Classical dynamics and thermalization in holographic matrix models

David Berenstein, UCSB
Leiden, 11.10.12

Holography

Equivalence between certain quantum gauge theories in d dimensions and quantum gravity in higher dimensions (>d)

Also called AdS/CFT correspondence or gauge/gravity duality

Moral

Complete equivalence:

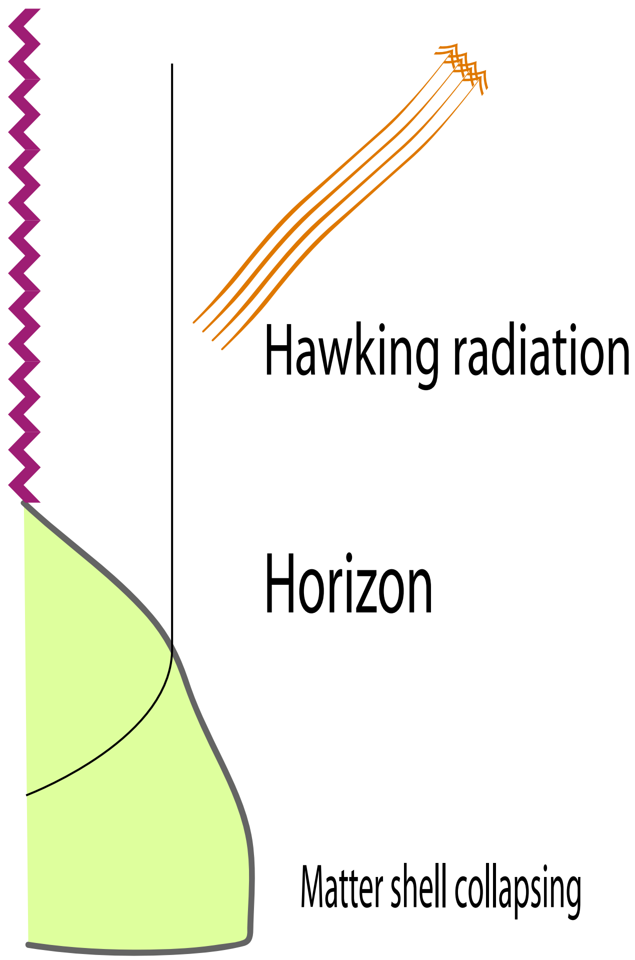
Everything that can happen on the gravity side happens in gauge theory.

Expressed in different variables.

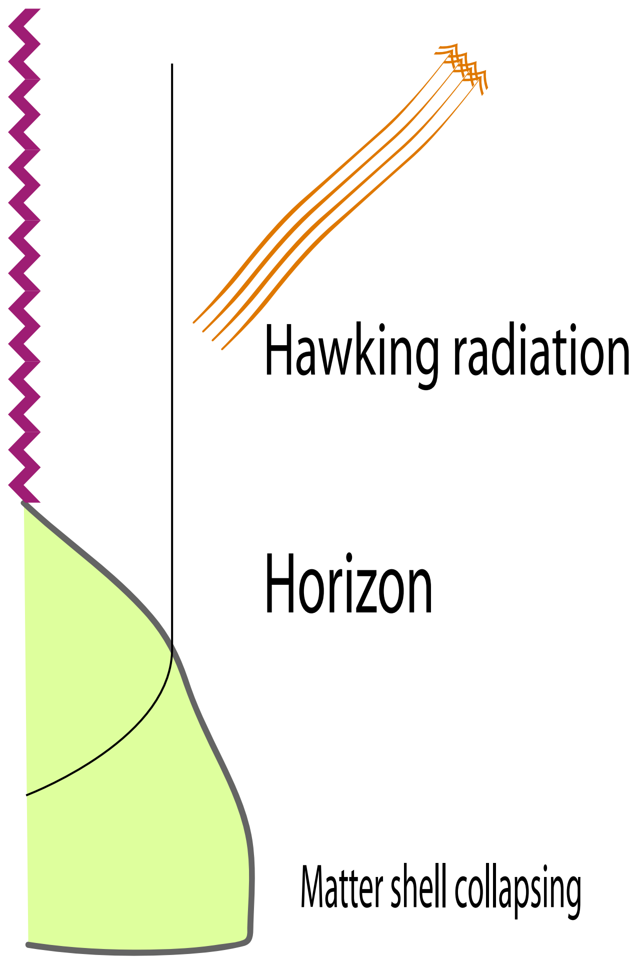
Questions about gravity

- If gravity is emergent, what is geometry? (How do we decode the hologram?)
- How does the Black Hole information paradox get resolved in detail?
- How fast does information scramble in black holes ?(Fast scrambling conjecture of Sekino and Susskind)
- Is there an enhanced conformal symmetry near black hole horizons? (Why can we apply Cardy's formula to compute black hole entropy?)

Black hole life cartoon.



Black hole life cartoon.



Need to reproduce history of formation and evaporation of black hole on dual: requires studying time dependence of physics for a long time.

Searching for a tractable model

- Real time strongly coupled QFT is essentially impossible to simulate.
- We can try looking for weakly coupled real time QFT: still impossible to simulate (black holes are thermal, so we need multi-body quantum physics to study thermalization, not just 2-2 scattering).
- Classical field theory suffers from the UV catastrophe, so it always needs quantum physics unless there are only finitely many degrees of freedom.
- Idea (compromise): study classical dynamics of gauge theories in 0+1 dimensions. These are gauged multi-matrix models. Hope that we can connect the dots.

Plan for rest of talk

- Two matrix models with known holographic duals.
- Picking initial conditions and evolving: linear and non-linear regime.
- Observables: eigenvalues and traces.
- Tests of thermalization.
- Visualizing geometry.
- Emergent conformal symmetry?

BFSS matrix model

Dimensional reduction of U(N) SYM in d=9+1 to 0+1

$$S_{BFSS} = \frac{1}{2g^2}\int dt\,\left((D_t X^I)^2 + \frac{1}{2}[X^I, X^J]^2\right) + \mathrm{fermions}$$

Banks, Fischler, Shenker, Susskind ‘96

There are 9 dynamical matrices and one matrix constraint.

Moduli space

Vacua are commuting matrices

$$[X^I, X^J] = 0$$

Up to gauge transformations the matrices are diagonal: eigenvalues are positions of D0-branes.

Produces an $\mathbb{R}^9$

For each D0 brane

- There is an effective metric: off diagonal modes have a mass that measures the euclidean distance in 9 flat dimensions.

- When D0 branes are far, off-diagonal modes are integrated out (high cost in energy). When they are near each other they can become active.

- The model describes the DLCQ quantization of M-theory in flat space. The discrete momentum is the rank of the matrices.

- Technicality: there are bound states at threshold of n D0 branes. These are interpreted as a single graviton of momentum n. (Wave functions unknown)

- There is an effective metric: off diagonal modes have a mass that measures the euclidean distance in 9 flat dimensions.

- When D0 branes are far, off-diagonal modes are integrated out (high cost in energy). When they are near each other they can become active.

- The model describes the DLCQ quantization of M-theory in flat space. The discrete momentum is the rank of the matrices.

- Technicality: there are bound states at threshold of n D0 branes. These are interpreted as a single graviton of momentum n. (Wave functions unknown)

Near Horizon of D0 branes

$$ds^2 = H^{-1/2}(r)(dx_{||}^2) + H^{1/2}(r)(dr^2 + r^2 d\Omega_{8-p}^2)$$

$$\int_0^{r_0} H^{1/4}(r)dr = \infty$$

There is an infinitely long throat for p <= 3

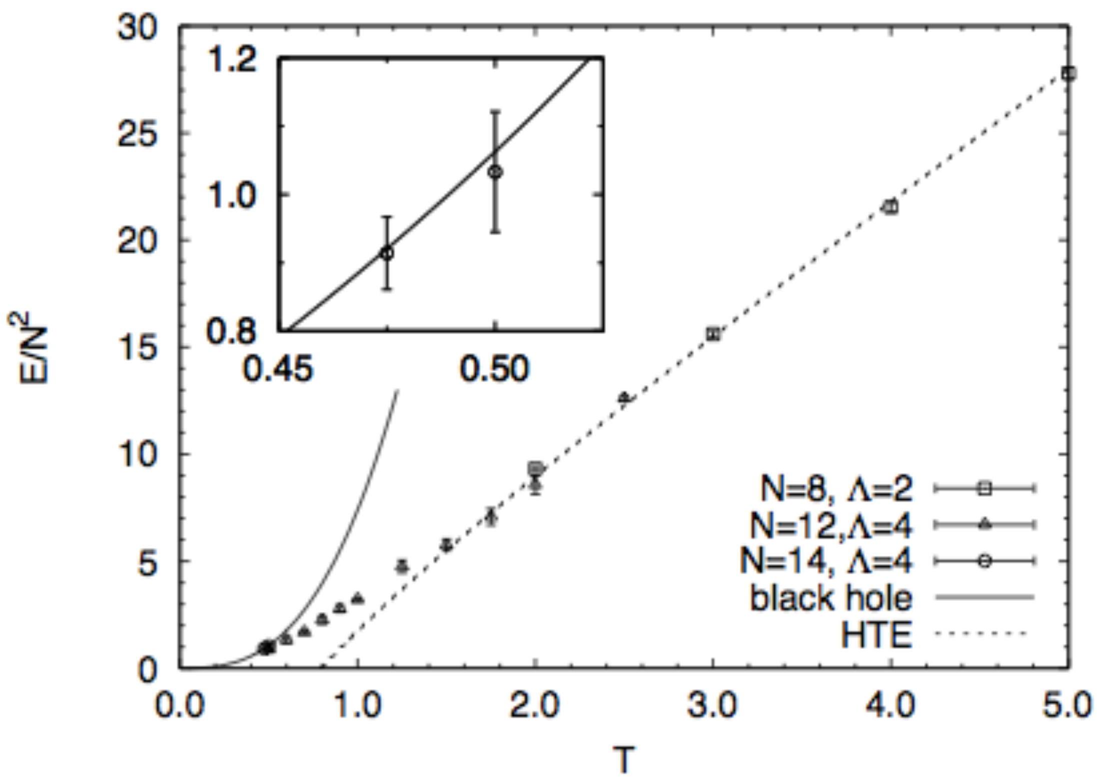
Effective curvature goes to zero as r goes to zero for p<3: gravity describes well IR, grows when r gets bigger - stringy UV.

Low energy thermal described by a Black hole (deconfined IR):
can compute specific heat from gravity.

High T regime should be a stringy black hole - best described by D-brane dynamics (Horowitz-Polchinski in reverse).

