Monte Carlo Renormalization Group studies of many-fermion models

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The conformal window in SU(3) with $N_f$ fundamental fermions

2-loop perturbation theory prediction:
- $N_f < 16$ AF
- $N_f \leq 8$ chirally broken, confining
- $N_f > 8$ IRFP in gauge coupling, conformal at $m=0$

Higher order and non-perturbative effects can change the bottom of the conformal window

Questions for lattice studies:
- Minimal $N_f$ where the conformal regime develops
- Running of the coupling just below the conformal window (technicolor)
- Properties of the IRFP (anomalous dimension of the mass)
The lattice phase diagram (arrows: UV to IR)

Below the conformal window ($N_f$ small)

- Strong lattice artifacts
- $\chi$ SB
- QCD-like

Quenched limit
$g$ is relevant

$g=0, m=0$ perturbative FP
$g, m$ are relevant

$\beta$
The lattice phase diagram

In the conformal phase (but \(N_f < 16\))

Phase transition at \(m = 0\) separating \(\chi\) SB and conformal phases. Could be a UVFP

There is no phase separation at finite \(m\)

The quasi-conformal phase and \(\chi\) SB phases are smoothly connected

(at least at large \(m\))

Strong lattice artifacts can introduce new transitions or mask the UVFP

IRFP; its location is not physical, depends on the RG; The IRFP is not directly observable
What can lattice simulation do?

In the conformal phase (but $N_f<16$)

1) Establish the existence of an IRFP / conformal phase

2) Verify $\chi$ SB or confinement in the scaling regime (before lattice artifacts take over)

Connect the perturbative FP and strong coupling regions (MCRG or Schroedinger fn)
The conformal window in SU(3) with $N_f$ fundamental fermions

I use a Monte Carlo renormalization group method, based on Wilson RG to study the flow lines, phase diagrams

• Start with well understood models:
  – $N_f = 0$ and $4$ : QCD like
  – $N_f = 16$ : conformal with IRFP

• Continue with more interesting cases
  – $N_f = 8$ : looks QCD like
  – $N_f = 12$ : getting difficult; probably QCD-like, maybe walking
  – $N_f = 10 - 15$ : identify the bottom of the conformal window
  – different fermion representations, gauge group: the method is applicable without change

• I use nHYP smeared staggered fermions:
  – highly reduced taste breaking
  – $m=0$ is $m_q=0$ for staggered
The MCRG / 2-lattice matching method

The method identifies pairs of bare couplings \((K, K')\) where
\[
a(K) = a(K') / 2
\]
This predicts the (differential) bare step scaling function
\[
s_b(K;2) = K-K'
\]

- At an RG fixed point \(s_b(K^*)=0\)
  - the bare step scaling function identifies a FP just like the renormalized one!
- \(s_b\) is related to the scaling dimension of the coupling
  - for AF models \(s_b = 3 \ln(2)/(4\pi^2) b_0 + O(g^2)\)
  - \(s_b\) in the mass predicts the anomalous dimension of the mass
    \[
    K- K^* = (K' - K^*) 2^{1/y}
    \]
Renormalization Group transformation

A real space block transformation averages out the short distance modes

\[ \{ U_{n,\mu} \} \rightarrow \{ V_{2n,\mu} \} \]
\[ a \rightarrow 2a, \xi \rightarrow \xi /2 \]
\[ S(\beta) \rightarrow S(\beta') \]

The change in the action $\beta \rightarrow \beta'$ defines the RG flow in the (infinite) dimensional parameter space.
Fixed points are either at $\xi=\infty$ or $\xi=0$
If $K^{(n)} = K'^{(n-1)}$ --> $a(K) = a(K')/2$

$s_b(K) = K - K'$

The location of the IRFP depends on the RG transformation
--- > RG can be optimized

Tuning the free parameter in the RG transformation can pull the FP and its RG in
RG flow lines & 2-lattice matching

Two actions are identical if all operator expectations values agree

The plaquette after 1-4 levels of blocking

\[ 32^4 \rightarrow 16^4 \rightarrow 8^4 \rightarrow 4^4 \rightarrow 2^4 \] (symbols) compared to \[ 16^4 \rightarrow 8^4 \rightarrow 4^4 \rightarrow 2^4 \] (lines)

<table>
<thead>
<tr>
<th>( n_b )</th>
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<th>3</th>
<th>4</th>
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<td>( a(\bar{\alpha}=7.0) )</td>
<td>( = )</td>
<td>( a(\bar{\alpha}=6.51)/2 )</td>
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Two lattice matching

Identify matched couplings \((\beta, \beta')\) by comparing expectations values after \(n_b\) \((n_b - 1)\) RG blocking steps

- Use several blocking levels for consistency
- Optimize blocking to bring FP close
- Use several operators to check scaling
  - Plaquette, 3 6-link loops, an 8-link loop

\[ \text{Optimization/Consistency check} \]
Two lattice matching

The step scaling function is  \( s_b = \lim_{n_b \to \infty} (\beta - \beta') \)

