

Schiff Moments

J. Engel

May 9, 2017

Connection Between EDMs and T Violation

Consider non-degenerate ground state $|\text{g.s.} : J, M\rangle$. Symmetry under rotations $R_y(\pi)$ for vector operator like $\vec{d} \equiv \sum_i e_i \vec{r}_i$ implies:

$$\langle \text{g.s.} : J, M | d_z | \text{g.s.} : J, M \rangle = - \langle \text{g.s.} : J, -M | d_z | \text{g.s.} : J, -M \rangle .$$

$R^{-1}R$ $R^{-1}R$

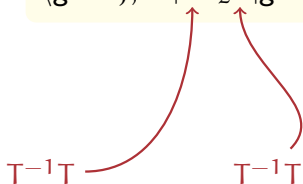
Connection Between EDMs and T Violation

Consider non-degenerate ground state $|g.s. : J, M\rangle$. Symmetry under rotations $R_y(\pi)$ for vector operator like $\vec{d} \equiv \sum_i e_i \vec{r}_i$ implies:

$$\langle g.s. : J, M | d_z | g.s. : J, M \rangle = - \langle g.s. : J, -M | d_z | g.s. : J, -M \rangle .$$

T takes M to $-M$, like $R_y(\pi)$. But \vec{d} is *odd* under $R_y(\pi)$ and *even* under T, so for T conserved

$$\langle g.s. : J, M | d_z | g.s. : J, M \rangle = + \langle g.s. : J, -M | d_z | g.s. : J, -M \rangle .$$



Connection Between EDMs and T Violation

Consider non-degenerate ground state $|g.s. : J, M\rangle$. Symmetry under rotations $R_y(\pi)$ for vector operator like $\vec{d} \equiv \sum_i e_i \vec{r}_i$ implies:

$$\langle g.s. : J, M | d_z | g.s. : J, M \rangle = - \langle g.s. : J, -M | d_z | g.s. : J, -M \rangle .$$

T takes M to $-M$, like $R_y(\pi)$. But \vec{d} is *odd* under $R_y(\pi)$ and *even* under T, so for T conserved

$$\langle g.s. : J, M | d_z | g.s. : J, M \rangle = + \langle g.s. : J, -M | d_z | g.s. : J, -M \rangle .$$

Together with the first equation, this implies

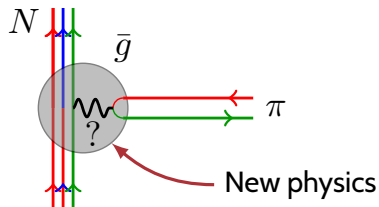
$$\langle d_z \rangle = 0 .$$

If T is violated, argument fails because T takes $|g : JM\rangle$ to states with $J, -M$, but *different energy*.

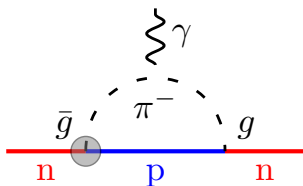
One Way Things Get EDMs

Starting at fundamental level and working up:

Underlying fundamental theory generates three T-violating πNN vertices in chiral PT:

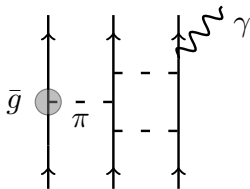


Then neutron gets EDM from chiral-PT diagrams like this:



How Diamagnetic Atoms Get EDMs

Nucleus gets one from nucleon EDM
and **T-violating NN interaction**:



$$V_{\text{PT}} \propto \left\{ \left[\bar{g}_0 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \frac{\bar{g}_1}{2} (\tau_1^z + \tau_2^z) + \bar{g}_2 (3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \right] (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) - \frac{\bar{g}_1}{2} (\tau_1^z - \tau_2^z) (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \right\} \cdot (\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2) \frac{\exp(-m_\pi |\mathbf{r}_1 - \mathbf{r}_2|)}{m_\pi |\mathbf{r}_1 - \mathbf{r}_2|}$$

+ contact term

Finally, atom gets one from nucleus. Electronic shielding makes relevant nuclear object the “Schiff moment” $\langle S \rangle \approx \langle \sum_p r_p^2 z_p + \dots \rangle$.

Job of nuclear theory: calculate dependence of $\langle S \rangle$ on the \bar{g}_i (and on the contact term and nucleon EDM).

How Does Shielding Work?

