1D stellar models, treatment of problematic physics

• Overview of input stellar physics
• The treatment of mixing
  • Convection
  • Rotation
  • Thermohaline mixing and numerics
• What about binaries
Overview of input Stellar Physics

The modeling of stars requires a good description of

- the properties of the **plasma**
  - EOS, opacity
- the **nuclear physics**
  - reaction rates, neutrinos emission, screening factors
- **energy transport**
  - convection
  - radiation (atmosphere, atmosphere)
  - neutrino (for SN explosions)
- **chemical transport**
  - non standard mixing processes
- of the evolution of the mass
  - winds or mass accretion

"The reason why most people in astrophysics do cosmology is because stars are too complicated" (an anonymous Australian fellow)
The equation of state

problems in regions of high density because of non-ideal effects

- very low mass stars
- white dwarfs

effects associated with pressure ionization, Coulomb shielding important

crystallization of the ions, phase separation, plasmon neutrinos → affects age determination
Neutron stars

- All these EOS predict different maximum mass for a neutron star. Observations of NS in binary systems should help discriminate between the various possibilities.
  - Impact on the explosion mechanisms especially for mergers.
Stellar atmosphere

Above the photosphere, diffusive approximation breaks down.

- need for realistic atmosphere models. Strongly affect the location of the star in the HRD, mass loss rate, yields,....
- complex problem because of molecules, dust and surface convection below $\tau < 2/3$
- problematic because $\alpha_{MLT}^{\text{atmos}} \neq \alpha_{MLT}^{\text{interior}}$

Siess 2001, ASPC
Opacities

Problematic in atmosphere of cool stars where molecules are abundant

→ strong impact on the
  ▷ mass loss and thus on
  ▷ the fate of the star (e.g. SAGB)
  ▷ on the yields

New opacity tables or analytical fits are now available to account for the formation of molecules in C-rich stars and should be accounted for

Nuclear reaction rates

- For the energetics and stellar evolution, OK but main uncertainties remain:
  - $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$: sets CO ratio after He burning and impact the nucleosynthesis of heavier elements, e.g. Ne, Na, Mg, and Al (in massive stars)
  - $^{12}\text{C} + ^{12}\text{C}$: crucial to determine the minimum initial mass for a neutron star
  - 3 $\alpha$ reactions: a new rate has been proposed but inconsistent with stellar evolution (no thermal pulse, very short RGB!)

- Highly uncertain for advanced nucleosynthesis, in particular r-process.
- Pycnonuclear reactions: at very high density, for neutron stars mainly, very uncertain

- Neutrino physics
  - the cooling track of WD $\rightarrow$ modify age determination
  - hydrodynamics and nucleosynthesis of SN2 explosion
Energy transport by convection

The MLT rest upon strong assumptions

1. it assumes blobs have all the same size and same velocity
2. that the upward and downward streams are symmetrical (no flux of kinetic energy)
3. all convective elements dissolve after traveling a distance \( \Lambda = \alpha H_p \)
4. it is a local theory (i.e. no overshoot, ...)
5. it is time-independent
6. and of course it is 1D

Despite all these caveats, it works remarkably well. It is able

- to fit the distribution of stars in the HRD (isochrone fitting)
- to account for global surface composition changes (formation of carbon stars, Li depletion, ...
The limits of MLT

1 convection is asymmetric
e.g. solar granulation

Narrow cold sinking plumes vs
extended rising bubbles

Near tip AGB
• convection becomes almost supersonic and
• \( \tau_{\text{conv}} < \tau_{\text{KH}} \rightarrow \text{problem!} \)
$\alpha_{\text{MLT}}$ is NOT constant

Fig. 5. $\alpha_{\text{MLT}}$ for standard mixing-length theory (Böhm-Vitense 1958) with $\Lambda = \alpha_{\text{MLT}} H_P$ ($\Lambda$: mixing-length, $H_P$: local pressure scale height). The presentation of the data is analogous to Fig. 3.