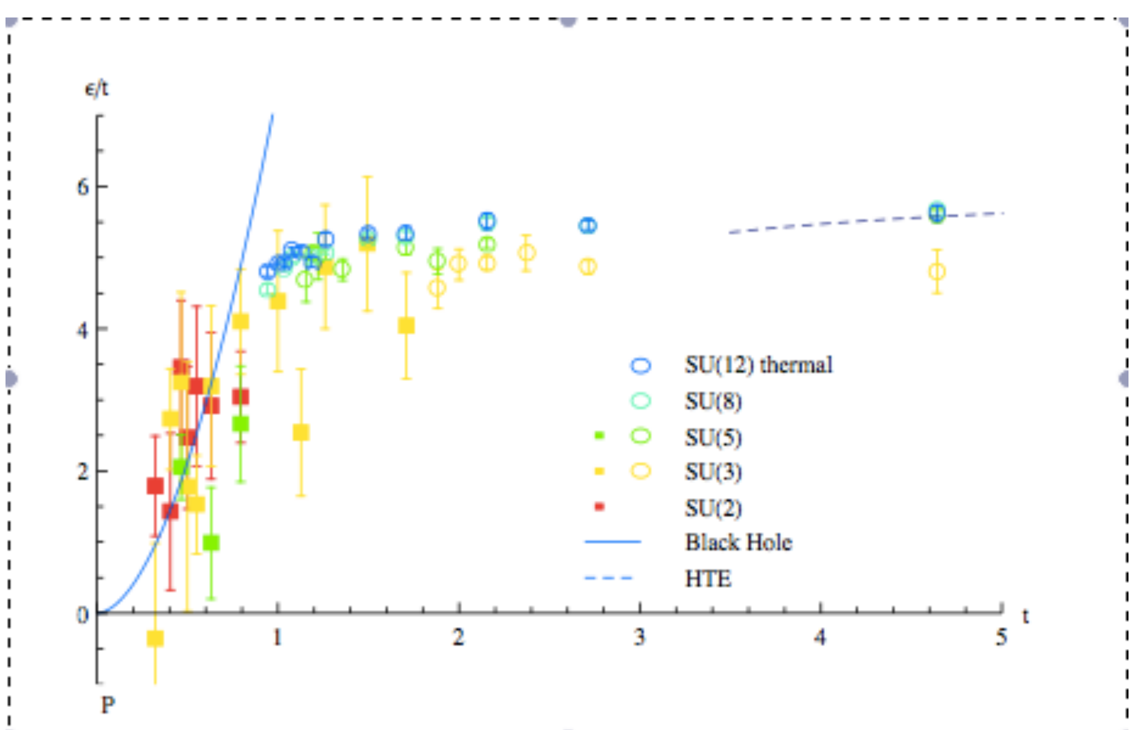
Classical simulations may describe a stringy black hole.

Monte-Carlo: can get equation of state



Agnastopoulos, Hanada, Nishimura,Takeuchi

arXiv:0707.4454 [hep-th]



Catterall-Wiseman

arXiv:0803.4273

BMN model (B w/ Maldacena + Nastase ‘02)

- Mass deformation of BFSS
- Describes DLCQ on a plane wave rather than flat space (has flux)
- No moduli space, but preserve number of susy: solutions don’t disperse to infinity (guarantees existence of thermodynamic ensemble: good for numerics, problem for BFSS)
- Absence of moduli space indicates gravitational potential on plane wave.
- Arises from SU(2) invariant compactification of N=4 SYM on a three sphere. (Kim, Klose, Plefka).

BMN model deformation

Split 9X into 3 X +6 Y

$$S_{BMN} = S_{BFSS} - \frac{1}{2g^2}\int dt\,\left(\mu^2 (X^i)^2 + \frac{\mu^2}{4}(Y^a)^2 + 2\mu i\,\epsilon_{\ell jk} X^\ell X^j X^k\right)$$

$$+\text{fermions}$$

$$V^{(X)}_{BMN} = \frac{1}{2g^2}\text{tr}\left[\left(i[X^2,X^3]+\mu X^1\right)^2 + \left(i[X^3,X^1]+\mu X^2\right)^2 + \left(i[X^1,X^2]+\mu X^3\right)^2\right] .$$

Can choose units where

$$\mu = 1$$

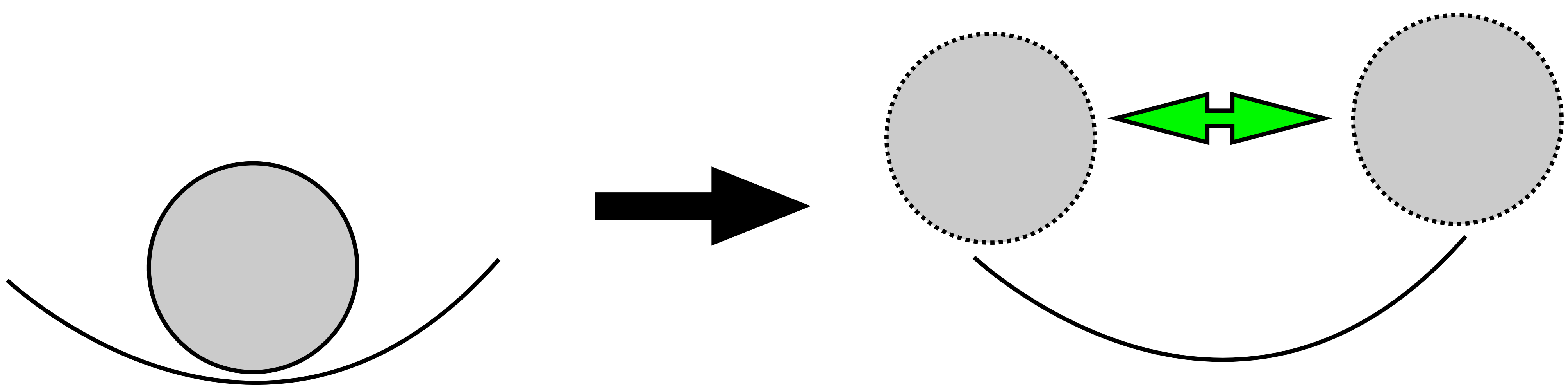
For 1x1 matrices the system is free

Describes 9 harmonic oscillators (3X, 6Y) with different masses.

Initial conditions also give an $\mathbb{R}^9$

Like in BFSS, the trace mode decouples, so we can rigidly move objects by exciting these zero modes.

This is how we identify the geometry.



Vacua are characterized by

$$[X_i, X_j] = i\epsilon_{ijk} X_k$$

These are SU(2) representations: they are classical solutions and there is no moduli space.

A single irreducible is called a fuzzy sphere.

A fuzzy sphere can also be interpreted either as a graviton or as a spherical M2 brane.

How to look at it depends on the strength of interactions.

$$\hbar$$

Making (dual) black holes

Picking initial conditions

Work in BMN model where vacua are classical.

For simplicity, take 1 fuzzy sphere + 1 D0 brane and make them collide.

This is like colliding 2 gravitons at high energy.

Check for thermalization

Picking initial conditions

Work in BMN model where vacua are classical.

For simplicity, take 1 fuzzy sphere + 1 D0 brane and make them collide.

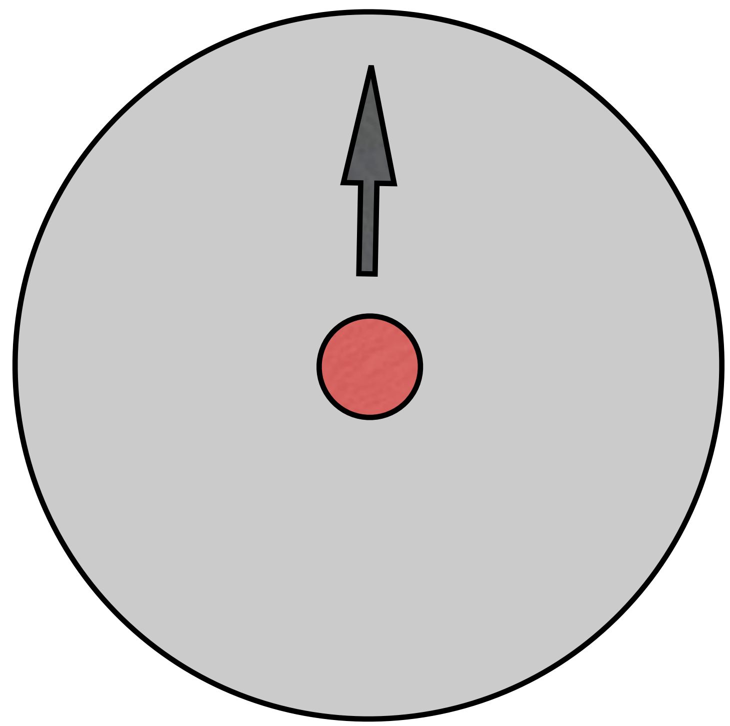
This is like colliding 2 gravitons at high energy.

Check for thermalization

- Remember: black holes are thermal

- Look for thermalization in classical dynamics

- Input Planck’s constant in initial conditions for fluctuating fields (add small random perturbations of a known classical initial state)



Take fuzzy sphere +
a d0 brane and make them collide.

Simplest classical solution

Take direct sums of fuzzy sphere + a single brane in motion.

This is an exact classical solution for all times: the branes cross each other
in a periodic motion and nothing happens.

When we add perturbations, they can be treated linearly for some time.

Modes connecting D0-brane to fuzzy sphere have a mass that is periodic and time dependent.

On part of the trajectory some modes are tachyonic: related to the Nielsen-Olesen instability in QCD

Linearized analysis for two spheres colliding

D.B., D. Trancanelli, arxiv:1011.2749

Some modes can grow exponentially very fast.

$$\ddot{q}_\ell(t) + (m_\ell^\pm(t))^2 q(t) = 0\,.$$

Trick to diagonalize: use angular momentum symmetry in terms of fuzzy spherical harmonics.

Linearized analysis for two spheres colliding

D.B., D. Trancanelli, arxiv:1011.2749

Some modes can grow exponentially very fast.

$$\ddot{q}_\ell(t) + (m_\ell^\pm(t))^2 q(t) = 0\,.$$

Trick to diagonalize: use angular momentum symmetry in terms of fuzzy spherical harmonics.

$$(\omega^-_{\ell,\ell+1})^2 = -b + (b-\ell-1)^2\,,$$
$$(\omega^+_{\ell,-\ell-1})^2 = b + (b+\ell+1)^2\,.$$

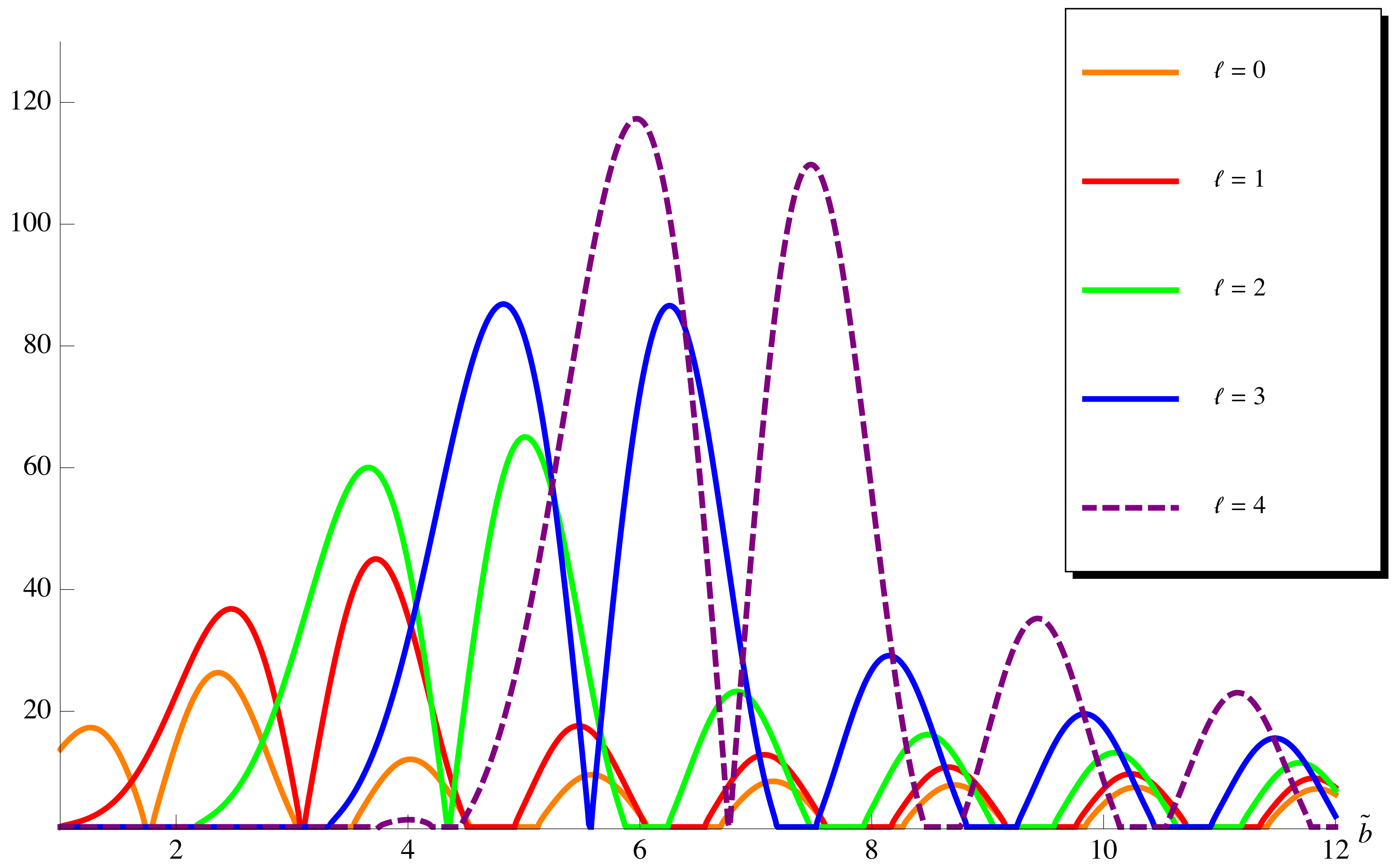
$$b(t) = \tilde{b}\sin(t)$$

$$\begin{pmatrix} q_1(t+2\pi) \\ q_2(t+2\pi) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix}$$

Eigenvalues of matrix determine if stable (eigenvalue unitary), or unstable (eigenvalues real).

