Book of Abstracts

Order structures, Jordan algebras, and geometry

Lorentz Center

May 29 to June 2, 2017
**Jordan geometries, and their friends**
Wolfgang Bertram
(Université de Lorraine, France)

**Abstract:** I intend to present my research on geometries corresponding to Jordan algebraic structures, and on related concepts, such as associative geometries and, if time permits, I will pay special attention to the ordered case (possible generalizations of formally real Jordan algebras, and their corresponding geometries having a partial cyclic order).

**Order structures, Jordan algebras and geometry**
Cho-Ho Chu
(Queen Mary University of London, United Kingdom)

**Abstract:** We revisit the workshop topics and highlight some unresolved problems.

**Partially ordered vector spaces**
Anke Kalauch
(TU-Dresden, Germany)

**Abstract:** We discuss basic properties of Archimedean partially ordered vector spaces with order units. We present their functional representation, i.e. their order dense embedding into a space of continuous functions on a compact Hausdorff space. Here Riesz homomorphisms and Riesz* homomorphisms play a role, we give some geometric characterization. The functional representation turns out to be a special case of a vector lattice cover of a pre-Riesz space. We apply the order dense embedding to obtain properties of ideals and bands.

**The cone of positive operators, quotient metrics and convex bodies**
Gabriel Larotonda
(Universidad de Buenos Aires and Instituto Argentino de Matemática “Alberto P. Calderón”, Argentina)

**Abstract:** The manifold \( M = GL^+(H) \) of positive invertible operators in a separable complex Hilbert space has two nice properties: geodesics are defined for all time (in fact, they are one-parameter subgroups), and given a cloud of finite points \( A_i \in M \) there exist several ways to define and compute a barycenter of them (e.g. Cartan barycenter, Karcher means; the notion has been lately extended to measures in \( M \), via the Wasserstein distance, see [LawLim17]). These two properties can be thought of as consequences of the completeness of a certain Riemannian metric in \( M \) [La07], but perhaps it is more adequate (and more relevant) to see them as consequences of the fact that the geodesic structure of \( M \) induces what is called a nonpositively curved geometry in a wider (non-necessarily Riemannian) sense as introduced and studied by Alexandrov, Cartan, Busemann, Gromov and Toponogov among others, see [Ba95,Nee02,ConLa10]. In this talk we would like to give some perspective on the idea that \( M \) is an homogeneous space of the general linear group, and in fact its metric can be obtained as a quotient metric in \( GL/U \), where \( U \) is the group of unitary operators. With the nonpositively curved geometry of \( M \) at hand, we will also discuss the existence of a nearest point projection map \( \pi : M \to C \) with nice geometrical features, in particular we will focus on the ideas introduced by Mostow in [Mo55] and those introduced by Porta-Recht in [Pr94].
Local Midpoints on Smooth Manifolds

Jimmie Lawson

(Louisiana State University, USA)

Abstract: We consider three methods for obtaining midpoints, primarily midpoints of geodesics of sprays, but also midpoints of symmetry (in symmetric spaces), and metric midpoints (in Riemannian and Finsler manifolds). We derive general conditions under which these approaches yield the same result. We also derive a version of the Lie-Trotter formula based on the midpoint operation and use it to show that continuous maps preserving (local) midpoints are smooth. Of particular interest is the consideration of these results in the cone of positive elements of a $C^*$-algebra with identity, which is a symmetric space with a canonical Finsler structure for which the Finsler distance metric agrees with the Thompson metric.

Connections between order structures, Jordan algebras, and metric geometry: an overview

Bas Lemmens

(University of Kent, United Kingdom)

Abstract: The famous Koecher-Vinberg theorem characterises the finite dimensional formally real Jordan algebras in terms of symmetric cones. For infinite dimensional formally real Jordan algebras no such characterisation exists, as most of these Jordan algebras are realised as Banach spaces rather than Hilbert spaces. In this talk I will outline a few alternative ways one might be able to characterise the formally real Jordan algebras in terms of the geometry of cones. Besides recalling some important concepts such as Hilbert’s and Thompson’s metrics, which are useful Finsler metrics on cones, I will mention some related open problems and aim to provide some background for some of the other talks in workshop.
Multivariate non-expansive means on symmetric cones
Yongdo Lim
(Sungkyunkwan University, Korea)

Abstract: We are concerned with multivariate non-expansive (resp. contractive) means and an $L^1$-ergodic theorem on a normal cone equipped with the Thompson metric. Based on the important construction schemes of multivariate matrix means of positive definite matrices, namely the proximal average and the Cartan mean (the least squares average) for the Cartan-Hadamard metric, we construct a one parameter family of contractive means interpolating continuously and monotonically the harmonic, arithmetic and Cartan barycenters. A version of Birkhoff ergodic theorem for these contractive means and its extension to symmetric cones are discussed.

