure gradient**

Electron pressure gradient, shot 54285, t=0.3282s

Marginal density for volume

Marginal density for plasma effective radius, shot 54285, t=0.4286s

Magnetic Mapping:

- Interferometer #1 los
- Thomson Yagi
- Diagnost loop

Total kinetic energy

Diagonstic loop

Uncertainty of magnetic mapping

Volume [m³]

Plasma volume

Plasma radius

Magnetic Mapping:

- Plasma volume
- Plasma radius

Diagonstic loop

Uncertainty of magnetic mapping
Flux surfaces and handling of uncertainty of flux surfaces is fundamental to integration of diagnostics!
Current Tomography: Magnetic Model

\[ p(\{I_i\} \mid D^{\text{Mag}}) \]
Measurements and Forward Function

1. Pickups

\[ P_i = B_R(R_i, Z_i) \cos(\Theta_i) + B_Z(R_i, Z_i) \sin(\Theta_i) \]

2. Saddle coils

\[ S_i = \frac{d_i}{8} (\Psi(R_i^{(2)}, Z_i^{(2)}) - \Psi(R_i^{(1)}, Z_i^{(1)})) \]

3. Flux Loops

\[ F_i = \Psi(R_i, Z_i) \]

\[ \begin{align*}
\vec{D}^{\text{Pred}} &= \vec{M} \vec{I} + \vec{C} \\
\vec{I} &\quad \text{- vector of beam currents} \\
\vec{M}_{ji} &\quad \text{- Maps beam current i to measurement j} \\
\vec{C} &\quad \text{- PF contribution to measurements}
\end{align*} \]
Posterior using Magnetic Measurements

Likelihood: Gaussian noise

\[
p(D_{\text{Mag}}^T | I) = \frac{1}{(2\pi)^{N_D/2} |\Sigma_D|^{1/2}} \exp\left( -\frac{1}{2} (M I + C - D_{\text{Mag}}^T) \Sigma_D^{-1} (M I + C - D_{\text{Mag}}^T) \right)
\]

Prior: Multivariate Normal over free currents

\[
p(I) = \frac{1}{(2\pi)^{N_I/2} |\Sigma_I|^{1/2}} \exp\left( -\frac{1}{2} (I - m_I)^T \Sigma_I^{-1} (I - m_I) \right)
\]

\[\Rightarrow\]

Posterior: Multivariate normal over free currents

\[
p(I | D_{\text{Mag}}^T) = \frac{1}{(2\pi)^{N_I/2} |\Sigma|^{1/2}} \exp\left( -\frac{1}{2} (I - m)^T \Sigma^{-1} (I - m) \right)
\]

where

\[m = (M \Sigma_D M + \Sigma_I)^{-1} M \Sigma_D (D_{\text{Mag}}^T - C)\]

\[\Sigma = (M \Sigma_D M + \Sigma_I)^{-1}\]

Forward function: \[D_{\text{Pred}} = M I + C\]
CAR prior:
Impose Prior Correlations between neighbouring Beams

Conditional Autoregressive Model (CAR)

\[ I_i \mid I_{-i} \sim N\left(\sum_j \beta_{ij} I_j, \tau\right) \]

\[ \beta_{ij} = \frac{1}{4} \quad \text{If i,j neighbouring beams} \]

\[ \beta_{ij} = 0 \quad \text{otherwise} \]
Flux Surfaces

Boundary

Internal Flux Surfaces

Poloidal Flux Samples
Posterior Distributions for Some Quantities

Magnetic Axis

X-Point

Total Plasma Current

Uncertainty: 6-8 cm

Uncertainty: 2-3 cm

Uncertainty: 0.5%

Plasma Volume

Plasma Surface Area

Uncertainty: 1.2%

Uncertainty: 0.8%
Can be \textit{approximately} found by using vacuum toroidal flux, ignoring flux contribution from poloidal current.
Including more diagnostics

MSE:
\[
\tan \gamma^i = \frac{B_Z^i A_1^i + B_R^i A_2^i + B_T^i A_3^i}{B_Z^i A_4^i + B_R^i A_5^i + B_T^i A_6^i}
\]

1. MSE linearised:
\[
\tan \gamma^i = \frac{B_Z^i A_1^i + B_T^i A_3^i}{B_T^i A_6^i}
\]

2. Polarimetry:
\[
\Delta \psi^i \propto \int_{\text{los}^i} n_e(l) B_{\parallel}(l) dl
\]

3. Interferometry:
\[
\overline{n}_e^i \propto \int_{\text{los}^i} n_e(l) dl
\]

Linearised MSE linear in jtor, ne linear if jtor fixed, polarim. linear with ne fixed...
CT with linearised MSE, interf., polarim

CT magnetics

CT magnetics, MSE, polar.

CT magnetics and MSE

CT magnetics, MSE, interf., polarim

OBS! Same prior for all three cases.
2D toroidal current distribution

Toroidal current distribution, pulse 75050, t=62s

Toroidal current distribution $j_{tor}(R, Z=0)$, pulse 75050, t=62s

$j_{tor}$ [kA/m²]
Strike point distributions

Inner strike point pdf over s-coord. pulse 75210, t=53.20s

Outer strike point pdf over s-coord. pulse 75210, t=53.20s
q-profiles and location of rational surfaces
Electron density profiles

Inversion from MAP
CT with MSE MAP and interferometry inversion, pulse=75050, t=64s

Inversion with uncertainty in flux surface mapping
CT with MSE interferometry inversion on CT posterior, pulse=75050, t=64s
Conclusions

• 6 diagnostic groups combined in a Bayesian way.

• Flux surfaces, q-profiles, ne-profiles etc without equilibrium assumptions

• Grad-Shafranov assumption could therefore be tested.

• Results specifically independent on whether pressure is isotropic or not.

• Uncertainties on all quantities inferred, incl. strike point positions, magnetic axis, LCFS etc.

• Real time speed.

• Basic foundation for adding further diagnostics and possibly solving many mapping problems.