Paul Carter

‘Single and double pulses in the FitzHugh--Nagumo system’

The FitzHugh--Nagumo system is known to exhibit stable, spatially monotone traveling pulses, as well as traveling pulses with oscillatory tails. It has been observed numerically that these pulses can undergo a sharp transition upon numerical continuation, reemerging as double pulses. We describe a construction of this transition from single to double pulses using blow-up techniques and canard theory. We also discuss future work involving similar global constructions of spike-adding transitions in neuronal bursters.

Alan Champneys

"Bistability and the theory of life, rainforests and (almost) everything"

This talk describes work with current PhD students Bert Wuyts and Nicolas Verschuren on reaction-diffusion problems for which the local interaction naturally produces bistability between a low-intensity and a high-intensity state. The first problem, with Bert, concerns data on rainforest/savanna bistability. Here there have been recent studies in high profile journals that suggest a possible climate-induced tipping point leading to desertification of the rainforest. By looking at additional data on closeness to human impact, we show that the data in pristine regions instead displays spatially separated regions of rainforest and savanna, controlled by average rainfall levels. We using reaction-diffusion models with local bistability of tree density together with fire diffusion predicts sharp fronts (Maxwell points) in a reduced form of the model, but there are complexities associated with nonlinear diffusion. The full version of the model is singularly perturbed by weak diffusion. It is not clear if there are additional propensity for localised patterns near the forest/savanna boundary.

The second problem, with Nicolas, concerns how cells develop polarity, the first step to them forming inhomogeneous structures that go to make up the rich diversity of cell types seen on earth. A fundamental mechanism involving small G-proteins in active and inactive forms is revisited and shown to lead to a non-conservative Schnakenberg-like model. It has subcritical Turing patterns which leads to bistability between patterned and nonpatterned states which leads to snaking-like behaviour. The problem has two small parameters - diffusion ratio and size of nonconservative perturbation. either sharp pulses or localised structures. Partial explanation of pulse formation in these limits is available via numerical methods and formal asymptotic calculations. These point to certain subtleties and singularities that appear to be generic features of a class of singularly perturbed reaction diffusion problems.

Paul Cornwall

‘A symplectic view of stability for traveling waves in a FitzHugh-Nagumo system’

The Maslov index is a powerful tool in the stability analysis of nonlinear waves. As a generalization of Sturm-Liouville theory, it provides the ideal result of stability analyses; namely, spectral information is encoded in geometric properties of the wave itself. Although theorems exist relating the Maslov
index to stability, calculating the index is difficult in practice, since one must know half of the solutions to the variational equation along the wave. We tackle this problem in a doubly-diffusive FitzHugh-Nagumo system. Since the solution space of interest is tangent to the unstable manifold of a critical point, the timescale separation allows us to calculate the Maslov index and prove that fast traveling waves are stable.

**Freddy Dumortier**

“Torus canards and slow log-determinant integral”

In view of providing proofs related to the occurrence of torus canards in 3d-vectorfields, we look for technical tools that could play a role. We therefore first recall some tools that were crucial in the study of canard cycles near curves of singularities, emphasizing their intrinsic nature. We then discuss how they can be adapted in an appropriate way.

**Saeed Farjami**

“Computing the Stable Manifold of a Saddle Slow Manifold”

The behavior of systems with fast and slow timescales is organized by families of locally invariant slow manifolds. Recently, numerical methods have been developed for the approximation of attracting and repelling slow manifolds. However, the accurate computation of saddle slow manifolds, which are typical in higher dimensions, is still an active area of research. A saddle slow manifold has associated stable and unstable manifolds that contain both fast and slow dynamics, which makes them challenging to compute. We give a precise definition for the stable manifold of a saddle slow manifold and design an algorithm to compute it; our computational method is formulated as a two-point boundary value problem and uses pseudo-arclength continuation with Auto. We explain how this manifold acts as a separatrix and determines the number of spikes in the transient response generated by a stimulus with fixed amplitude and duration.