Jordan isomorphisms and preserver problems
Lajos Molnar
(University of Szeged, Hungary)

Abstract: In this talk we give a survey of preserver problems whose solutions can be considered as characterizations of Jordan isomorphisms on operator algebras, matrix algebras or on its substructures (unitary groups, positive cones, etc). We will present several classical results and also recent ones.

A Cartan-Hadamard Theorem for Banach-Finsler Manifolds
Karl Hermann Neeb
(FAU Erlangen-Nürnberg, Germany)

Abstract: This talk is based on my paper with the same title. In that paper we study Banach-Finsler manifolds endowed with a spray which have seminegative curvature in the sense that the corresponding exponential function has a surjective expansive differential in every point. In this context we generalize the classical theorem of Cartan-Hadamard, saying that the exponential function is a covering map. We apply this to symmetric spaces and thus obtain criteria for Banach-Lie groups with an involution to have a polar decomposition. Typical examples of symmetric Finsler manifolds with seminegative curvature are bounded symmetric domains and symmetric cones endowed with their natural Finsler structure which in general is not Riemannian.

Introductory talk regarding Jordan algebras and symmetric cones
Takaaki Nomura
(Kyushu University, Japan)

Abstract: This talk presents some of the basic facts about Jordan algebras and symmetric cones. I will also try to include some characterization theorems of symmetric cones among homogeneous open convex cones.
An introduction to JB-algebras
Lina Oliveira
(Universidade de Lisboa, Portugal)

Abstract: This is an introductory talk on the classical theory of JB-algebras, starting from the general setting of Jordan algebras. The selection of topics will include aspects of the theory such as order, projections, spectral theory and duality. The presentation aims to be self-contained.

Lecture 1: Jordan algebras and symmetric cones in finite and infinite dimension
Harald Upmeier
(University of Marburg, Germany)

Abstract: A convex open cone $\Omega$ in a real vector space (either finite-dimensional or a Banach space) is called symmetric if its group $G$ of linear transformations acts transitively, and every point $p \in \Omega$ admits a (non-linear) symmetry $s_p$. A fundamental observation (due to M. Koecher in the finite-dimensional setting) is the fact that symmetric cones are algebraically described in terms of so-called Jordan algebras (euclidean Jordan algebras in finite dimension and JB-algebras in the Banach setting). The first lecture describes this relationship, and its applications to the geometry of the underlying cone and its boundary. A large variety of examples, both in finite and infinite dimension, are presented, as well as an outline of the classification. Passing to the complexification, one obtains the so-called tube domains which generalize the upper half-plane and play a fundamental role in representation theory and number theory.

Lecture 2: Jordan triples and symmetric manifolds in finite and infinite dimension
Harald Upmeier
(University of Marburg, Germany)

Abstract: A bounded domain $D$ in a complex vector space (either finite-dimensional or a Banach space) is called symmetric if its group $G$ of biholomorphic automorphisms acts transitively, and every point $p \in D$ admits a symmetry $s_p \in G$. Generalizing the relationship between symmetric cones and Jordan algebras, the second lecture describes the algebraic description of symmetric domains in terms of the so-called Jordan triple systems. Again, there exist important examples both in finite dimension (classification of hermitian symmetric spaces) and in Banach spaces (e.g. the unit ball of a $C^*$-algebra). Via the Cayley transform, the symmetric cones and tube domains are equivalent to symmetric bounded domains. In addition, we will describe the compact-type dual of symmetric domains, which for example comprise the Grassmann manifolds in finite and infinite dimension.
Symmetric cones and antitone maps
Cormac Walsh
(École Polytechnique, France)

Abstract: Every symmetric cone admits a map that is antitone and homogeneous of degree $-1$, namely the inverse map of the associated Euclidean Jordan algebra. In fact, among finite dimensional cones, the existence of such a map characterises the symmetric cones. I will explain the proof of this result. It involves considering the Funk metric on the cone, which is a non-symmetric metric. Any map with the properties above necessarily reverses this metric. So, by studying the the action of the map on the horofunction boundary of the Funk metric, one can deduce information about the geometry of the cone.

Hilbert and Thompson isometries of cones in JB-algebras
Marten Wortel
(North West University, South Africa)

Abstract: In this talk we will consider the Hilbert and Thompson isometries of cones in JB-algebras. We give an overview of the recent results in this area and explain the link between these isometries and the maximum deviation preserving maps. We then characterize these Hilbert and Thompson isometries and we will explain some of the techniques used in this characterization.