Ludwig et al 1999, 346, 111
Consequences of changing alpha on AGB evolution

Different treatments of convection change envelope temperature
- luminosity $\rightarrow$ mass loss rate $\rightarrow$ evolution!
- if HBB: nucleosynthesis changes CNO cycle and NeNa, MgAl
MLT does not predict overshooting

but it needed to reproduce

- main sequence width
- blue loops
- cluster turn off

- age of binary stars
- s-process in AGB stars, ...


S-process in AGB stars: the need for mixing

- Neutrons produced by $^{12}\text{C}(p,\gamma)^{13}\text{N}(\beta^+)^{13}\text{C}(\alpha,n)$
- Injection of protons from the envelope in the pulse region

Abundance determination in AGB can help constraint overshoot parameter.
Theoretical justification

\[ F_{conv} = \rho v c_P \delta T \]

MLT says

\[ \frac{\delta T}{T} \propto \nabla - \nabla_{ad} \]

\[ \frac{dv}{dt} = -g \left( \frac{\delta \rho}{\rho_e} \right) \]

\[ \text{buoyancy} \]

At the convective boundary:

\[ \nabla = \nabla_{ad} \Rightarrow \delta T = \delta \rho = 0 \Rightarrow \frac{dv}{dt} = 0 \quad \text{NOT} \quad v = 0 \]

So at the edge of the convective zone \( v \neq 0 \)

The inertia of the convective cell is the cause of overshooting
Implementations of overshooting

Most commonly used in stellar evolution code are

- **force mixing** in a region of width \( d_{over} H_P \) beyond the Schwarzschild limit
- use an exponentially decaying diffusion coefficient (Herwig)

\[
D(r > R_{\text{core}}) = D_{\text{conv}}(R_{\text{core}}) \exp \left( \frac{r - R_{\text{core}}}{f_{over} H_P} \right)
\]

**WARNING**: parameterization based on hydro simulation of convection at the surface of A type star

- **Roxburgh's** integral condition (Vandenberg): find the radius where the heat flux by convection becomes zero

\[
\int_0^{r_c} \frac{1}{T} \left[ \frac{dT}{dt} \left( 4\pi r^2 \rho v_s \right) \right] = \int_0^{r_c} (L - L_{\text{rad}}) d \left( \frac{1}{T} \right) = 0
\]

easy to implement BUT overshoot overestimated because viscous dissipation neglected
Neutrality gradient approach

Find the grid point where

$$\nabla_{rad} = \nabla_{ad}$$
Neutrality gradient approach

Find the grid point where

$$\nabla_{\text{rad}} = \nabla_{\text{ad}}$$
Neutrality gradient approach

Find the grid point where

\[ \nabla_{\text{rad}} = \nabla_{\text{ad}} \]
Neutrality gradient approach

Find the grid point where

\[ \nabla_{rad} = \nabla_{ad} \]

gradient neutrality is reached

unstable region
Time Dependent Convection (TDC)

When the evolutionary timescale becomes comparable to the convective timescale $\tau_{\text{conv}}$, MLT breaks down. This happens e.g.

- when studying the interaction between convection and pulsation, i.e. when $P < \tau_{\text{conv}}$
- near the tip of the AGB where $\tau_{\text{KH}} < \tau_{\text{conv}}$
- in evolved stars where $\tau_{\text{nuc}} < \tau_{\text{conv}}$ : convective URCA process
  - in the 1000 yr preceding SNIa explosion, the URCA process is crucial to know the thermal structure of the pre-SNIa model
    → can delay the explosion
  - in the O-Ne core of super AGB stars
**URCA process and URCA shell**

**idea**: EC reactions + neutrinos emission → cooling and pressure drop (electrons absorbed) → core collapse

Cooling is effective in a thin shell where the reactions are balanced: → the URCA shell

\[ ^{20}\text{F},^{20}\text{Ne} \text{ pair} \]

\[ ^{20}\text{F} \rightarrow ^{20}\text{Ne} + e^- + \nu^- \]

\[ ^{20}\text{F} \leftrightarrow ^{20}\text{Ne} \]

\[ e^- \rightarrow e^- + \nu + \nu^- \]

\[ ^{20}\text{Ne} + e^- \rightarrow ^{20}\text{F} + \nu \]