**Grégory Faye**

Center Manifolds without a Phase Space

In this talk, we present center manifold theorems that allow one to study the bifurcations of small solutions from a trivial state in systems of functional equations posed on the real line. Despite the nonlocal nature of the problem, we do recover a local differential equation describing the dynamics on the set of small bounded solutions, exploiting that the translation invariance of the original problem induces a flow action on the center manifold. As an illustration, we apply our results to a singularly perturbed problem of slowly varying traveling waves in neural field equations. This is joint work with Arnd Scheel.
Hermen Jan Hupkes

‘Discretization Schemes vs Travelling Waves for Reaction-Diffusion Systems’

We study various temporal and spatial discretization methods for the Nagumo and the FitzHugh-Nagumo equations. In particular, we consider infinite-range spatial discretizations of the Laplacian, adaptive grid methods and full spatial-temporal discretizations using BDF schemes.

Our main goal is to understand in what sense the well-known existence, uniqueness and stability results for travelling fronts and pulses transfer to these discretized settings.

The main focus is on the functional differential operators that arise after linearizing around travelling waves in various well-understood limits. In particular, we discuss how the discretization schemes affect the spectral properties of these operators.

These schemes give rise to highly singular perturbations that we attempt to understand via weak-limit methods based on the pioneering work of Bates, Chen and Chmaj (2003).

[ Joint work with E. van Vleck (U. Kansas) and W. Schouten (U. Leiden) ]

Samuel Jelbart

“Relaxation Oscillation and Canard Behaviour in Slow-Fast Systems Without a Slow/Fast Variable Splitting”

Classical geometric singular perturbation theory (GSPT) is devoted to the study of slow-fast systems for which the scale separation is reflected in a slow/fast variable splitting. In this talk I will show how GSPT can be used to gain insight into the behaviour of some simple slow-fast systems for which there is no (global) slow/fast variable splitting, with particular emphasis on those dynamical aspects which are exclusive to such systems. In particular, relaxation oscillation and canard behaviour which cannot occur in systems exhibiting a slow/fast variable splitting is presented.

Christopher Jones

‘Stability indices and GSP’

One of the great successes of Geometric Singular Perturbation theory (GSP) has been the use of fast/slow structure to compute indices for measuring the extent of instability for nonlinear waves. This has largely been based on the Evans Function. We consider here how the multi-timescale decomposition might afford calculations of other indices that are used, including the Maslov Index and Geometric Phase. Both of these topological/geometric indices have aesthetically pleasing and well developed theories, but they suffer from having few analytic examples. The suggestion made here is that GSP might be leveraged to provide a sold set of applications that can then guide the further theoretical development.
Edgar Knobloch

‘Relaxation oscillations and singular perturbation theory in fluid mechanics’

I will describe examples of relaxation oscillations in fluid mechanics, focusing in particular on binary fluid convection and the dynamics of Faraday waves, that could (should!) be analyzed using singular perturbation theory methods extended to partial differential equations. I will conclude by discussing the sensitivity to noise of convectively unstable systems, and how singular perturbation theory methods applied to a spatial dynamics description of the system provide insight into this phenomenon.

Hinke Osinga

"Slow and fast and global"

Global manifolds are the backbone of a dynamical system and key to the characterisation of its behaviour. They arise in the classical sense of invariant manifolds associated with saddle-type equilibria or periodic orbits and also in the form of finite-time invariant manifolds in systems that evolve on multiple time scales. Dynamical systems theory relies heavily on the knowledge of such manifolds, because of the geometric insight that they can offer into how observed behaviour arises. In applications, global manifolds need to be computed and visualised so that quantitative information about the overall system dynamics can be obtained. This requires accurate numerical methods and a precise understanding of how the computations depend on various model parameters. Numerical methods based on two-point boundary value problem continuation have the major advantage that they remain well posed in parameter regimes where geometric singular perturbation theory does not necessarily apply. We utilise numerical findings in such parameter regimes to identify special events that characterise transitions between evolutions on slow and fast time scales. These techniques are particularly useful when studying changes in the global system dynamics, such as mixed-mode oscillations, global re-injection mechanisms, transient bursting, and phase sensitivity. This talk will focus on two case studies that represent the most recent developments in this area.