Theorem (Schiff)

The nuclear dipole moment causes the atomic electrons to rearrange themselves so that they develop a dipole moment opposite that of the nucleus. In the limit of nonrelativistic electrons and a point nucleus the electrons' dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes.

How Does Shielding Work?

Proof

Consider atom with non-relativistic constituents (with dipole moments \vec{d}_k) held together by electrostatic forces. The atom has a "bare" edm $\vec{d} \equiv \sum_k \vec{d}_k$ and a Hamiltonian

$$H = \sum_k \frac{p_k^2}{2m_k} + \sum_k V(\vec{r}_k) - \sum_k \vec{d}_k \cdot \vec{E}_k$$

$$= H_0 + \sum_k (1/e_k) \vec{d}_k \cdot \vec{\nabla} V(\vec{r}_k)$$

$$= H_0 + i \sum_k (1/e_k) [\vec{d}_k \cdot \vec{p}_k, H_0]$$

K.E. + Coulomb

dipole perturbation

How Does Shielding Work?

The perturbing Hamiltonian

$$H_d = i \sum_{\mathbf{k}} (1/e_{\mathbf{k}}) \left[\vec{\mathbf{d}}_{\mathbf{k}} \cdot \vec{\mathbf{p}}_{\mathbf{k}}, H_0 \right]$$

shifts the ground state $|0\rangle$ to

$$\begin{aligned} |\tilde{0}\rangle &= |0\rangle + \sum_{\mathbf{m}} \frac{|\mathbf{m}\rangle \langle \mathbf{m}| H_d |0\rangle}{E_0 - E_{\mathbf{m}}} \\ &= |0\rangle + \sum_{\mathbf{m}} \frac{|\mathbf{m}\rangle \langle \mathbf{m}| i \sum_{\mathbf{k}} (1/e_{\mathbf{k}}) \vec{\mathbf{d}}_{\mathbf{k}} \cdot \vec{\mathbf{p}}_{\mathbf{k}} |0\rangle (E_0 - E_{\mathbf{m}})}{E_0 - E_{\mathbf{m}}} \\ &= \left(1 + i \sum_{\mathbf{k}} (1/e_{\mathbf{k}}) \vec{\mathbf{d}}_{\mathbf{k}} \cdot \vec{\mathbf{p}}_{\mathbf{k}} \right) |0\rangle \end{aligned}$$

How Does Shielding Work?

The induced dipole moment \vec{d}' is

$$\begin{aligned}\vec{d}' &= \langle \tilde{0} | \sum_j e_j \vec{r}_j | \tilde{0} \rangle \\ &= \langle 0 | \left(1 - i \sum_k (1/e_k) \vec{d}_k \cdot \vec{p}_k \right) \left(\sum_j e_j \vec{r}_j \right) \\ &\quad \times \left(1 + i \sum_k (1/e_k) \vec{d}_k \cdot \vec{p}_k \right) | 0 \rangle \\ &= i \langle 0 | \left[\sum_j e_j \vec{r}_j, \sum_k (1/e_k) \vec{d}_k \cdot \vec{p}_k \right] | 0 \rangle \\ &= - \langle 0 | \sum_k \vec{d}_k | 0 \rangle = - \sum_k \vec{d}_k \\ &= - \vec{d}\end{aligned}$$

So the net EDM is zero!

Recovering from Shielding

The nucleus has finite size. Shielding is not complete, and nuclear T violation can still induce atomic EDM D_A .

Post-screening nucleus-electron interaction proportional to Schiff moment:

$$\langle S \rangle \equiv \left\langle \sum_p e_p \left(r_p^2 - \frac{5}{3} \langle R_{\text{ch}}^2 \rangle \right) z_p \right\rangle + \dots$$

If, as you'd expect, $\langle S \rangle \approx R_{\text{Nuc}}^2 \langle D_{\text{Nuc}} \rangle$, then D_A is down from $\langle D_{\text{Nuc}} \rangle$ by

$$O(R_{\text{Nuc}}^2/R_A^2) \approx 10^{-8}.$$

Fortunately, the large nuclear charge and relativistic wave functions offset this factor by $10Z^2 \approx 10^5$.

Overall suppression of D_A is only about 10^{-3} .

Theory for Heavy Nuclei

$\langle S \rangle$ largest for large Z , so experiments are in heavy nuclei.