**Convective cell**

**beta decay region**

**URCA shell**

**electron capture region**

**Energy goes away**

**Density drops**

Nuclear energy production

Chemical transport

Convection
Time Dependent Convection

The philosophy of TDC models (Unno, Kuhfuss, Gould, Stellingwerf, Yecko et al. 1998, Bono et al. 1999, Xion et al. 2006, …) lies in 4 main points

1. separation of variables $x = \langle x \rangle + x'$
2. linearization of the Navier-Stokes equations and simplification (anelastic + Boussinesq approximations)
3. the correlations are treated as a function of turbulent energy

$$\mathcal{E}_t = \langle \frac{u'^2}{2} \rangle$$
4. the closure of the system is given by an equation for the evolution of $\mathcal{E}_t$

They account for overshooting, kinetic energy flux but need to introduce additional free parameters
Rotation

Rotation has a strong impact on the mass loss rate

$$\frac{\dot{M}(\Omega)}{\dot{M}(0)} = \frac{(1 - \Gamma)^{\frac{1}{\alpha} - 1}}{[1 - \frac{4}{9}\left(\frac{v}{v_{\text{crit}}}\right)^2 - \Gamma]^{\frac{1}{\alpha} - 1}}$$

$$v_{\text{crit}}^2 = \frac{2GM}{3R_{\text{pol}}} \quad \text{and} \quad v = R\Omega$$

$$\Gamma = \frac{L}{L_{\text{edd}}} = \frac{\kappa L}{4\pi GcM}$$

where $\alpha$ is a force multiplier which depends on the wind theory and Teff

Deformation of the structure

The idea is to decompose the potential in spherical harmonics and average the stellar structure equation over an equipotential.

\[
\Psi(r, \theta) = \frac{GM_{\Psi}}{r} - \frac{4\pi G}{3r^3} P_2(\cos \theta) B + \frac{1}{2} \omega^2 r^2 \sin^2 \theta \left( + \Psi_{tides} \right)
\]

\[
g_{\Psi}(r, \theta) = \left[ \left( \frac{\partial \Psi}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial \Psi}{\partial \theta} \right)^2 \right]^{1/2} \quad \text{and} \quad \langle g \rangle = \frac{1}{S_{\Psi}} \int_{\Psi} g(r, \theta) dS
\]

and \( B \) is function of stellar variables (radau equation).

The stellar structure equations then write

\[
\frac{\partial P}{\partial m} - \frac{GM_{\Psi}}{4\pi r^4_{\Psi}} f_P = 0 \quad \text{and} \quad \frac{d \ln T}{d \ln P} = \min \left[ \nabla_{\text{conv}}, \nabla_{\text{rad}} \frac{f_T}{f_P} \right]
\]

\[
f_P = \frac{4\pi r_{\Psi}^4}{GM_{\Psi} S_{\Psi}} \frac{1}{\langle g^{-1} \rangle} \quad \text{and} \quad f_T = \left( \frac{4\pi r_{\Psi}^2}{S_{\Psi}} \right)^2 \frac{1}{\langle g \rangle \langle g^{-1} \rangle}
\]

With this method, rotating stars have a larger radius and are cooler. The method is valid for slow rotators and for specific rotational laws.
Rotational mixing

- The deformation of the structure → temperature gradient between the pole and equator → induces **meridional currents** → redistribution of
  - heat,
  - composition
  - angular momentum

- The star is differentially rotating → shear → turbulence → mixing

  ➢ Rotation is the source of many instabilities
  ➢ solberg-Hoiland instability
  ➢ dynamical shear instability
  ➢ secular shear instability
  ➢ GSF, ABCS instabilities
  ➢ ....

more details is Ana's talk

Decressin et al 2009
1. Is the process advective vs diffusive?