Bjorn Sandstede

“Fast waves in discrete and continuous FitzHugh-Nagumo systems”

I will give an overview of results on the existence and stability of fast travelling pulses in the discrete and the continuous FitzHugh-Nagumo systems. For the discrete case, I will report on the existence of stable fast pulses that are constructed by gluing fronts and backs of the discrete Nagumo system together: the main technical challenge is ill-posedness of the associated travelling-wave systems. For the continuous case, I will discuss the existence of fast pulses with spatially oscillatory tails and show that these waves are stable: the stability criterion and the expansion of the critical eigenvalue near the origin turn out to be very different from the case of pulses with spatially monotone tails. These results are joint works with Paul Carter, Hermen Jan Hupkes, and Björn de Rijk.
Arnd Scheel

‘Singular perturbation and pinning’

I will describe perturbation problems arising in the context of front propagation at small speeds, all connected to pinning phenomena in various flavors. Technically, the problems lead to fast-slow structures in nonlocal (forward-backward) dynamical systems, and to singular perturbation problems in the presence of essential spectrum. The small speed typically enters as the small singular parameter.

Frits Veerman

‘Breathing pulses in singularly perturbed reaction-diffusion systems’

Pulse solutions to singularly perturbed reaction-diffusion systems have been numerically observed to display ‘breathing’ behaviour, an oscillatory modulation of the pulse amplitude while the pulse position remains fixed. The singularly perturbed structure of the system, and of the pulse itself, allows us to carry out an explicit weakly nonlinear stability analysis to investigate the existence of such stable ‘breathing’ pulses. The analysis provides explicit expressions that enable direct calculation of the sub- or supercriticality of the associated Hopf bifurcation. The broad applicability of this approach on a general class of reaction-diffusion systems serves as an inspiration to analytically investigate other (multi-)pulse destabilisation scenarios.

Theodore Vo

‘Generic Torus Canards’

Torus canards are solutions of slow/fast systems that alternate between attracting and repelling manifolds of limit cycles of the fast subsystem. Since their discovery in 2008, torus canards have been shown to mediate the transition from rapid spiking to bursting in several paradigm computational neural models. So far, torus canards have only been studied numerically, and their behaviour inferred based on classical averaging and geometric singular perturbation methods. This approach, however, is not rigorously justified since the averaging method breaks down near a fold of periodics - - exactly where the torus canards originate. In this work, we combine techniques from Floquet theory, averaging theory and geometric singular perturbation theory to develop an averaging method for folded manifolds of limit cycles. In so doing, we devise an analytic scheme for the identification and topological classification of torus canards. We demonstrate the predictive power of our results in a model for hormone induced calcium oscillations in liver cells, where we use our torus canard theory to explain the mechanisms that underlie a novel class of bursting rhythms.

Michael Ward

‘Spots, Traps, and Cells: Asymptotic Analysis of Localized Solutions to some Linear and Nonlinear Diffusive Systems’

We will give a broad overview of some mathematical challenges and results for localized patterns arising in three specific classes of singularly perturbed diffusive processes. In the context of linear
diffusive processes, with applications to Brownian particle dynamics, we will discuss some recent results for the Berg-Purcell problem of calculating the effective trapping rate for a spherical target whose surface contains many small absorbing traps. Next, we will consider a new class of PDE-ODE models in 2-D domains whereby localized "cells"
with multi-component intracellular dynamics are coupled together through a bulk diffusion field. The possibility of triggered synchronous oscillations in the cells mediated by the bulk diffusion field, resulting in both quorum-sensing and diffusing-sensing behavior, will be discussed. Finally, we will give some new results for the analysis of localized spot patterns for certain reaction-diffusion systems in 2-D, focusing on a refined linear stability analysis for periodic and multi-spot patterns and the identification of anomalous scaling laws for spot amplitude temporal oscillations. Common features in the analysis of all of these singularly perturbed problems will be emphasized, including the key role of certain matrices involving various Green's functions, and the derivation and study of new classes of discrete variational problems arising from the asymptotic reductions.

Martin Wechselberger

“Two-stroke relaxation oscillators”

In classic van der Pol-type oscillator theory, a relaxation cycle consists of two slow and two fast orbit segments per period (slow-fast-slow-fast). A possible alternative relaxation oscillator type consists of one slow and one fast segment only. Le Corbeiller (1960) termed this type a two-stroke oscillator (compared to the four-stroke vdP oscillator). I will provide examples of such oscillators and discuss these problems from a geometric singular perturbation theory (GSPT) point of view. It is worth mentioning that Fenichel's seminal work on GSPT (JDE 1979) discusses this more general setting, but it has not received much attention in the literature.