Ab initio methods are making rapid progress, but

- ▶ Interaction (from chiral EFT) has problems beyond $A = 50$.
- ▶ Many-body methods not quite ready to tackle soft nuclei such as ^{199}Hg , or even those with rigid deformation such as ^{225}Ra .

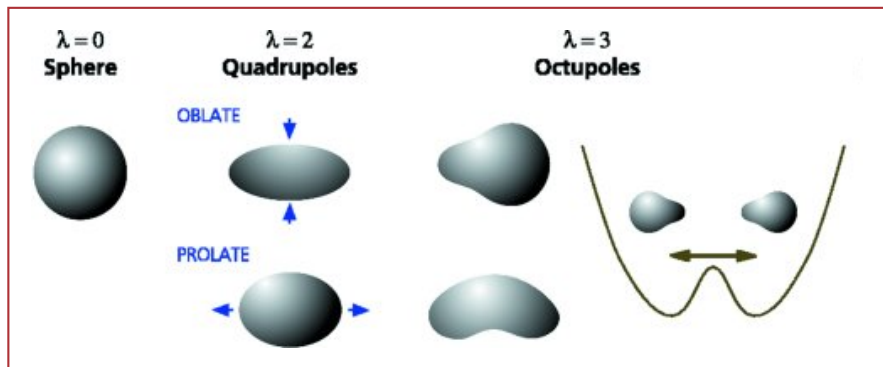
so

for now we must rely on **nuclear density-functional theory**:

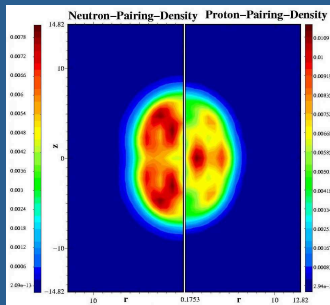
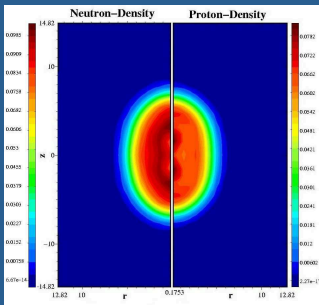
mean-field theory with phenomenological “density-dependent interactions” (Skyrme, Gogny, or successors) **plus** corrections, e.g.:

- ▶ projection of deformed wave functions onto states with good particle number, angular momentum
- ▶ inclusion of small-amplitude zero-point motion (RPA)
- ▶ mixing of mean fields with different character (GCM)
- ▶ ...

Nuclear Deformation



Zr-102: normal density and pairing density
 HFB, 2-D lattice, SLy4 + volume pairing
 Ref: Artur Blazkiewicz, Vanderbilt, Ph.D. thesis (2005)



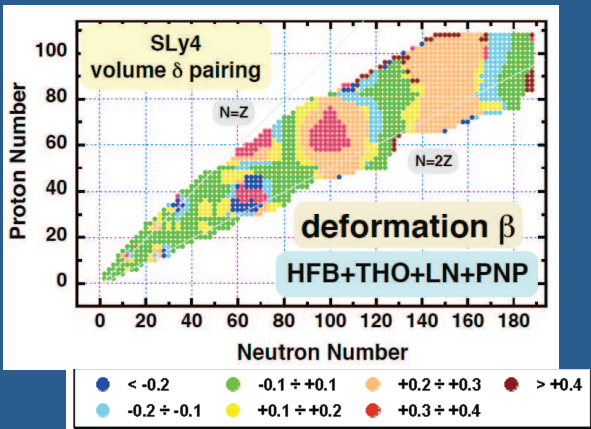
HFB: $\beta_2^{(p)}=0.43$

exp: $\beta_2^{(p)}=0.42(5)$, J.K. Hwang et al., Phys. Rev. C (2006)

Applied Everywhere

Nuclear ground state deformations (2-D HFB)

Ref: Dobaczewski, Stoitsov & Nazarewicz (2004) arXiv:nucl-th/0404077



Varieties of “Recent” Schiff-Moment Calculations

Need to calculate

$$\langle S \rangle \approx \sum_m \frac{\langle 0 | S | m \rangle \langle m | V_{PT} | 0 \rangle}{E_0 - E_m} + \text{c.c.}$$

where $H = H_{\text{strong}} + V_{PT}$.