Most unstable mode typically has highest L

Amplification factor on one oscillation.



Amplitude of oscillation

Expectations

Off diagonal modes connecting two fuzzy spheres grow exponentially classically.
Once they get large enough the rest of the system back-reacts.

Hopefully one ends up with an interesting evolution that thermalizes after that.

Numerics

C. Asplund, D.B., D. Trancanelli arXiv:1104.5469

C. Asplund, D.B., E. Dzienkowski, work in progress

Add quantum fluctuation seeds: generate randomly from gaussian distribution normalized to harmonic oscillator wave functions.

$$X^0 = \begin{pmatrix} L_n^0 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^1 = \begin{pmatrix} L_n^1 & \delta x_1 \\ \delta x_1^\dagger & 0 \end{pmatrix}, \quad X^2 = \begin{pmatrix} L_n^2 & \delta x_2 \\ \delta x_2^\dagger & 0 \end{pmatrix},$$

$$P^0 = \begin{pmatrix} 0 & 0 \\ 0 & v \end{pmatrix}, \quad P^{1,2} = 0 = Q^{1,\ldots,6}, \quad Y^a = \delta y^a.$$

$$\delta x, \delta y \simeq \sqrt{\hbar/n}$$

Matrices are chosen to satisfy Gauss' law

Evolve via Leapfrog algorithm

$$X_{t+\delta t} = X_t + P_{t+\frac{\delta t}{2}}\delta t\,, \qquad P_{t+\frac{\delta t}{2}} = P_{t-\frac{\delta t}{2}} - \frac{\partial V}{\partial X}\bigg|_t \delta t$$

Good at preserving constraints

Gauge invariant Observables

Take eigenvalues of matrices:

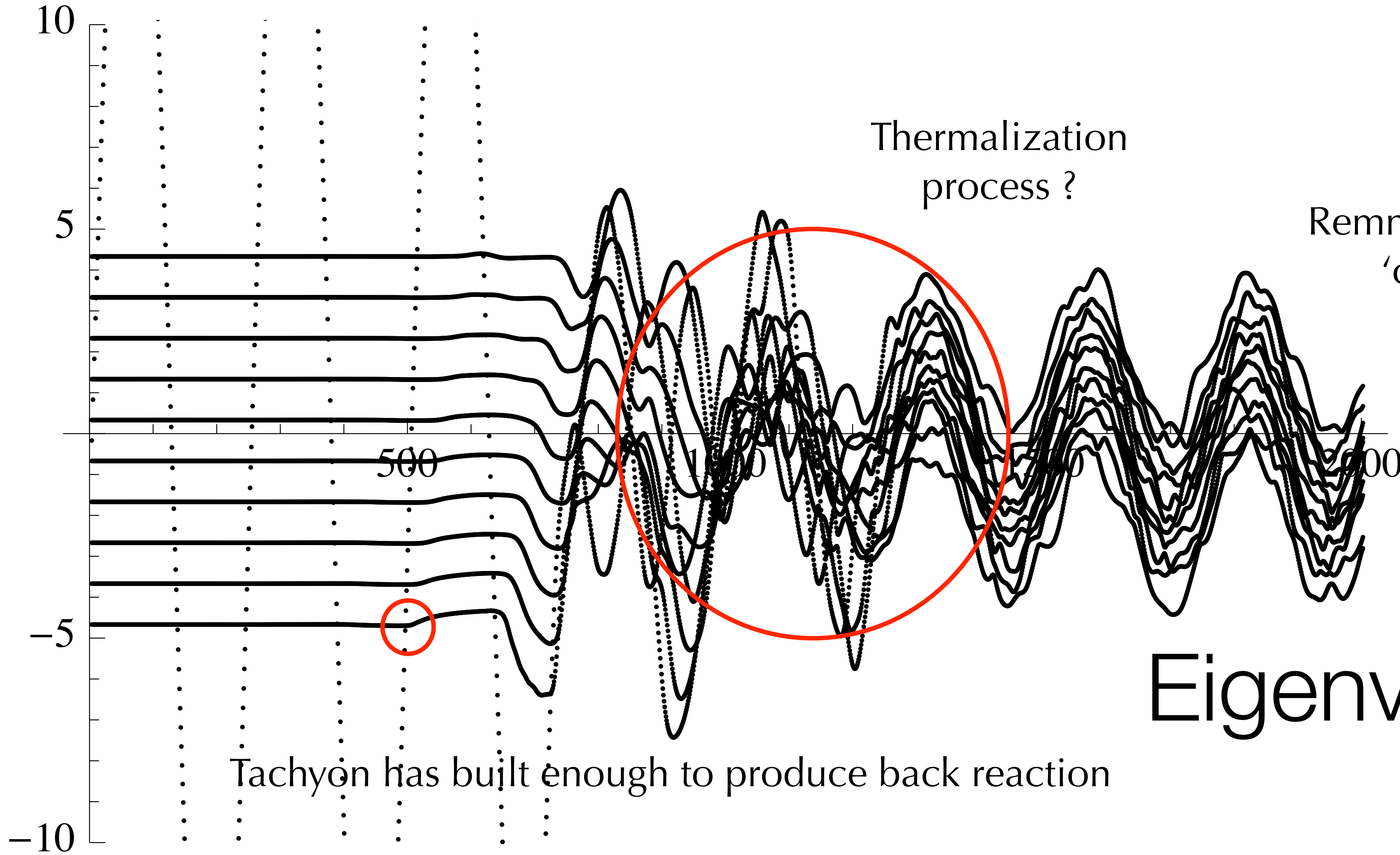
D-brane like (open string)

Take traces of products of matrices:

Collective degrees of freedom (closed strings)

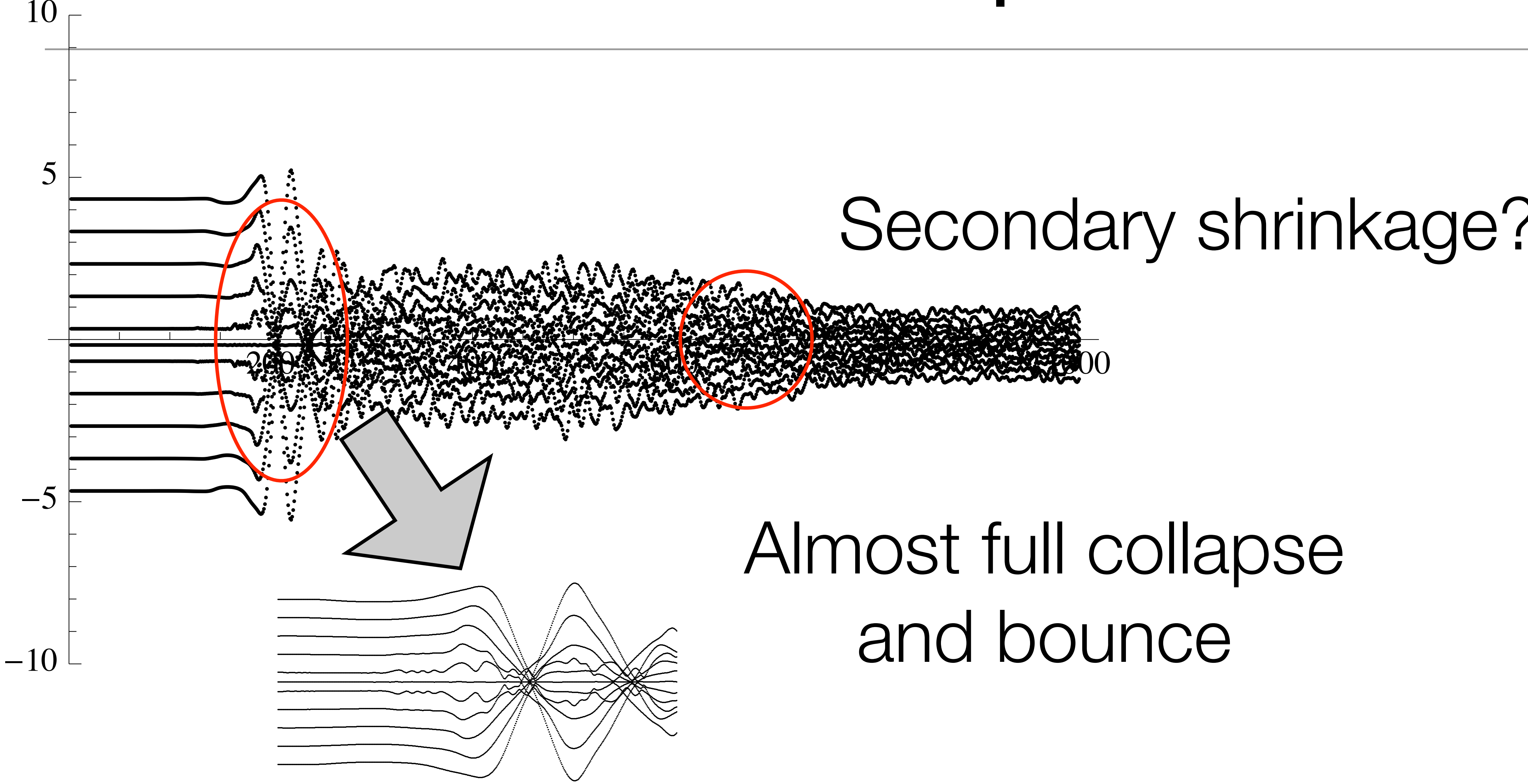
Results:

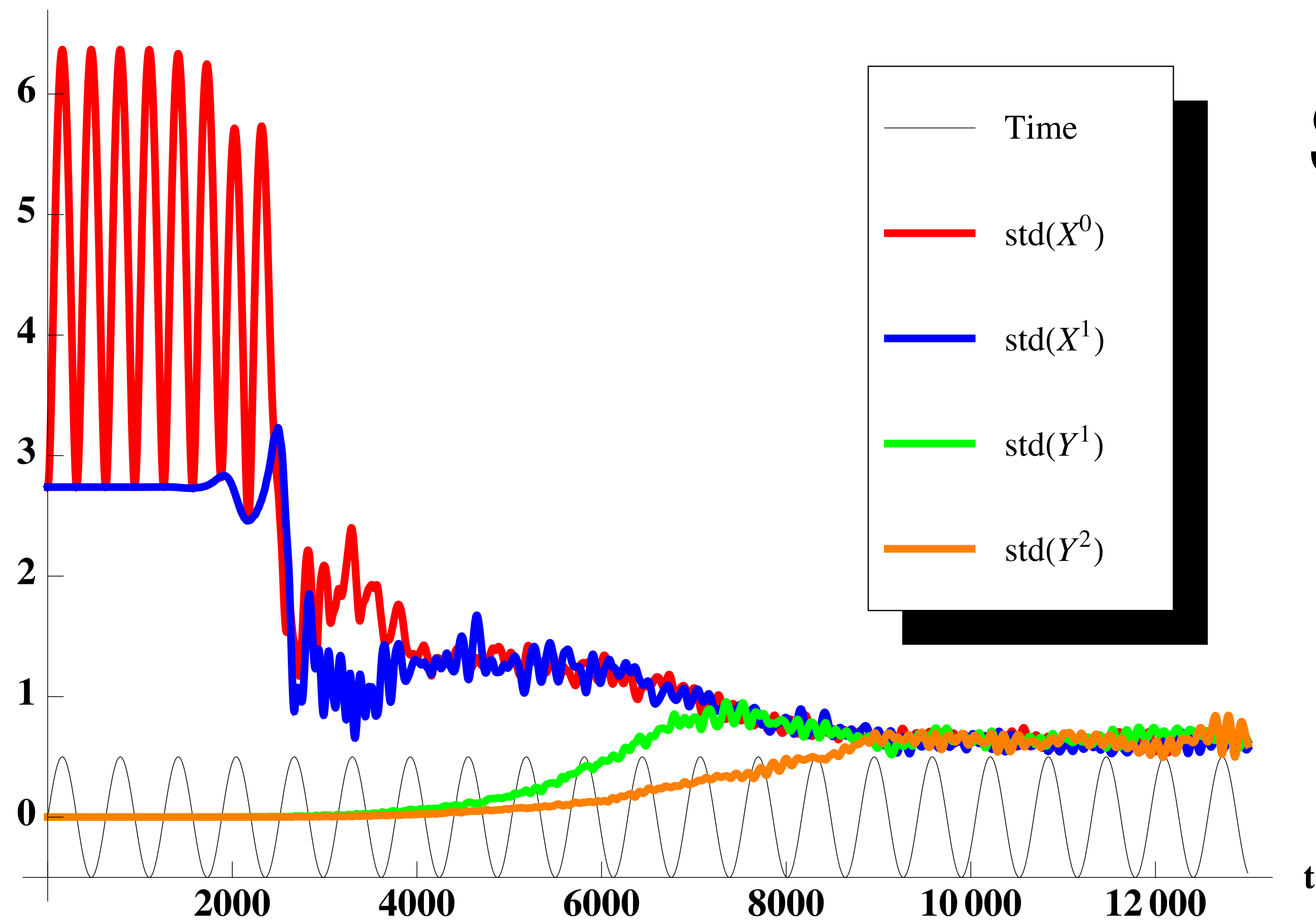
Kick single eigenvalue: tachyon still forms for short time on crossing sphere



Eigenvalue spectrum

Another axis of sphere





Standard deviation of eigenvalues: size.

Trace of X,Y decoupled: serves as physical clock.

Secondary shrinkage is from growth of Y matrices (grow via parametric resonance)

More recent similar results by

Riggins+ Sahakian, **arXiv:1205.3847**

Work in BFSS with collapsing (and bouncing) fuzzy sphere as set of initial conditions.

Thermalization

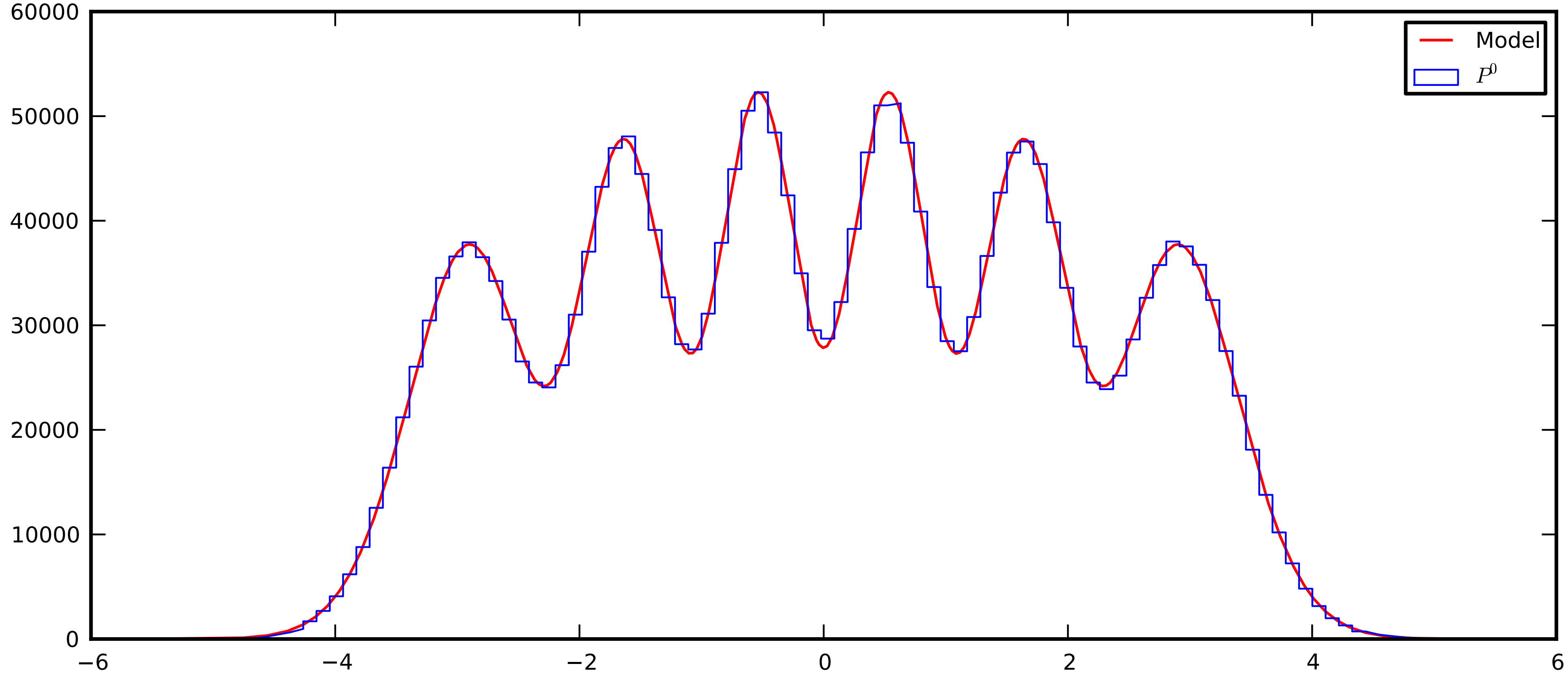
Tests of thermality

$$H \simeq \frac{P^2}{2} + V(X)$$

Thermal implies time averaged distribution of some quantities (momenta) should match the Gibbs ensemble.

$$\mathcal{P}(P) \simeq \exp(-\beta \frac{P^2}{2})$$

This is the standard gaussian matrix model ensemble.



Need to study it with traceless matrices.

The ridges include the finite N exact soln.

Effective temperature is given by second moment.

N	$\langle \mathrm{Tr}(P_0^2)\rangle_0$	$\langle \mathrm{Tr}(P_1^2)\rangle_0$	$\langle \mathrm{Tr}(P_2^2)\rangle_0$	$\langle \mathrm{Tr}(Q_1^2)\rangle_0$	$\langle \mathrm{Tr}(Q_2^2)\rangle_0$	$\langle \mathrm{Tr}(Q_3^2)\rangle_0$	$\langle \mathrm{Tr}(Q_4^2)\rangle_0$	$\langle \mathrm{Tr}(Q_5^2)\rangle_0$	$\langle \mathrm{Tr}(Q_6^2)\rangle_0$
4	23.2 ± 0.6	23.3 ± 0.4	23.2 ± 0.5	21.3 ± 0.5	21.3 ± 0.5	21.2 ± 0.6	21.2 ± 0.4	21.3 ± 0.4	21.0 ± 0.4
11	26.9 ± 0.3	27.2 ± 0.2	27.0 ± 0.3	26.6 ± 0.2	26.5 ± 0.3	26.6 ± 0.2	26.6 ± 0.3	26.6 ± 0.2	26.5 ± 0.2
23	32.2 ± 0.3	32.2 ± 0.2	32.1 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	32.0 ± 0.2

$$T_{effX} \neq T_{effY}?$$

N	$\langle \mathrm{Tr}(P_0^2)\rangle_0$	$\langle \mathrm{Tr}(P_1^2)\rangle_0$	$\langle \mathrm{Tr}(P_2^2)\rangle_0$	$\langle \mathrm{Tr}(Q_1^2)\rangle_0$	$\langle \mathrm{Tr}(Q_2^2)\rangle_0$	$\langle \mathrm{Tr}(Q_3^2)\rangle_0$	$\langle \mathrm{Tr}(Q_4^2)\rangle_0$	$\langle \mathrm{Tr}(Q_5^2)\rangle_0$	$\langle \mathrm{Tr}(Q_6^2)\rangle_0$
4	23.2 ± 0.6	23.3 ± 0.4	23.2 ± 0.5	21.3 ± 0.5	21.3 ± 0.5	21.2 ± 0.6	21.2 ± 0.4	21.3 ± 0.4	21.0 ± 0.4
11	26.9 ± 0.3	27.2 ± 0.2	27.0 ± 0.3	26.6 ± 0.2	26.5 ± 0.3	26.6 ± 0.2	26.6 ± 0.3	26.6 ± 0.2	26.5 ± 0.2
23	32.2 ± 0.3	32.2 ± 0.2	32.1 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	32.0 ± 0.2

$$T_{effX} \neq T_{effY}?$$

Need to be careful: there is a constraint.

$$\delta([X, P_X] + [Y, Q_Y])$$

N	$\langle \mathrm{Tr}(P_0^2) \rangle_0$	$\langle \mathrm{Tr}(P_1^2) \rangle_0$	$\langle \mathrm{Tr}(P_2^2) \rangle_0$	$\langle \mathrm{Tr}(Q_1^2) \rangle_0$	$\langle \mathrm{Tr}(Q_2^2) \rangle_0$	$\langle \mathrm{Tr}(Q_3^2) \rangle_0$	$\langle \mathrm{Tr}(Q_4^2) \rangle_0$	$\langle \mathrm{Tr}(Q_5^2) \rangle_0$	$\langle \mathrm{Tr}(Q_6^2) \rangle_0$
4	23.2 ± 0.6	23.3 ± 0.4	23.2 ± 0.5	21.3 ± 0.5	21.3 ± 0.5	21.2 ± 0.6	21.2 ± 0.4	21.3 ± 0.4	21.0 ± 0.4
11	26.9 ± 0.3	27.2 ± 0.2	27.0 ± 0.3	26.6 ± 0.2	26.5 ± 0.3	26.6 ± 0.2	26.6 ± 0.3	26.6 ± 0.2	26.5 ± 0.2
23	32.2 ± 0.3	32.2 ± 0.2	32.1 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	32.0 ± 0.2

$$T_{effX} \neq T_{effY}?$$

Need to be careful: there is a constraint.