- Finite volume effects are controlled when actions are compared on identical volumes
- Requires relatively small statistics
- Has been used successfully (long time ago)
  - for pure gauge SU(3) to determine the \( \beta \) function
  - U(1) gauge to determine the order of the phase transition
  - For spin model to test/find critical exponents
SU(3) pure gauge - test case

RG matching: $32^4 \rightarrow 16^4$ and $16^4 \rightarrow 8^4$

Step scaling function can be calculated in many ways
- Schrödinger fn; Wilson loop ratios,
- physical observables $r_0$, $T_c$

![Graph showing the step scaling function for $N_f=0$](image)

Perturbative value $s(\beta) = 0.59$

- Errors shown only for MCRG
- Excellent agreement between $r_0$, $T_c$ and MCRG
- 2-loop SF is between 1-loop SF and $r_0$/MCRG
- $s_b = 0$ would indicate an IRFP
- Since at $\beta = 6$ we can test confinement, we know there is no physical IRFP
$N_f=4$ flavor

When $m_q=0$, it is just like quenched QCD, $g^2$ is the only relevant operator.
Perturbatively
\[ s_b(\beta) = \beta_1 - \beta_2 = 0.46 + O(g^2) \quad (\beta = 6 / g^2) \]

Numerical simulations:
16$^4$ and 8$^4$ matching of 5 operators
\~100 configurations are sufficient

Looks like pure gauge: no IRFP
$N_f=8$ flavors

Very similar to $N_f=4$;
No IRFP, the step scaling function approaches the perturbative value
**N_f=16 flavor SU(3) model**

**Do we see a difference?**

On the critical surface around an IRFP every coupling flows into the FP
→ every coupling matches with every other one when $n_b \rightarrow \infty$

With finite $n_b$ the optimal matching minimizes the slowest flowing operator pulls the IRFP close to $\beta \rightarrow s_b = \beta - \beta' = 0$

With $N_f=16$ the gauge coupling is nearly marginal; matching follows its flow,

$s_b = \beta - \beta' = 0$ is expected
$N_f=16$ flavor SU(3) model

Do we see a difference?

$16^4 \rightarrow 8^4$ MCRG

Matching of the plaquette at $m=0$

- Blocked plaquette increases with $n_b$
  
  $\rightarrow$ flow runs to the IRFP

- Matching consistent with $\beta = \beta'$

$-s_b(\beta) \approx 0$ : $g$ is nearly marginal
$N_f=16$ flavor SU(3) model

Matching in the mass at fixed $\beta = 5.8$

$m_2 = m_1 \cdot 2^{1/\nu}$

- use the same gauge observables (probably not the best choice)

-at $\alpha_{opt}$ both $n_b=2(1)$ and $3(2)$ predicts the same matching pair
The critical exponent for the mass

At several couplings, mass values

\[ m_2 = m_1 \, 2^{1/\nu} \]

\[ \nu = 1.0(1) \]

Free field exponent (close to GFP)
N_f=12 flavors

Use the same techniques as before; $16^4 \rightarrow 8^4$, some $32^4 \rightarrow 16^4$

Matching of the plaquette

With optimized blocking

- Plaquette decreases with $n_b$
- $s_b > 0$

Step scaling function

Step scaling function connects to perturbative, remains positive
(walking???)
N\textsubscript{f}=12 flavors

- Why is this preliminary?
  - \(s_b\) depends on blocking parameter \(\alpha\)
    
    Optimal blocking parameter is set by requiring
    
    \(s_b(n_b=3)=s_b(n_b=2)\) on 16\textsuperscript{4}.
    
    On 32\textsuperscript{4} I will have
    
    \(s_b(n_b=4)=s_b(n_b=3)=s_b(n_b=2)\) - more reliable
    
    - Matching dies at \(\beta < 5.4\). It is either
      
      - end of the scaling window
        
        —> is the system in a \(\chi\) SB phase there?
      
      - other block transformation is needed
        
        (there are many options)

At present

MCRG implies that

- \(N_f=12\) is below the conformal window
- \(\beta\) function runs ~2x slower than perturbative
Summary of results

- $N_f=0$ results for the step scaling function are consistent with other methods.
- $N_f=4,8$ results are consistent with perturbative predictions at the Gaussian FP.
- $N_f=16$ results imply the existence of an IRFP; the critical exponent of the mass is near 1.
- $N_f=12$ results are similar to $N_f=4,8$: no indication of IRFP; mass scales with exponent consistent with 1. Larger volumes are needed to confirm this conclusion.
Conclusion

MCRG is an effective alternative method to study the phase structure and scaling properties of lattice QFT’s

- The method is very universal, straightforward to implement for any other system (sextet fermions are under study)
- MCRG requires only limited statistics
- MCRG can predict the anomalous dimension of the mass