\[
\rho \frac{d r^2 \Omega}{dt} \bigg|_m = \frac{1}{r^2} \left( \frac{\partial}{\partial r} \right)_t \left[ \rho r^4 D \frac{\partial \Omega}{\partial r} \right] \\
\rho \frac{d r^2 \Omega}{dt} \bigg|_m = \frac{1}{5 r^2} \left( \frac{\partial}{\partial r} \right)_t \left[ \rho r^4 \Omega v_{ES} \right] + \frac{1}{r^2} \left( \frac{\partial}{\partial r} \right)_t \left[ \rho r^4 D_v \frac{\partial \Omega}{\partial r} \right] 
\]

where \( v_{ES} \) is the Eddington Sweet circulation

2. The calculation of the diffusion coefficients is phenomenological and rest upon dimensional arguments

\[ D = \frac{d_{inst}^2}{\tau_{inst}} = v_{inst} \times d_{inst} \]

where \( v_{inst}, d_{inst} \) and \( \tau_{inst} \) are the characteristic velocity, length and time scales of the instability

3. It is also assumed that the total diffusion coefficient is the sum of all contributions which is not correct

4. Shall we use the same diffusion coefficients for chemicals and AM?
Thermohaline instabilities

Occurs e.g. when hot salted water rests on top of cold pure water.

This instability involves two components: a stabilizing one (temperature) that diffuses faster than the other destabilizing one (composition).

In stars, the mean molecular weight $\mu$ replaces the salinity.
Treatment of thermohaline mixing

The instability develops in radiative layers where $\nabla_\mu < 0$

Phenomenologically, the coefficient is given by (Ulrich/Kippenhahn)

$$D_{thl} = K \frac{\phi}{\delta} \frac{-\nabla_\mu}{\nabla_{ad} - \nabla}$$

where $K$ is a form factor
$K \propto \text{(length/diameter of finger)}$
$K \propto 1000$

Source of $\mu$ inversion

- In the H burning shell
  
  $^3\text{He} + ^3\text{He} \rightarrow 2p + ^4\text{He}$
  
  2 particles $\rightarrow$ 3 particles
Impact on the surface abundances

Thermohaline mixing connects the envelope with the HBS

$^7\text{Li}$, $^{12}\text{C}/^{13}\text{C}$, C
In super-AGB stars during carbon burning

- Flame does not reach the center
- Some carbon remains in the core

**Standard case**
- Reaction: \( \text{C} + \text{C} \rightarrow \text{Ne} + \text{O} \) on top of CO
- Mixture → mu-inversion

**With thermohaline mixing**
- The flame does not reach the center
- Some carbon is left in the core
The impact of numerics

Treatment of thermohaline mixing more is more sensitive to numerics because of the explicit dependence of the diffusion coeff. on the composition
Conclusion

- Mass loss: critical for SAGB and massive stars need better description of the dependence of the mass loss on $Z$
- Need for 3D simulations to derive a prescription for mixing
- Treatment of the atmosphere and opacities are crucial for the yields and what about binary stars?

- Situation even more complicated because of
Tidal deformation of the star

Same approach as the Kippenhan one for rotating stars but with the tidal potential included.

Impact on the mass transfer rate

Modification of the standard Roche potential

In the stars are not rotating synchronously and/or the system is eccentric the Roche radius is modified

\[ A = \frac{f^2(1 + e)^4}{(1 + e \cos \nu)^3} \]

\[ f = \frac{\Omega}{\omega_{orb}} \]

\[ \frac{R_{L}^{\text{Egg}}}{a} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} \]

Impact on the mass transfer rate

Critical surface velocity

mass transfer goes along with deposition of angular momentum

Only a small amount of mass needs to be accreted to bring the gainer to Keplerian velocity (Packet 1981)

so what happens next?

is matter accreted?

is the evolution conservative?

is the evolution homogeneous?
The deposition of matter with a different composition can induce additional mixing, e.g. thermohaline mixing.

Also the illumination from the companion star can alter the structure and what about the physics of mass and angular momentum transfer (accretion via a disc, winds or via direct impact), the torques applied to the stars, tides, magnetic fields, ...

So as Achim would say “stick to single stars ...”

Thanks