$$\delta([X, P_X] + [Y, Q_Y])$$

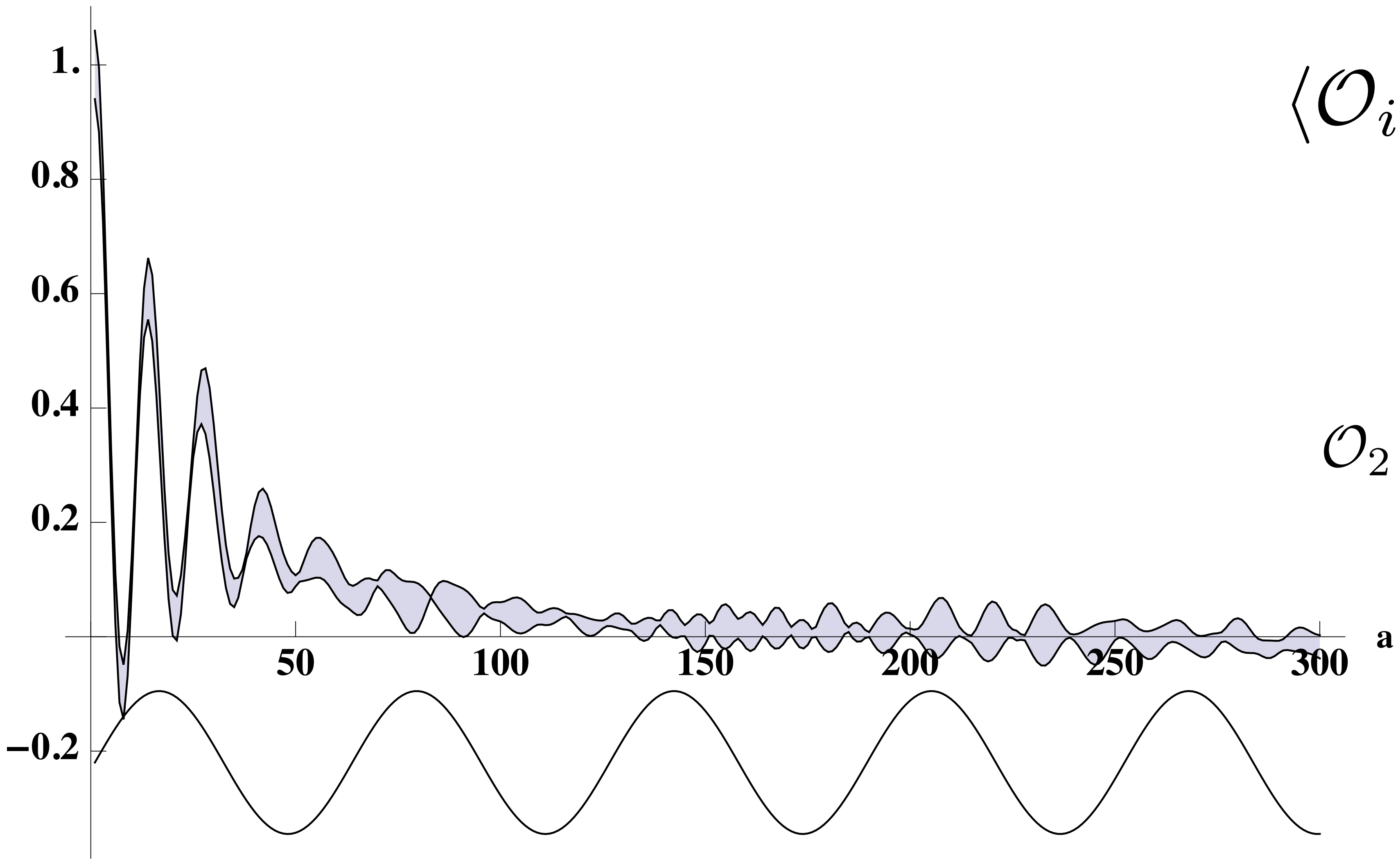
Symmetry breaking of X,Y eom's

$\Rightarrow$ symmetry breaking between X, Y,

$\Rightarrow$ symmetry breaking between P, Q

Fast thermalization?

Test via Normalized Autocorrelation functions



$$\langle \mathcal{O}_i(t)\mathcal{O}_i^\dagger(t+a)\rangle$$

$$\mathcal{O}_2 = \mathrm{tr}[(X^1 + iX^2)^2]$$

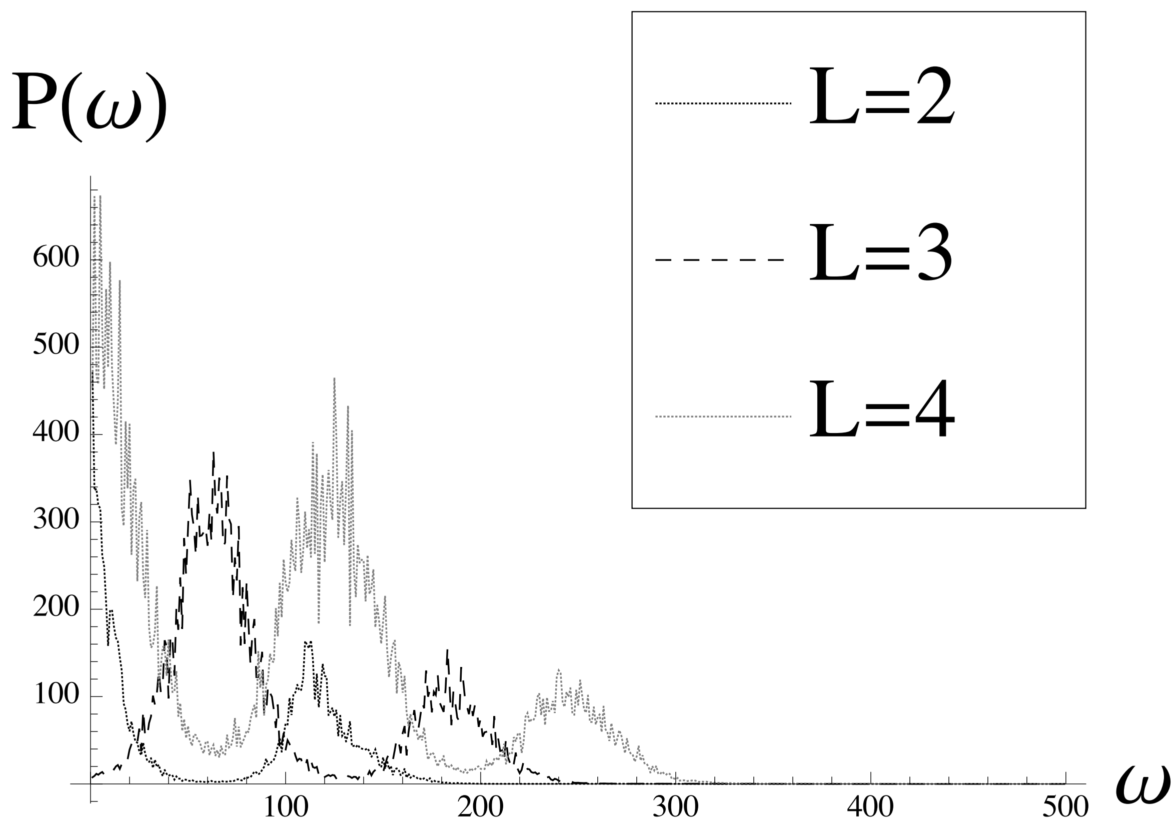
Autocorrelations

Better in Fourier space.

Autocorrelation function is
Fourier transform of power spectrum

$$\langle \mathcal{O}_i(t)\mathcal{O}_i^\dagger(t+a)\rangle = \int dw P(w) \exp(iwa)$$

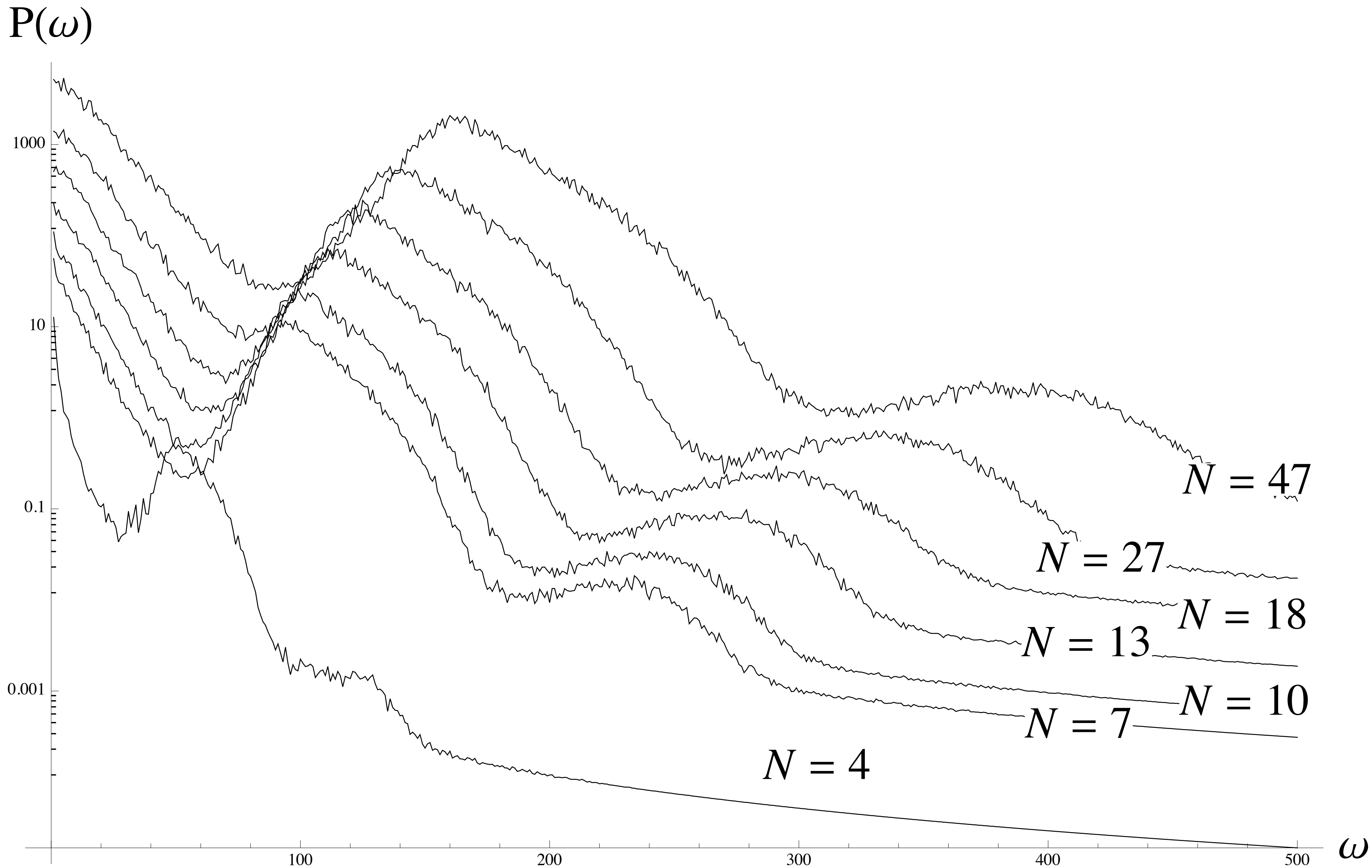
Look at BFSS (classical model with no scale)



Broadband spectrum indicates chaos.

Chaos in dim. reduction of YM is well known since 80's

(Chirikov et al. Mantiyan et al. - In russian)



Can compare different N: neither frequency nor amplitude normalized. Fit to max, and rescale power.