- ▶ H_{strong} represented either by Skyrme density functional or by simpler effective interaction, treated on top of separate mean field.
- ▶ V_{PT} either included nonperturbatively or via the explicit sum over intermediate states above.
- ▶ Nucleus either forced artificially to be spherical or allowed to deform.

¹⁹⁹Hg via Explicit RPA in Spherical Mean Field

1. Skyrme HFB (mean-field theory with pairing) in ¹⁹⁸Hg.
 2. Polarization of core by last neutron and action of V_{PT} , treated as explicit corrections in quasiparticle RPA, which sums over intermediate states.
-

$$\langle S \rangle_{\text{Hg}} \equiv \alpha_0 g\bar{g}_0 + \alpha_1 g\bar{g}_1 + \alpha_2 g\bar{g}_2 \quad (\text{e fm}^3)$$

	α_0	α_1	α_2
SkM*	0.009	0.070	0.022
SkP	0.002	0.065	0.011
SIII	0.010	0.057	0.025
SLy4	0.003	0.090	0.013
SkO'	0.010	0.074	0.018
Dmitriev & Senkov RPA	0.0004	0.055	0.009

Range of variation here doesn't look too bad. But these calculations are not the end of the story...

Deformation and Angular-Momentum Restoration

If deformed state $|\Psi_K\rangle$ has good intr. $J_z = K$, one averages over angles to get:

$$|J, M\rangle = \frac{2J+1}{8\pi^2} \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) |\Psi_K\rangle$$

Matrix elements (with more detailed notation):

$$\langle J, M | S_m | J', M' \rangle \propto \iint \sum_n d\Omega d\Omega' \times (\text{some D-functions}) \\ \times \langle \Psi_K | R^{-1}(\Omega') S_n R(\Omega) | \Psi_K \rangle$$

$$\xrightarrow[\Omega \approx \Omega']{\text{rigid defm.}} (\text{Geometric factor}) \times \underbrace{\langle \Psi_K | S_z | \Psi_K \rangle}_{\langle S \rangle_{\text{intr.}}}$$

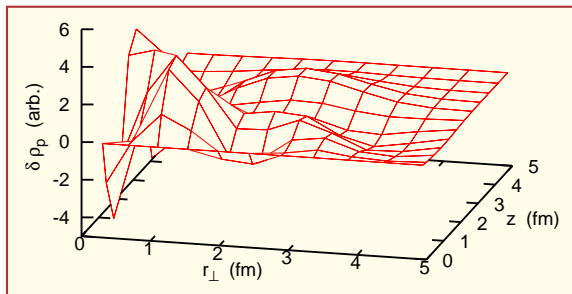
For expectation value in $J = \frac{1}{2}$ state:

$$\langle S \rangle = \langle S_z \rangle_{J=\frac{1}{2}, M=\frac{1}{2}} \implies \begin{cases} \langle S \rangle_{\text{intr.}} & \text{spherical nucleus} \\ \frac{1}{3} \langle S \rangle_{\text{intr.}} & \text{rigidly deformed nucleus} \end{cases}$$

Exact answer somewhere in between.

Deformed Mean-Field Calculation Directly in ^{199}Hg

Deformation actually small and soft – perhaps worst case scenario for mean-field. But in heavy odd nuclei, that's the best that has been done¹. V_{PT} included nonperturbatively and calculation done in one step. Includes more physics than RPA (deformation), plus economy of approach. Otherwise should be more or less equivalent.



Oscillating PT-odd density distribution indicates delicate Schiff moment.

¹Has some “issues”: doesn't get ground-state spin correct, limited for now to axially-symmetric minima, which are sometimes a little unstable, true minimum probably not axially symmetric ...

Results of “Direct” Calculation

Like before, use a number of Skyrme functionals:

		E_{gs}	β	$E_{exc.}$	α_0	α_1	α_2
SLy4	HF	-1561.42	-0.13	0.97	0.013	-0.006	0.022
SIII	HF	-1562.63	-0.11	0	0.012	0.005	0.016
SV	HF	-1556.43	-0.11	0.68	0.009	-0.0001	0.016
SLy4	HFB	-1560.21	-0.10	0.83	0.013	-0.006	0.024
SkM*	HFB	-1564.03	0	0.82	0.041	-0.027	0.069
Fav. RPA	QRPA	-	-	-	0.010	0.074	0.018

Hmm...

