Modelling particle resuspension in turbulent boundary layers

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Outline

- Background
  - Nuclear Severe Accidents

- Stochastic model for resuspension
  - Improvements to Rock’n’Roll Model
    - Role of non Gaussian aerodynamic forces moments
      - Fluctuations and timescales
    - DNS/ LES measurements in a turbulent boundary layer
  - Multilayer Resuspension Model

- Summary and Conclusions

Particle resuspension in a turbulent boundary layer
Background

- particles < 5microns in size
  - principle adhesive forces – Vader Waals intermolecular forces
- Severe accidents concerned
  - Light-water-cooled reactor (LWR)
    steam spikes (primary circuit), hydrogen deflagration (containment)
  - Helium-cooled high-temperature reactor (HTR)
    Accumulation of contaminated dust in the coolant circuit
    Loss of coolant accident (LOCA)
  - International Thermonuclear Experimental Reactor (ITER)
    Accumulation of contaminated dust in the vacuum vessel
    Coolant-water-ingress or Loss of vacuum accident (LOVA)

Particle resuspension in a turbulent boundary layer
Resuspension models

\[ \Gamma = \frac{a}{2} F_L + r F_D \quad \Rightarrow \quad F = \frac{1}{2} F_L + \frac{r}{a} F_D \]

At the point of detachment \((y_{dh})\) the adhesive ‘pull off’ force \( f_a = -F_{AR} \) at \((y_{dh})\),

\[ \langle F \rangle + f(t) + F_{AR}(y) = 0 \]

mean

\[ f_{dh} = f_a - \langle F \rangle \]

Particle resuspension in a turbulent boundary layer
Rock’n’Roll model

$y_{dh}$ is the displacement at detachment point

$$p = \frac{\int_0^\infty v W (v, y_{dh}) dv}{\int_0^\infty \int_{-\infty}^{y_{dh}} W (v, y) dy dv}$$

rate constant

number of particles released/ sec / number of particles attached to surface

$$p = \frac{\int_0^\infty \dot{f} W (f_{dh}, \dot{f}) df}{\int_0^\infty \int_{f_{dh}}^\infty W (f, \dot{f}) df d\dot{f}}$$

Particle resuspension in a turbulent boundary layer

$W(f, \dot{f}) = 2\pi \sqrt{\langle f^2 \rangle \langle \dot{f}^2 \rangle} \exp\left(-\frac{f^2}{2\langle f^2 \rangle}\right) \exp\left(-\frac{\dot{f}^2}{2\langle \dot{f}^2 \rangle}\right)$

Gaussian statistically independent pdf

$$p = \frac{1}{2\pi} \sqrt{\langle \dot{f}^2 \rangle / \langle f^2 \rangle} \exp\left(-\frac{f_{dh}^2}{2\langle f^2 \rangle}\right) \frac{1}{2} \left[1 + \text{erf} \left(\frac{f_{dh}}{\sqrt{2}\langle f^2 \rangle}\right)\right]$$
Influence of surface ‘microscale roughness’

- Rock’n’Roll model – stochastic model (with potential energy accumulation)

\[ \Lambda(t) = \int_0^\infty p(r'_a) e^{-\rho(r'_a)^t} \varphi(r'_a) dr'_a \]

Reeks and Hall (2001)

- \( \rho \) is the resuspension rate constant = probability per unit time for detachments from a surface

\[ f_a = \frac{3}{2} \pi \Delta \gamma r_a \] JKR \( r_a \) - asperity radius \( r'_a = \frac{r_a}{r} \) normalised asperity radius

\[ \varphi(r'_a) = \frac{1}{\sqrt{2\pi}} \frac{1}{r'_a} \frac{1}{\ln \sigma'_a} \exp \left( - \frac{[\ln(r'_a/\bar{r}'_a)]^2}{2[\ln \sigma'_a]^2} \right) \] \( \bar{r}'_a \) & \( \sigma'_a \)

Very broad range of adhesive forces

Biassi et al. (2001) defined a correlation between particle radius and reduction and spread factor based on experimental data. (e.g. Hall’s, Reeks & Hall, 2001)

- geometric mean \( \bar{r}'_a = 0.016 - 0.0023r^{0.545} \)
- geometric spread \( \sigma'_a = 1.8 + 0.136r^{1.4} \)