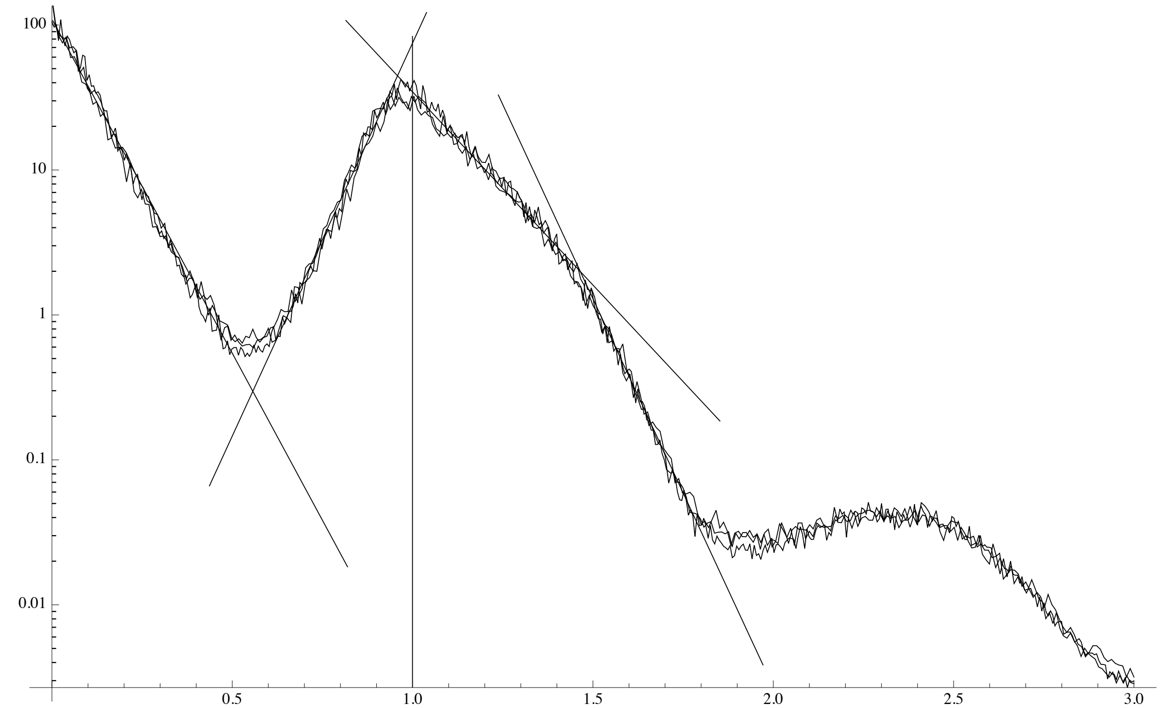


FIG. 8. The power spectrum of $\mathrm{tr}(X^1 + iX^2)^2(t)$ for various sizes of $N \times N$ matrices. The axis of frequency has been rescaled for each N, to the frequency ω_N, and we have also rescaled the power spectrum. The reference frequency for each N is located at 1 in the graph. Results shown for $N = 7, 10, 47$. We also have drawn additional suggestive straight lines superposed on the graph that serve as distinctive features of the power spectrum.

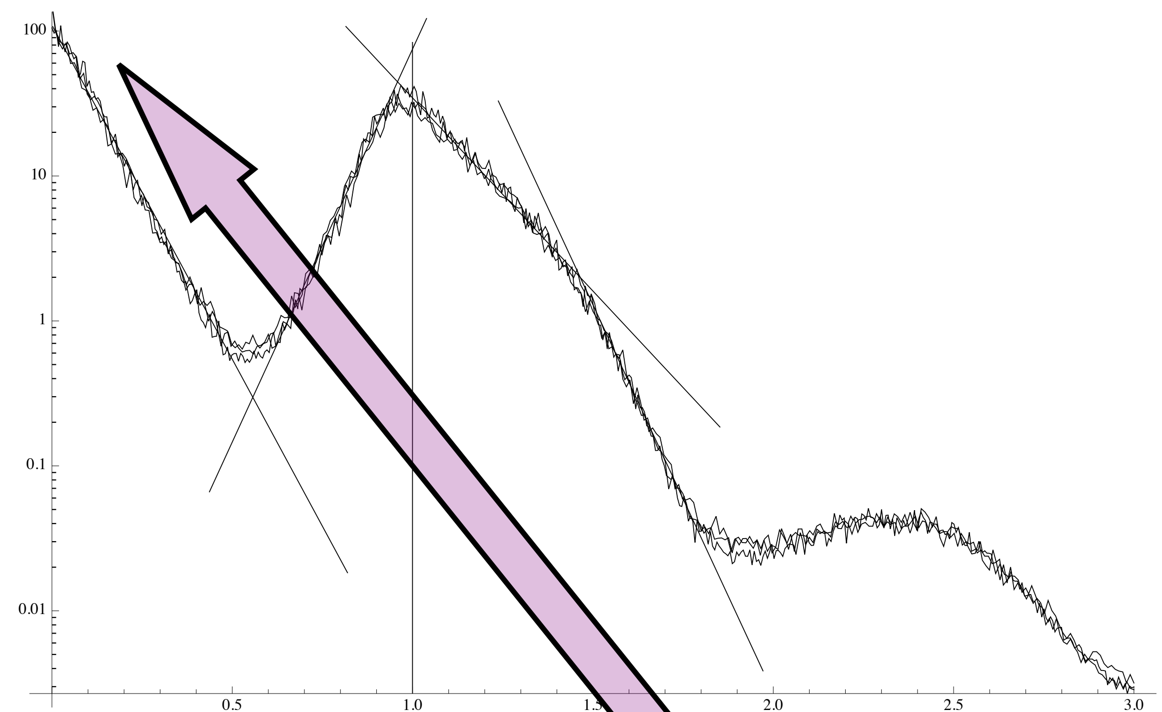


FIG. 8. The power spectrum of $\mathrm{tr}(X^1 + iX^2)^2(t)$ for various sizes of $N \times N$ matrices. The axis of frequency has been rescaled for each N, to the frequency ω_N, and we have also rescaled the power spectrum. The reference frequency for each N is located at 1 in the graph. Results shown for $N = 7, 10, 47$. We also have drawn additional suggestive straight lines superposed on the graph that serve as distinctive features of the power spectrum.

Notice Log spectrum seems to have an
absolute value singularity at 0:
this would imply power
law decays of correlation functions.

Interesting IR

- Power spectrum seems almost singular at zero.
- The log of power spectrum seems to have an absolute value singularity. Such singularity would imply polynomial decay of autocorrelation functions for asymptotically long times.
- Hydrodynamics: collective degrees of freedom whose time dependent autocorrelation functions are N independent.

Matching to black holes?

- Absolute value singularity can be approximated by square root

$$|\omega| \simeq \sqrt{\omega^2 + \epsilon}$$

Matching to black holes?

- Absolute value singularity can be approximated by square root

$$|\omega| \simeq \sqrt{\omega^2 + \epsilon}$$

Branch cuts in the complex plane?

Can also think as closely packed sequence of poles.

Similar to spectrum of quasi-normal modes for black hoels.

Hydro?

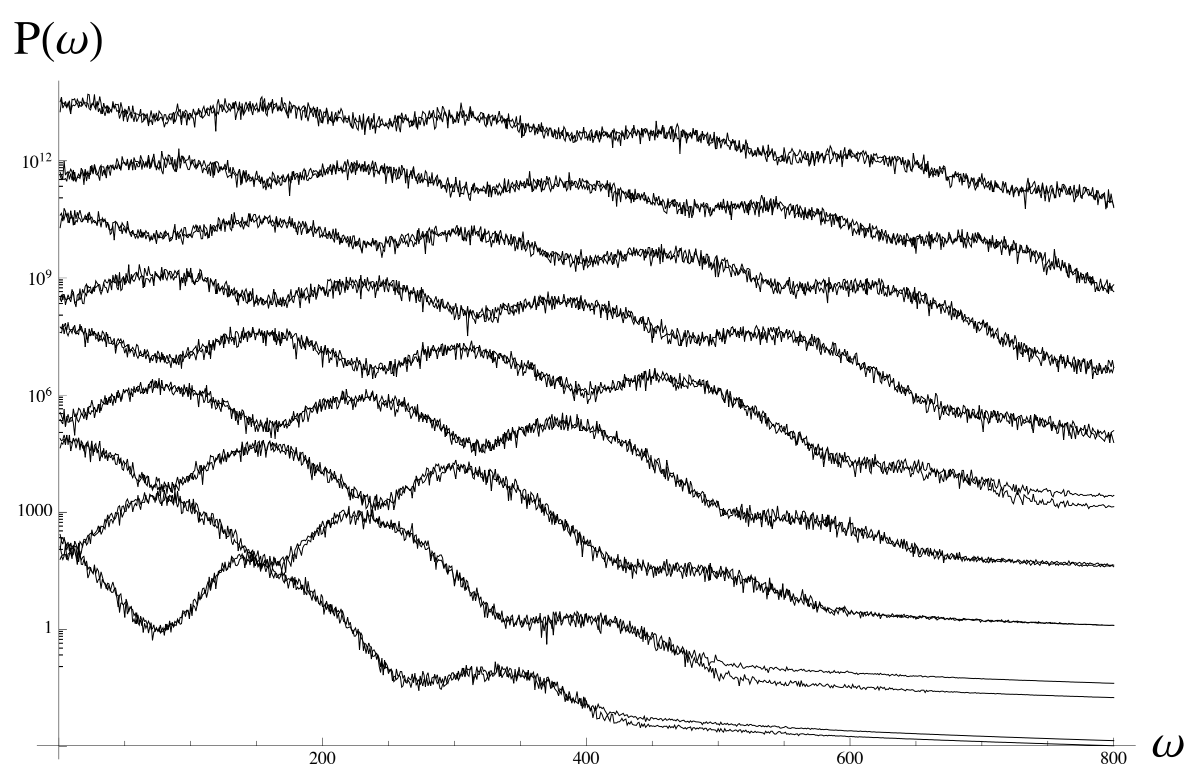
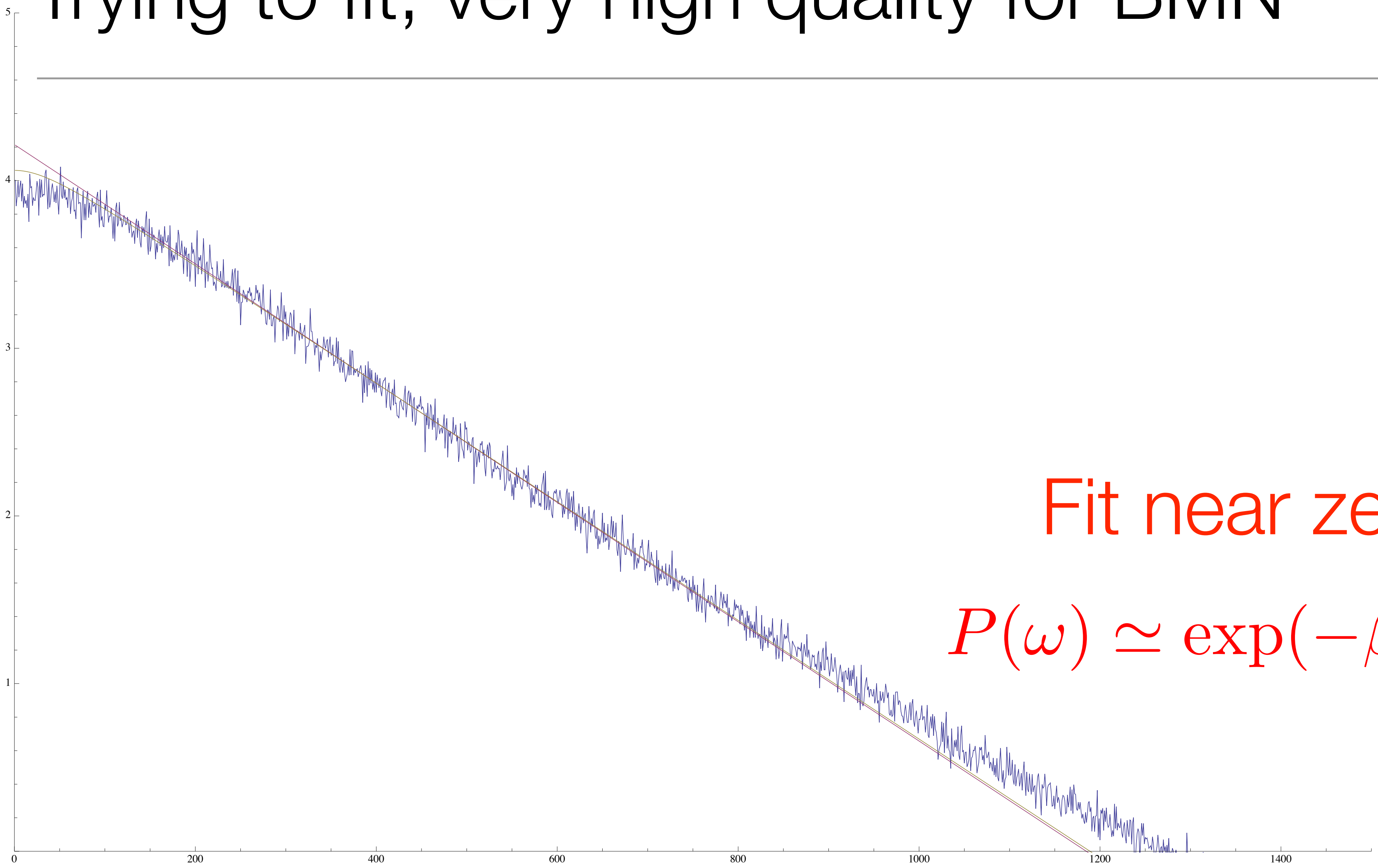


FIG. 10. Power spectrum in arbitrary units for $\mathcal{O}_L$, with $L = 2, \ldots 10$, with values of L increasing from bottom to top in the graph. For each L we show two such sets. This data is from $N = 27$.