What to Do About Discrepancy

- ▶ Revisit/recheck existing calculations.
- ▶ Improve treatment further:
 - ▶ Variation after projection
 - ▶ Triaxial deformation

Ultimate goal: mixing of many mean fields, aka “generator coordinates”

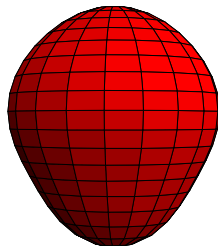
Still a ways off because of difficulties marrying generator coordinates to density functionals.

Schiff Moment with Octupole Deformation

Here we treat always V_{PT} as explicit perturbation:

$$\langle S \rangle = \sum_m \frac{\langle 0 | S | m \rangle \langle m | V_{PT} | 0 \rangle}{E_0 - E_m} + \text{c.c.}$$

where $|0\rangle$ is unperturbed ground state.



Calculated ^{225}Ra density

Ground state has nearly-degenerate partner $|\bar{0}\rangle$ with same opposite parity and same intrinsic structure, so:

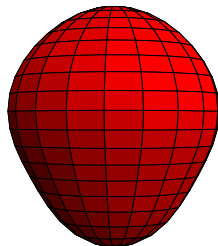
$$\langle S \rangle \longrightarrow \frac{\langle 0 | S | \bar{0} \rangle \langle \bar{0} | V_{PT} | 0 \rangle}{E_0 - E_{\bar{0}}} + \text{c.c.}$$

Schiff Moment with Octupole Deformation

Here we treat always V_{PT} as explicit perturbation:

$$\langle S \rangle = \sum_m \frac{\langle 0 | S | m \rangle \langle m | V_{PT} | 0 \rangle}{E_0 - E_m} + \text{c.c.}$$

where $|0\rangle$ is unperturbed ground state.



Calculated ^{225}Ra density

Ground state has nearly-degenerate partner $|\bar{0}\rangle$ with same opposite parity and same intrinsic structure, so:

$$\langle S \rangle \longrightarrow \frac{\langle 0 | S | \bar{0} \rangle \langle \bar{0} | V_{PT} | 0 \rangle}{E_0 - E_{\bar{0}}} + \text{c.c.} \propto \frac{\langle S \rangle_{\text{intr.}} \langle V_{PT} \rangle_{\text{intr.}}}{E_0 - E_{\bar{0}}}$$

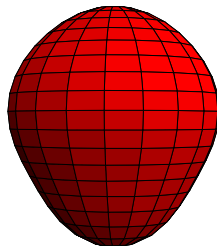
Why is this? See next slide.

Schiff Moment with Octupole Deformation

Here we treat always V_{PT} as explicit perturbation:

$$\langle S \rangle = \sum_m \frac{\langle 0 | S | m \rangle \langle m | V_{PT} | 0 \rangle}{E_0 - E_m} + \text{c.c.}$$

where $|0\rangle$ is unperturbed ground state.



Calculated ^{225}Ra density

Ground state has nearly-degenerate partner $|\bar{0}\rangle$ with same opposite parity and same intrinsic structure, so:

$$\langle S \rangle \longrightarrow \frac{\langle 0 | S | \bar{0} \rangle \langle \bar{0} | V_{PT} | 0 \rangle}{E_0 - E_{\bar{0}}} + \text{c.c.} \propto \frac{\langle S \rangle_{\text{intr.}} \langle V_{PT} \rangle_{\text{intr.}}}{E_0 - E_{\bar{0}}}$$

Why is this? See next slide.

$\langle S \rangle$ is large because $\langle S \rangle_{\text{intr.}}$ is collective and $E_0 - E_{\bar{0}}$ is small.

A Little on Parity Doublets

When intrinsic state $|\bullet\rangle$ is asymmetric, it breaks parity.