Particle resuspension in a turbulent boundary layer
Short and Long term Resuspension Rate

Resuspension rate vs. time [Reeks et al., 1988]

Particle resuspension in a turbulent boundary layer
Measurements of the statistics of $f(t)$, $\dot{f}(t)$ and $W(f, \dot{f})$

\[ F_D = 1.7 \cdot 6\pi \mu_f r u = 10.2\pi \frac{r^+ \mu_f^2 \rho_f}{u_\tau} u \quad \text{O’Neill’ (1968)} \]

### Large Eddy Simulation

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$h$</th>
<th>grid</th>
<th>Re$_\tau$</th>
<th>$\Delta t$</th>
<th>steps</th>
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<td>$2\pi$</td>
<td>2</td>
<td>$n$</td>
<td>1</td>
<td>$72 \times 72 \times 72$</td>
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<td>$6\pi$</td>
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<td>1</td>
<td>$384 \times 193 \times 384$</td>
<td>180</td>
<td>0.0034s</td>
<td>63738</td>
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### Direct Numerical Simulation

Particle resuspension in a turbulent boundary layer
Distributions of $z_1 = f / \langle f^2 \rangle$ from LES data

Particle resuspension in a turbulent boundary layer
Distributions of $z_2 = \dot{f} / \langle f^2 \rangle$ from LES data

Particle resuspension in a turbulent boundary layer
Resuspension rate constant, $p$

Joint distribution of fluctuating aerodynamic force and its derivative

$$z_1 = \frac{f}{\sqrt{\langle f^2 \rangle}} \quad z_2 = \frac{\dot{f}}{\sqrt{\langle f^2 \rangle}}$$

$$W(z_1, z_2) = \frac{z_1 + A_1}{A_2} \exp \left[-\frac{1}{2} \left( \frac{z_1 + A_1}{A_2} \right)^2 \right] \cdot \frac{B_1}{B_2 \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left( B_3 + B_1 \ln \left(z + \sqrt{z^2 + 1}\right) \right)^2 \right]$$

where $A_1, A_2, B_1, B_2, B_3$ and $B_4$ are all constants which depending on $y^+$.

$$z = \frac{z_2 - B_4}{B_2}$$

$$p = B_j \omega \frac{z_{dh} + A_1}{A_2} \exp \left[-\frac{1}{2} \left( \frac{z_{dh} + A_1}{A_2} \right)^2 \right] 1 - \exp \left[-\frac{1}{2} \left( \frac{z_{dh} + A_1}{A_2} \right)^2 \right]$$

Modified

$$p = \frac{1}{2\pi} \omega \exp \left(-\frac{1}{2} z_{dh}^2\right) \left[1 + \text{erf} \left( \frac{1}{\sqrt{2}} z_{dh} \right) \right]$$

Original R'n'R Model

$$\omega = \sqrt{\langle \dot{f}^2 \rangle} = \omega^* \frac{\nu z}{V}$$

$$z_{dh} = (f_a - \langle F \rangle) / \langle f^2 \rangle^{1/2}; \ z_a = f_a / \langle f^2 \rangle^{1/2}$$

Particle resuspension in a turbulent boundary layer
# Table of resuspension rate parameters

<table>
<thead>
<tr>
<th>LES</th>
<th>$B_{\text{dot}}$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$\omega^+$</th>
<th>$f_{\text{rms}} = \left( \frac{f^2}{F} \right)^{1/2} / \langle F \rangle$</th>
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<tbody>
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<td>$y^+ = 6$</td>
<td>0.366081</td>
<td>1.88837</td>
<td>1.510938</td>
<td>0.08553</td>
<td>0.335</td>
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<td>$y^+ = 2$</td>
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<td>1.86330</td>
<td>1.495317</td>
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<td>$y^+ = 1$</td>
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<td>1.90124</td>
<td>1.511536</td>
<td>0.10020</td>
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<td>DNS</td>
<td>$B_{\text{dot}}$</td>
<td>$A_1$</td>
<td>$A_2$</td>
<td>$\omega^+$</td>
<td>$f_{\text{rms}}$</td>
</tr>
<tr>
<td>$y^+ = 6$</td>
<td>0.358568</td>
<td>1.83605</td>
<td>1.478360</td>
<td>0.12714</td>
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<tr>
<td>$y^+ = 2$</td>
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<td>1.431301</td>
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<td>$y^+ = 0.1$</td>
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<td>1.81256</td>
<td>1.463790</td>
<td>0.16419</td>
<td>0.366</td>
</tr>
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Particle resuspension in a turbulent boundary layer
Particle resuspension in a turbulent boundary layer
Initial resuspension rate, $\Lambda(0)$