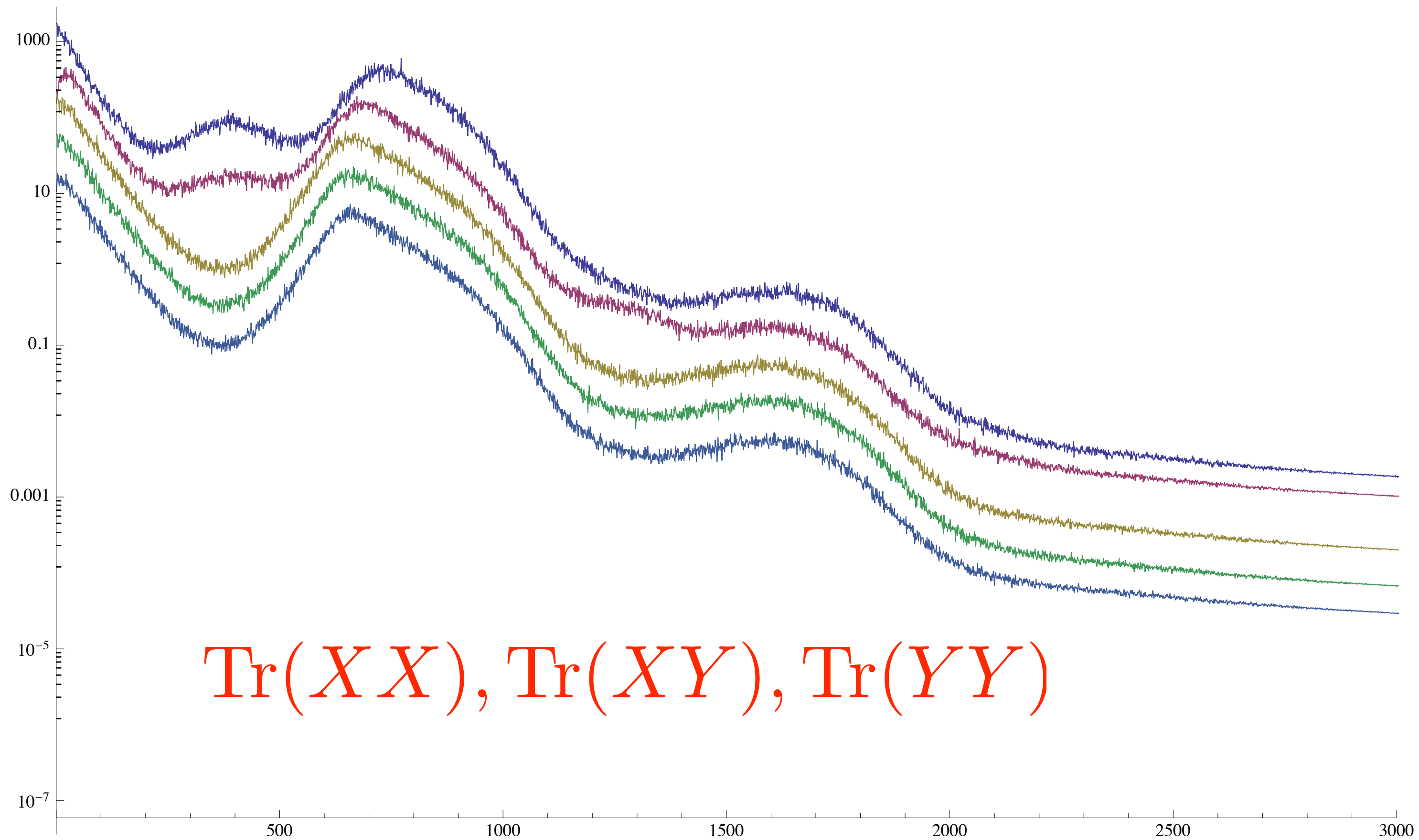
Trying to fit; very high quality for BMN



Fit near zero?

$$P(\omega) \simeq \exp(-\beta\sqrt{a^2+\omega^2})$$

Power spectrum again: BMN



Deformation adds peaks from mixing between modes with different symmetry.

Visualizations: how do we see extra dimensions?

Typical idea of matrix models: add eigenvalue.

One can always make the matrices bigger.

Typical idea of matrix models: add eigenvalue.

One can always make the matrices bigger.

By one (add probe)

Typical idea of matrix models: add eigenvalue.

One can always make the matrices bigger.

By one (add probe)

By direct sum.

Ask about the degrees of freedom connecting the one to the rest.

Typical idea of matrix models: add eigenvalue.

One can always make the matrices bigger.

By one (add probe)

By direct sum.

Ask about the degrees of freedom connecting the one to the rest.

$$\begin{pmatrix} X & * \\ *^\dagger & x \end{pmatrix}$$

Fermion mass matrix

$$\sum_i (X^i - x^i) \otimes \sigma^i$$

What matters is the spectrum of this one matrix (provided by dynamics)

defines a spectral Distance:

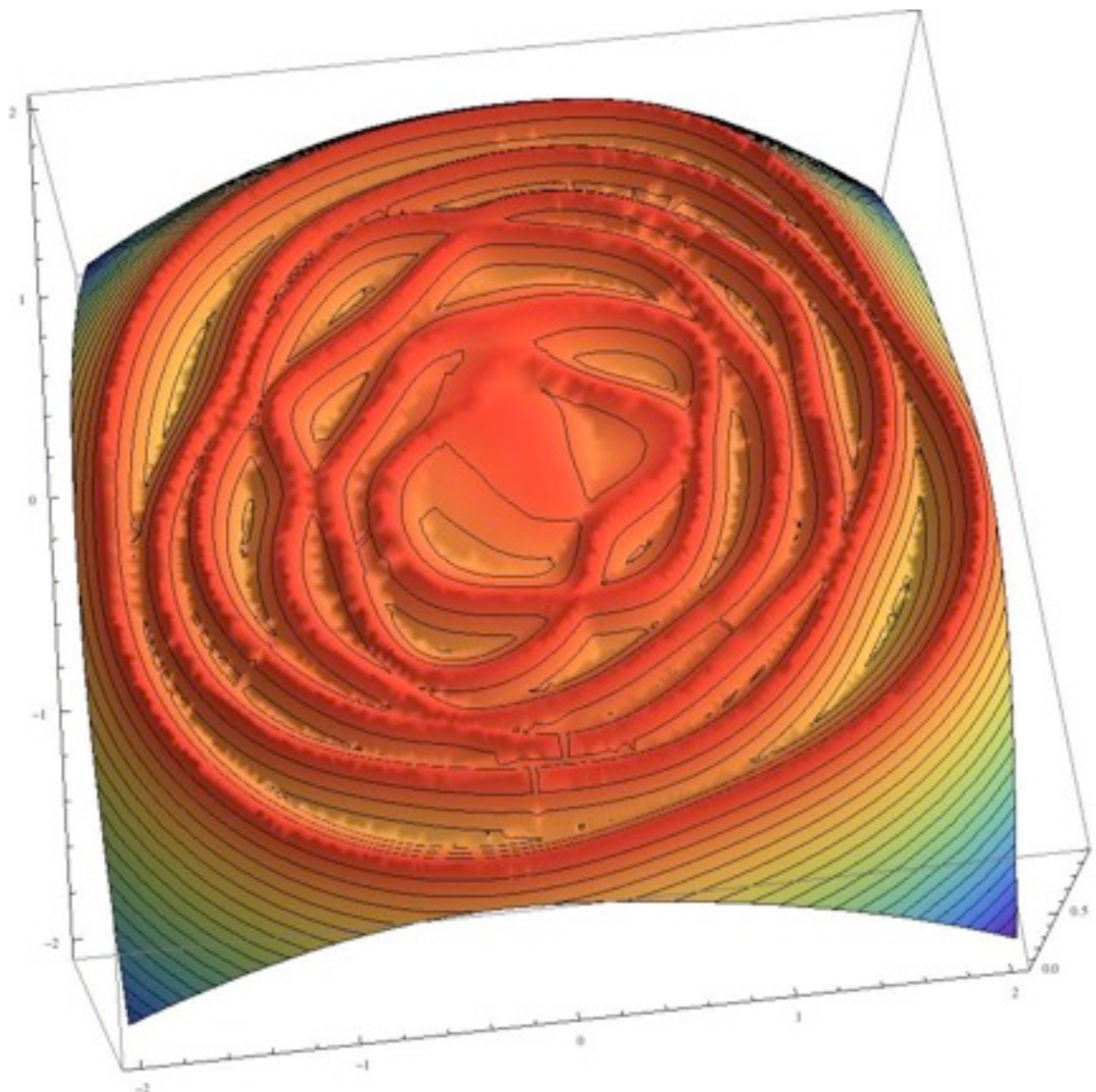
$$d(X,x) \simeq (\min(\mathrm{Abs}(\text{ Eigenvalues})))$$

D.B. + E. Dzienkowski arXiv:1204.2788

A 2D slice colored by spectral distance (21x21 matrices). Along X0X1 plane.

the movie (**Fermovlon.mov** // 38 mb) need to be downloaded separately

$$d(X, x) = \mathrm{Min}(\mathrm{Abs}(\mathrm{Eigenvalues}\ (H_{fer})))$$



High- definition
graph shows a lot of zero
distance surfaces: ridges

brane-antibrane polarization.