In the same way we get good J, we average over orientations to get states with good parity:

$$|\pm\rangle = \frac{1}{\sqrt{2}} (|\bullet\rangle \pm |\bullet\rangle)$$

These are nearly degenerate if deformation is rigid. So with $|0\rangle = |+\rangle$ and $|\bar{0}\rangle = |-\rangle$, we get

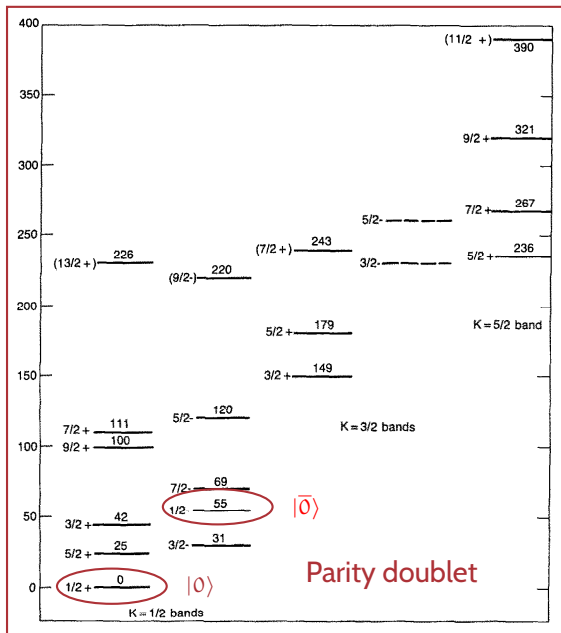
$$\langle S \rangle \approx \frac{\langle 0 | S_z | \bar{0} \rangle \langle \bar{0} | V_{PT} | 0 \rangle}{E_0 - E_{\bar{0}}} + \text{c.c.}$$

And in the rigid-deformation limit

$$\langle 0 | O | \bar{0} \rangle \propto \langle \bullet | O | \bullet \rangle = \langle O \rangle_{\text{intr.}}$$

again like angular momentum.

Spectrum of ^{225}Ra



²²⁵Ra Results

Hartree-Fock calculation with our favorite interaction SkO' gives

$$\langle S \rangle_{\text{Ra}} = -1.5 g\bar{g}_0 + 6.0 g\bar{g}_1 - 4.0 g\bar{g}_2 \text{ (e fm}^3\text{)}$$

Larger by over 100 than in ¹⁹⁹Hg!

Variation a factor of 2 or 3. But, as you'll see, we should be able to do better!

Current “Assessment” of Uncertainties

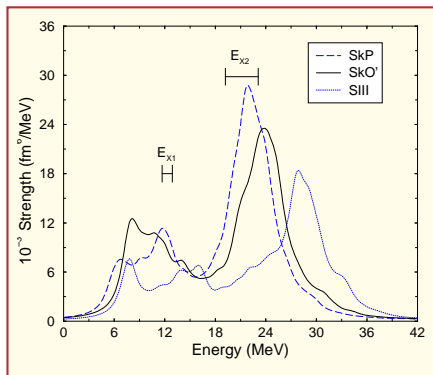
Judgment in recent review article (based on spread in reasonable calculations):

Nucl.	Best value			Range		
	α_0	α_1	α_2	α_0	α_1	α_2
^{199}Hg	0.01	± 0.02	0.02	0.005 – 0.05	-0.03 – +0.09	0.01 – 0.06
^{129}Xe	-0.008	-0.006	-0.009	-0.005 – -0.05	-0.003 – -0.05	-0.005 – -0.1
^{225}Ra	-1.5	6.0	-4.0	-1 – -6	4 – 24	-3 – -15

Uncertainties pretty large, particularly for α_1 in ^{199}Hg (range includes zero). How can we reduce them?

Reducing Uncertainty: Hg

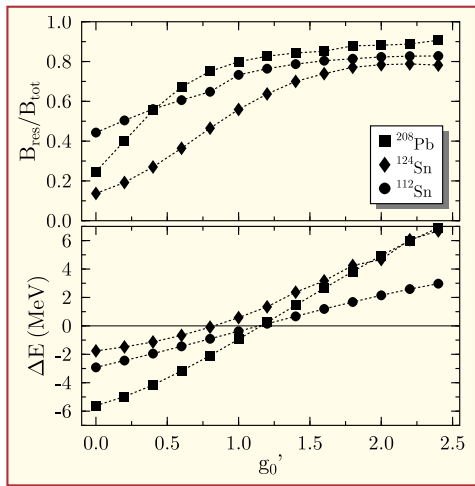
Improving many-body theory to handle soft deformation, though probably necessary, is tough. But can also try to optimize density functional.



Isoscalar dipole operator contains r^2z just like Schiff operator. Can see how well functionals reproduce measured distributions, e.g. in ^{208}Pb .

More on Reducing Uncertainty in Hg

V_{PT} probes spin density; functional should have good spin response. Can adjust relevant terms in, e.g. SkO', to Gamow-Teller resonance energies and strengths.