Particle resuspension in a turbulent boundary layer
Resuspension rates, Gaussian versus non-Gaussian statistics

geometric mean of normalised adhesive force \( z_a = \bar{f}_a / \langle f^2 \rangle^{1/2} = 3 \)

Particle resuspension in a turbulent boundary layer
Comparison of Original and Modified R’n’R model

Comparison with Hall’s experimental results

<table>
<thead>
<tr>
<th></th>
<th>( \omega^+ )</th>
<th>( \left(f^2\right)^{1/2}/\langle F \rangle )</th>
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</thead>
<tbody>
<tr>
<td>Modified (DNS)</td>
<td>0.164189</td>
<td>0.366</td>
</tr>
<tr>
<td>original</td>
<td>0.0413</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Particle resuspension in a turbulent boundary layer
Fraction resuspended: original versus modified R’n’R model

Particle resuspension in a turbulent boundary layer
Fraction resuspended: original versus modified R’n’R model

Particle resuspension in a turbulent boundary layer
Resuspension rate: original versus modified R’n’R model

Particle resuspension in a turbulent boundary layer
Multilayer modelling

Friess and Yadigaroglu (FY), 2001

Suppose we let \( n_i(\xi, t) d\xi \) denote the number of particles between \( \xi, \xi + d\xi \) in the \( i \text{-th} \) layer of a deposit composed of \( L \) layers, the layers being numbered sequentially from the top layer (totally exposed to the flow) downward as \( i = 1, 2, ..., L \). The set of ODE equations are thus

\[
\frac{\partial n_i(\xi, t)}{\partial t} = -p(\xi)n_i(\xi, t) + \psi(\xi) \int_0^\infty p(\xi')n_{i-1}(\xi', t) d\xi' \\
= -p(\xi)n_i(\xi, t) + \psi(\xi)\Lambda_{i-1}(t)
\]

\[
\Lambda_i(t) = \sum p n_i(\xi, t) \Delta \xi
\]

Particle resuspension in a turbulent boundary layer
Resuspension fraction vs. time

FY model predictions for STORM(SR11) phase 6 conditions

Particle resuspension in a turbulent boundary layer
Resuspension of each layer vs. time

FY model with rate constants based on non-Gaussian R’n’R model
STORM (SR11) Phase 6 condition

Particle resuspension in a turbulent boundary layer
Gaussian v non-Gaussian R’n’R models

\[ \omega^+ = 0.0413, f_{\text{rms}} = 0.2 \]

Particle resuspension in a turbulent boundary layer

STORM test SR11
Phase 6 conditions
Multilayer model results with BISE experiment

Resuspension fraction

Experimental data
- Monolayer, monodisperse
- 50 layers, monodisperse
- 100 layers, monodisperse

Particle resuspension in a turbulent boundary layer
Summary & Conclusions

- Significant improvement to the original Rock’n’roll model
  - LES / DNS measurements of the statistics of drag force and time derivative in viscous sub layer
    - Distributions of \( f / \langle f^2 \rangle^{1/2} \) and \( \dot{f} / \langle \dot{f}^2 \rangle^{1/2} \) non Gaussian
      - skewed with significant differences in the wings (intermittency)
      - independent of \( y^+ \)
    - frms \( f^{1/2} / \langle F \rangle \) independent of \( y^+ \), twice original frms
    - \( \omega^+ \) dependent on \( y^+ \), 4 x original
  - Examined the influence on resuspension
    - Non Gaussian gives more resuspension than Gaussian model (same frms and \( \omega^+ \))
    - Long term resuspension fraction and rates (difference reflected in different values of frms)
    - Long term resuspension rate weak dependence on \( \omega \)
      - very important in multilayer resuspension

Particle resuspension in a turbulent boundary layer