The Onion

This is a slice of a true onion:
not computer generated.
This is the image we get of the "inside the black hole"

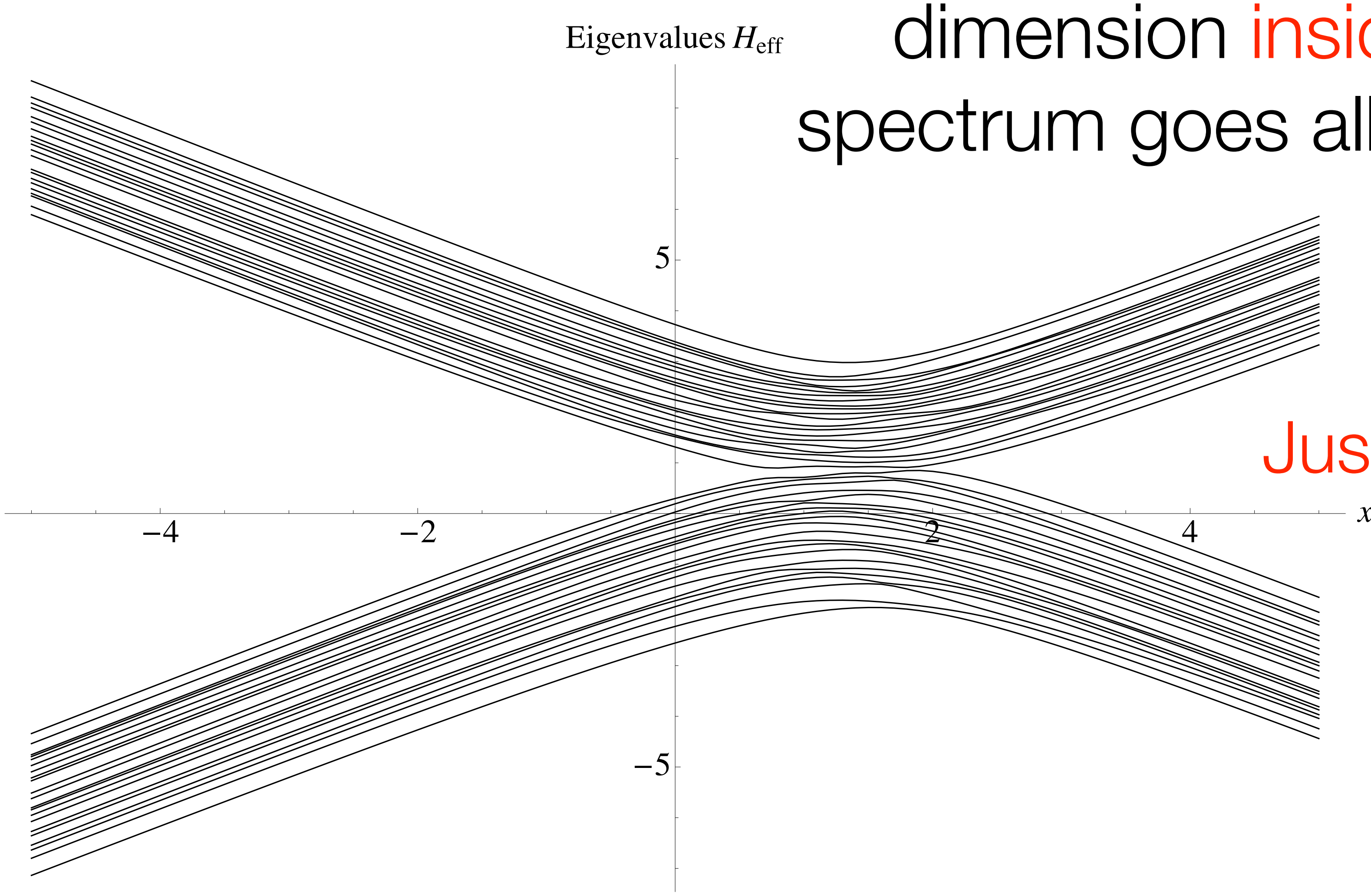
Emergent CFT?

Looking inside the black hole

Look at spectral density of fermionic modes of probe:
defines a notion of dimension

$$\rho(\lambda) \simeq \lambda^d \qquad\qquad D = d + 1$$

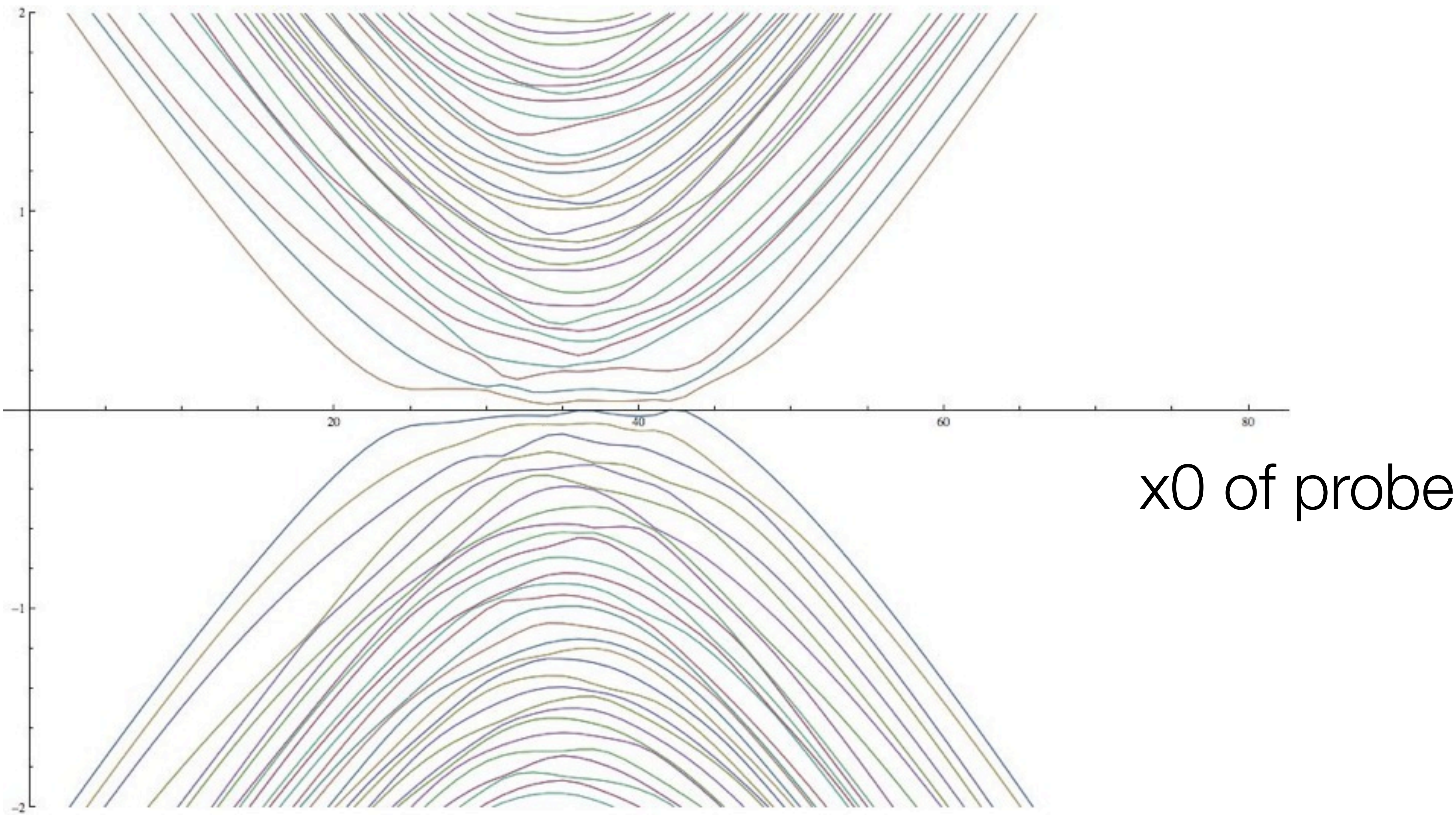
Numerically, the natural notion of dimension inside a BH is 2 (1+1) and spectrum goes all the way to zero (no gap)

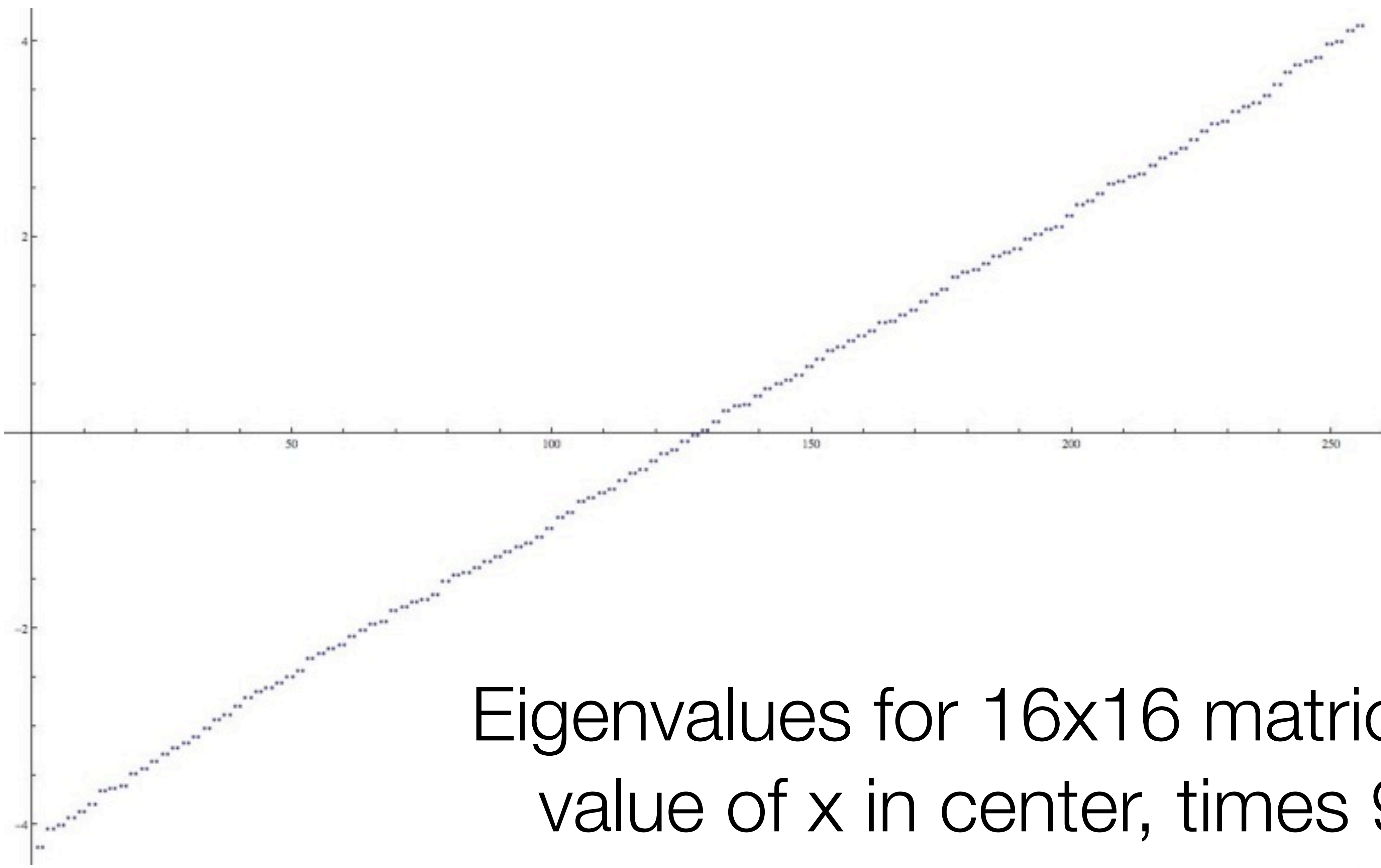


Just with 3x and Pauli matrices

Emergent 2D CFT?

Eigenvalues of OD complete fermion Hamiltonian with 9d gamma matrices 16x16





Eigenvalues for 16x16 matrices, for one value of x in center, times 9 gamma matrices (16x16)

Near linear spectrum is similar near zero to the dof of a 1+1 CFT

This is a spectrum of ‘long strings’: effective central charge goes as 1/N

Conclusions

- Fast thermalization (evidence)
- Interesting pattern of autocorrelations when system thermalizes: chaos + interesting IR
- Suggestive of spectrum of quasi-normal modes. Suggests extra hologrraphic radial direction.
- Emergent 1+1 CFT? (same typical density of states)