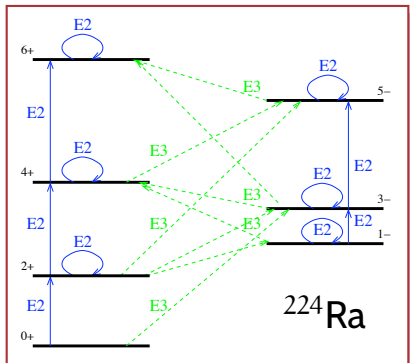
More generally, examine correlations between Schiff moment and lots of other observables.

Reducing Uncertainty: Ra

Important new developments here.



$\langle S \rangle_{\text{intr.}}$ correlated with octupole moment, which will be extracted from measured E3 transitions.

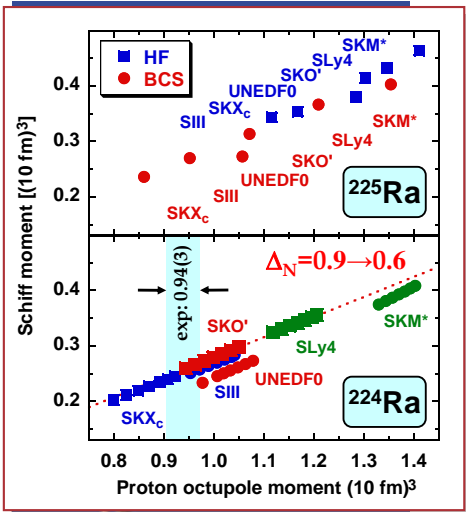


Gaffney et al., Nature

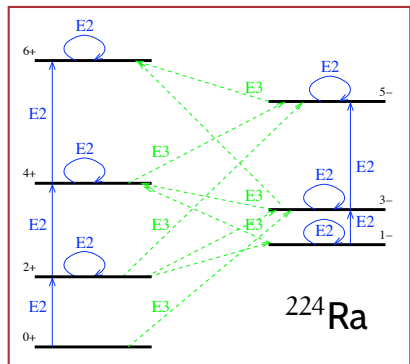
Transitions in ^{225}Ra to be measured soon?

Reducing Uncertainty: Ra

Important new developments here.



$\langle S \rangle_{\text{intr.}}$ correlated with octupole moment, which will be extracted from measured E3 transitions.

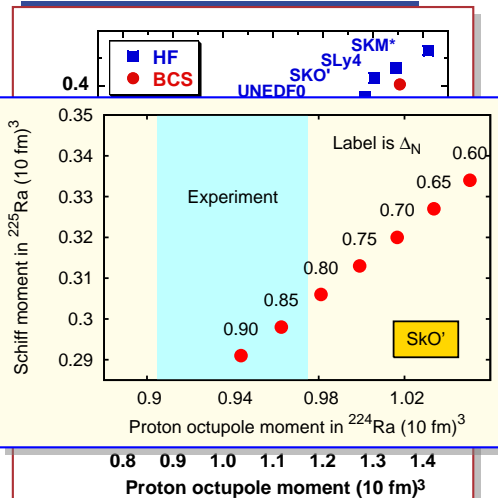


Gaffney et al., Nature

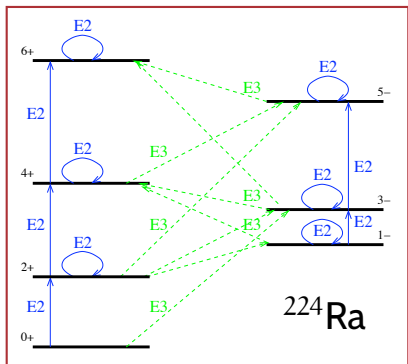
Transitions in ²²⁵Ra to be measured soon?

Reducing Uncertainty: Ra

Important new developments here.



$\langle S \rangle_{\text{intr.}}$ correlated with octupole moment, which will be extracted from measured E3 transitions.

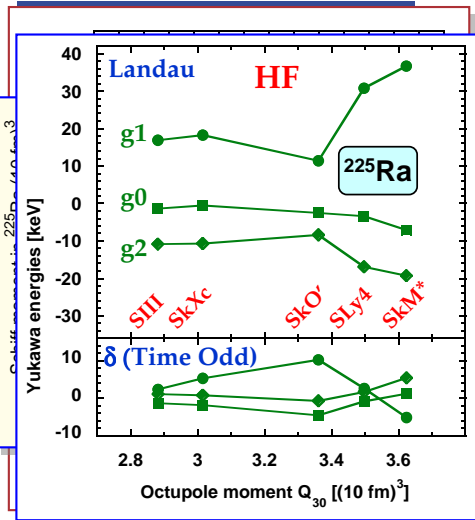


Gaffney et al., Nature

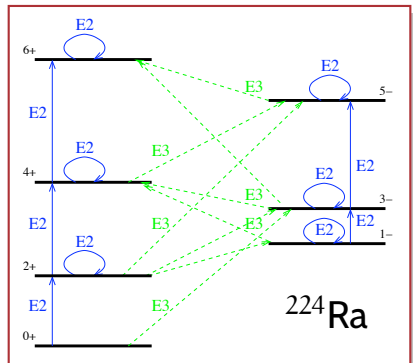
Transitions in ^{225}Ra to be measured soon?

Reducing Uncertainty: Ra

Important new developments here.



$\langle S \rangle_{\text{intr.}}$ correlated with octupole moment, which will be extracted from measured E3 transitions.

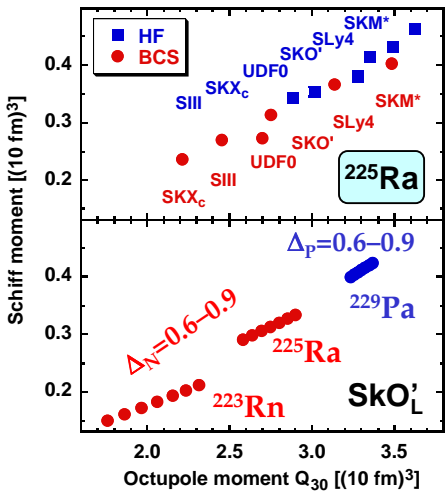


Gaffney et al., Nature

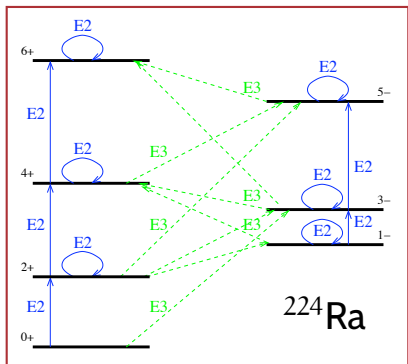
Transitions in ^{225}Ra to be measured soon?

Reducing Uncertainty: Ra

Important new developments here.



$\langle S \rangle_{\text{intr.}}$ correlated with octupole moment, which will be extracted from measured E3 transitions.



Gaffney et al., Nature

Transitions in ^{225}Ra to be measured soon?

More on Reducing Uncertainty in Ra

What about matrix element of V_{PT} ?

In one-body approximation

$$V_{PT} \approx \vec{\sigma} \cdot \vec{\nabla} \rho.$$

The closest simple one body operator is

$$O_{AC} = \vec{\sigma} \cdot \vec{r}.$$

Q: Can we measure $\langle \bar{0} | O_{AC} | 0 \rangle$ or something like it?

Doesn't occur in electron scattering, but does occur in weak neutral current. Neutrino scattering on Ra?

The Future

Calculations have become sophisticated, but we still have a lot of work to do.

In the near future, that work involve nuclear DFT.

- ▶ In Hg, a GCM calculation eventually needed.
And need correlation analysis, good proxies for Schiff distributions (e.g. isoscalar dipole distribution), V_{PT} distribution.
- ▶ In octupole-deformed nuclei, improved techniques probably won't change things drastically.
But again, need correlation analysis. Have good proxy for $\langle S \rangle_{int}$, need one for $\langle V_{PT} \rangle_{int}$.

The Future

Calculations have become sophisticated, but we still have a lot of work to do.

In the near future, that work involve nuclear DFT.

- ▶ In Hg, a GCM calculation eventually needed.
And need correlation analysis, good proxies for Schiff distributions (e.g. isoscalar dipole distribution), V_{PT} distribution.
- ▶ In octupole-deformed nuclei, improved techniques probably won't change things drastically.
But again, need correlation analysis. Have good proxy for $\langle S \rangle_{int}$, need one for $\langle V_{PT} \rangle_{int}$.

THE END.

Thanks for your kind attention.